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Wavefront measurement by phase retrieval through single defocus intensity patterns

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Abstract
An algorithm for wavefront measurement using a phase retrieval method is proposed. Only one defocus intensity pattern of the imaging optical system is needed for phase retrieval. Based on the extended Nijboer–Zernike (ENZ) approach, this algorithm draws on general inverse matrix theory, and a predictor–corrector method is also used to correct the linearization errors for medium-to-large aberrations. This method does not need multi-defocus intensity in the focal region, and it can exclude vibration during multi-intensity pattern capturing, making the whole process simple and brief. Some simulated results are presented which show this algorithm has good convergence against noise. A confirmatory experiment is carried out, and its results verifying this method are also given.

(Some figures may appear in colour only in the online journal)

1. Introduction

Optical instruments are widely used in industry. Many techniques have been introduced to manufacture and measure optical systems. Wavefront aberration is very important because it provides a direct understanding of the defects in the imaging optical system. A knowledge of the wavefront at the pupil is required to manufacture a perfect optical system. An interferometer has high precision and is widely used in wavefront measurement. Usually, it needs a coherent illumination source, such as a laser. Its testing environment needs almost constant temperature and a stable working platform. Also, a reference wavefront is necessary. When the optical system is sent to space or assembled on equipment, the interferometric method will not work well. An online testing method is needed. The relation between pupil function and point spread function (PSF) could be utilized. An approach has been proposed in the literature [1, 2] and the wavefront can be reconstructed from one or more images, typically of a point source, according to the relation between the PSF and the complex pupil function. This approach does not need a coherent source. However, noise in the image-capturing process and inherent non-unique problems tend to make the inversion process unreliable. Some approaches to retrieve the phase use a least-squares method or other optimization methods to solve this problem [3–7]. Another important method for phase retrieval is being investigated. Janssen *et al* [8] have studied Nijboer–Zernike theory and proposed an extended Nijboer–Zernike (ENZ) approach, providing a good approximation for the intensity distribution in or close to the focal plane [8, 9]. ENZ does not suffer from small aberration and amplitude uniformity restrictions. Based on the ENZ, another phase retrieval approach is introduced [10, 11]. This retrieval approach is stable against noise in the imaging capture process and has good accuracy. Because multiple images at the symmetric focal region are needed, the whole process would be cluttered. If there are no precise instruments for on-axis movement, or vibration disturbs the capturing intensity seriously, this process can not be finished properly. If phase retrieval needs only one defocused intensity pattern, the process would be efficient, and the cost of the instruments needed would decrease. More people could then share in this achievement.

2. Algorithm for phase retrieval

The objective of this paper is phase retrieval through a single defocus intensity pattern based on scalar ENZ theory. The retrieval process is based on general inverse matrix theory. The effect of vibration can be excluded, while the algorithm is still stable against noise.

The relation between the PSF and the exit pupil function is determined. The linear Zernike polynomial expansion of the exit pupil function is given as:

$$P(u,v)=\sum\beta_n^m Z_n^m(u,v)=\sum\beta_n^m R_n^m(\rho)\cos(m\vartheta). \quad (1)$$

Here, $P(u,v)$ is the exit pupil function, with u, v Cartesian coordinates. For convenience, only cosine Zernike terms are considered