Flat-stitching error analysis of large-aperture photon sieves

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Received 23 September 2013; revised 20 November 2013; accepted 26 November 2013; posted 27 November 2013 (Doc. ID 198121); published 23 December 2013

Large aperture can be easily achieved by employing photon sieves fabricated on the stitching membrane, which are suitable for packaging and folding. We have done research to find out the image quality degradation caused by stitching. A simulation has been done to investigate the behavior of the stitching wavefront. 10×10 pieces have been used as a model, and their wavefront error has been evaluated using the wavefront method discussed in this work. Besides, we find that the errors in the outer zones are more remarkable than in the inner ones of the same photon sieve. Testing and alignment directions based on this method are also mentioned in this work. © 2013 Optical Society of America

OCIS codes: (050.1970) Diffractive optics; (110.3000) Image quality assessment; (110.4100) Modulation transfer function; (220.1140) Alignment.

http://dx.doi.org/10.1364/AO.53.000090

1. Introduction

Membrane diffraction structures have been employed in space optical systems as entrance apertures since they can be manipulated in much more flexible ways when the aperture is large. Systems called transmission membrane telescopes can image by focusing transmission light. Membrane, because of its flexibility, can be easily folded, packaged, and unfolded in space. For this reason, researchers have engaged in launching it into space for imaging purpose [1–4]. In 2001, Kipp et al. provided a definition of a photon sieve in *Nature* for the first time [5]. As a novel optical element, a photon sieve employs plenty of random distribution holes instead of zones in zone plate. They can not only focus extreme short-wavelength light but also achieve better resolution and controlling of the side lobe. In space systems, one

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piece of membrane alone obviously cannot satisfy the demands of an aperture of dozens of meters, so stitching is necessary. Researchers have made more efforts in the focusing ability of photon sieves up to now, but few studies in image quality and stitching. Cao and Jahns from Hagen University built up the analytical model for the focusing of a pinhole photon sieve [6,7]. A broadband antihole photon sieve telescope has been signed in the U.S. Air Force Academy recently [8]. But as an entrance pupil, a photon sieve bends rays and focus, and the following elements be used for imaging. So, in the design stage, focusing ability should not be the only thing evaluated. We use wavefront error (WFE) to evaluate the image quality after stitching, and we offer a method, which can analyze and test discrete phase optics with the wavefront method directly. We design a scaled space using a photon sieve with diameter $D = 100 \text{ mm} \times 100 \text{ mm}$, focal length f = 14100 mm, primary wavelength $\lambda = 700$ nm, working through 532-808 nm. We use Matlab and Zemax to simulate

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the subpiece and analyze the WFE introduced by stitching. The tolerance of stitching has been achieved in this way. Also, this research can direct the system design and the choice of stitching method. Besides, according the wavefront map obtained from the test shop, we can also sense the subpiece which has a departure. By analyzing the wavefront map, orientation and degree of errors can be determined. This can be regarded as a common test method in similar situations of diffraction elements.

2. Stitching Error Analysis of Photon Sieves

A. Design of Photon Sieves

For unfolding purpose, we design an amplitude photon sieve $[\underline{9,10}]$. For a given wavelength λ and focal length f, the *n*th transparent zone radius r_n of a zone plate $[\underline{2}]$ is

$$r_n^2 = 2nf\lambda + n^2\lambda^2. \tag{1}$$

The corresponding width w_n is

$$w_n = \frac{\lambda f}{2r_n}.$$
 (2)

The configuration is shown here: the sum of zone numbers is 251, the diameter of the outermost hole is 0.1052 mm, the sum of holes is 529,774, the diameter of the Airy disk at the focal plane is 0.1708 mm, the half-width of peak is 0.09 mm, and the finest size it can recognize is 0.0854 mm. The throughput efficiency depends on the wavelength λ , and it is 6.75% when λ is 700 nm. The subpleces of the sieve are fixed on the aluminum sash. But in this stage, we assume that temperature has little effect on the stitching, and other influences are ignored in the computer simulation discussed in this paper. Figure <u>1</u> is the pattern of this element (the center part of it).

B. Imperfect Stitching Wavefront

Encountering the photon sieve, a plane wave will be modulated by the phase structure, and the first order of the diffraction light will converge as a sphere. But stitching errors tear the perfect wavefront, and WFEs will be brought in. What we have done is focus on these WFEs. The fine structures of the photon sieve used in this work are to enhance the quality of imaging, in fact. But an efficient method of modeling and simulation could not be achieved until now because of not only the huge quantity of structure data but also the difficulty of so much Fourier transform. So we simplify the model and manipulate it as a zone phase modulator to satisfy the demand of calculation and simulation. Besides, unfolding the photon sieve in space needs an auxiliary mechanism whose accuracy is not suitable to be considered in detail at this stage of design. So what we investigate is stitching in two dimensions. This means that every



Fig. 1. Pattern of a photon sieve (center part).

subpiece of the photon sieve has no warping, wrinkle, and torsion errors.

First, we derive the photon sieve phase equation of the simple 2×2 stitching type (Fig. 2).

The subpleces B, C, and D have stitching in a perfect condition for the four subpleces, and these three define a reference origin O. The subplece A, which has errors, determines a reference point O'. Any error in this plane can be described by three independent parameters: rotation angle θ of the subplece relative to point O' and transverse deviation *a* and longitudinal deviation *b* of O' relative to O. For point (x, y), the phase of the wavefront is $\Phi(x, y)$. The coordinate of *A* with errors becomes (X, Y) from the perfect (x0, y0). So

$$\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}.$$
(3)

In Zemax, a surface adds phase to the ray according to the following polynomial expansion [11]:



Fig. 2. Parameters defined in flat stitching.

$$\Phi = M \sum_{i=1}^{N} A_i \rho^{2i}, \qquad (4)$$

where N is the number of polynomial coefficients in the series; A_i is the coefficient on the 2*i*th power of ρ , which is the normalized radial aperture coordinate; and M is the diffraction order. A ray arriving at the focal point has a $n\lambda$ optical path difference (OPD) compared with the focal length:

$$\sqrt{\rho^2 + f^2} - f = n\lambda, \tag{5}$$

$$\rho^2 = n^2 \lambda^2 + 2n\lambda f, \qquad (6)$$

$$n^2 \lambda^2 \ll 2n\lambda f,\tag{7}$$

$$\rho^2 = 2n\lambda f, \tag{8}$$

$$\frac{\pi}{\lambda f}\rho^2 = 2n\pi = \Phi,\tag{9}$$

$$\Phi(X,Y) = \frac{\pi}{\lambda f} \{ [(X-a)\cos\theta + (Y-b)\sin\theta]^2 + [(Y-b)\cos\theta - (X-a)\sin\theta]^2 \}, \quad (10)$$

$$\Phi(X,Y) = \frac{\pi}{\lambda f} [(X-a)^2 + (Y-b)^2],$$
(11)

$$\frac{\partial \Phi(X,Y)}{\partial X} = \frac{k}{f}(X-a), \tag{12}$$

$$\frac{\partial \Phi(X,Y)}{\partial Y} = \frac{k}{f}(Y-b), \tag{13}$$

$$\frac{\partial^2 \Phi(X, Y)}{\partial X \partial Y} = 0. \tag{14}$$

In Eq. <u>11</u>, f is the focal length of the sieve and $k = 2\pi/\lambda$. The partial derivatives can be used in grid phase surface simulation in Zemax.

There is no parameter θ in the equation of $\Phi(X, Y)$. That means rotation errors (relative to O) have no effect on the wavefront phase. Since the deviation *a* and *b* has been determined, the wavefront phase could be described in these two parameters directly. This property that rotation causes no harm to the wavefront can be explained by the features of central symmetry.

C. Wavefront with Random Errors in Equal Orders of Magnitude

We use Eq. $\underline{11}$ equation derived in the last section to simulate the wavefront. By changing the order of the random stitching errors, we calculate the tolerance and draw a conclusion of which order of errors can be accepted in our design.

By using the grid phase surface in Zemax [11], we can describe the complex wavefront information of the photon sieve. What the grid phase needs for input is diffraction order, interpolation method, base surface type, and a ".DAT" file generated by computer which has the phase information of this surface. The .DAT file contains the values which define the (integer) number of points in the *x* and *y* directions, the (floating point) increment in the x and y directions, an (integer) flag indicating the units of the data, and the (floating point) decenter of the grid points relative to the base surface in x and y. Read the sections titled "Grid Sag" and "Grid Phase" in the chapter "Surface Types" of the Zemax manual for more details. Comparing with a binary surface, we choose the first order of diffraction, bicubic spline interpolation, and plane base surface. We can input the derivative we have calculated above, such as $\partial \Phi / \partial x$, $\partial \Phi / \partial y$, $\partial 2\Phi / \partial x \partial y$, or also can use the derivative provided by Zemax automatically. The Rayleigh criterion allows not more than one-quarter wavelength of OPD over the wavefront with respect to a reference sphere about a selected image point, in order that the image may be "sensibly" perfect. Corresponding to the $\lambda/4$ criterion (P-V OPD), the RMS OPD is 0.07λ , the Strehl ratio is 0.8, and 68% of energy converges in the Airy disk.

We preprocess our grid model in Matlab. Grid phase data has been calculated and rendered as a. DAT file. In consideration of the accuracy and efficiency of the simulation, first a 100 mm \times 100 mm model composed of 2001 \times 2001 discrete points is considered. The wavefront of this model is shown in Fig. 3, compared with an ideal wavefront of the same configuration.

From the figure, we can find that there are lots of differences in the details of these two wavefronts. But the overall shapes and curvatures are the same, besides the burrs caused by discrete sampling. The P-V OPD of the ideal wavefront is 0.0011λ , and the RMS OPD is 0.0003λ . Correspondingly, the P-V OPD of the grid phase wavefront is 0.0017λ , and the RMS OPD is 0.0003λ . These fit well, obviously, so the grid phase wavefront is an available and valuable method.



Fig. 3. Wavefront of model: (a) ideal and (b) discrete.

With its specialty well known, the grid phase surface is engaged in analyzing the wavefront with stitching errors. What we are concerned with is the magnitude order of errors, so in one situation the numerical values of the errors are random and the orders are the same. We investigate three kinds of situations: 1, 10, and 100 μ m orders of errors

in a 10×10 stitching formation. Through the modulation transfer function (MTF) curve and wavefront map, we can see the errors clearly. The simulation results are shown in Figs. 4-6.

The impact on the wavefront and transform function caused by stitching errors is shown in these figures. With the increasing of the errors' order, the image quality reflected by the MTF curve gets worse. We list the spot diameters on the focal plane and the WFEs of the three different orders in Table <u>1</u>.

The P-V WFE of the 10 μ m order is 0.2455 λ , and the RMS WFE is 0.0451 λ . This satisfies the image quality criterion, and we should choose a method of stitching according to this order of errors.

D. WFEs in the Specific Zones

The hole size of the photon sieve is finer in the outer zones than in the inner ones. In that case, the stitching impacts are different in the various zones. For a



Fig. 4. Simulation results of stitching errors in 1 µm order: (a) MTF curve and (b) wavefront map.



Fig. 5. Simulation results of stitching errors in 10 µm order: (a) MTF curve and (b) wavefront map.



Fig. 6. Simulation results of stitching errors in 100 µm order: (a) MTF curve and (b) wavefront map.

Table 1. Simulation Results of Different Error Orders

Orders of Stitching Errors		1 μm	10 µm	100 µm
Spot diameters (μm)	RMS Geometric	61.043 205.170	74.288 300.872	$1017.39 \\ 6174.13$
WFE (λ)	P-V RMS	$0.0306 \\ 0.0053$	$0.2455 \\ 0.0451$	$3.0078 \\ 0.4651$

 10×10 stitching formation with uniform accuracy, we introduce higher-order errors in one zone of the stitching formation and calculate what happens in the level of wavefront. In Table 2, gray pieces have stitching errors in the 1 µm order and dark pieces have errors in the 10 µm order.

Through the table above <u>1</u>, we can come to the conclusion that the outer zones' structures are more sensitive to the stitching errors than the inner ones. So, when we choose the supply structure, stitching method, and unfolding mechanics, outer-zone accuracy should be given priority.

3. Alignment Direction

Stitching errors can generate specific wavefronts. Conversely, stitching errors can be diagnosed from a specific wavefront.

We summarize every single situation of four-piece stitching which has specific errors in different orientations. The orientation convention follows the sign of the *a* and *b* mentioned in Fig. 2, and here we use "+" and "-" to indicate their signs. Table $\underline{3}$ shows the lookup table.

From the simulation results, we can find that in the wavefront map, the gray scale going down and color turning lighter means the subpiece's departure; the gray scale going up and color turning deeper means the subpiece's approach. The light color shown in the wavefront map means the error is caused by the departure of the subpieces located beside the stitching, and the alignment should be changed to make them closer. The deeper color also can reveal the error and direct the operations in the corresponding way.

In a more complicated situation (Fig. 7), we can diagnose that the pieces with notations 1-15 have significant stitching errors: pieces 2, 3, 5, and 14 shift to the lower right corner; pieces 6, 9, 10, and 12 shift to the upper right corner; piece 1, 4, and 15 shift to the upper left corner; and pieces 7, 8, 11, and 13 shift

Table 2.	Simulation	Results of	of Specific	Errors in	Different	Zones
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Errors	1 μm/10 μm	1 µm/10 µm	1 µm/10 µm	ГО 1 µm/10 µm	1 μm/10 μm
P-V WFE RMS WFE	0.4773 0.0500	0.2423 0.0333	$0.2267 \\ 0.0260$	$0.1868 \\ 0.0194$	0.0633 0.0066



Directions	(+,+)	(+,-)	(-,+)	(-,-)
Wave-front map				1
Directions	(+,+)	(+,-)	(-,+)	(-,-)
Wave-front map			1	
Directions	(+,+)	(+,-)	(-,+)	(-,-)
Wave-front map	2			
Directions	(+,+)	(+,-)	(-,+)	(-,-)
Wave-front map		1	r,	



Fig. 7. Wave front of complex stitching.

to the lower left corner. According to the conclusion we got in Table 2, the errors of pieces 2, 5, and 14 are more remarkable than those of pieces 3 and 12. In that case, we can diagnose the pieces whose errors are more obvious than the average. Repeating the test, stitch, and test process, we could adjust the whole photon sieve and achieve a smooth grayscale wavefront, which means an appropriate image quality.

4. Conclusion

We get the wavefront by simulation using the photon sieve model we designed for a large-aperture space application. According to the image quality demand and criterion, the order of the stitching errors should be controlled and must be less than 10 μ m. We also find that the outer zones of the photon sieves are more sensitive to errors than the inner ones. A lookup table has been established to determine the errors'directions of stitching, so testing and correction can be done visually. The wavefront method discussed in this paper can also be employed to achieve the image quality of the optical system with binary or diffraction elements and then to direct the further designing.

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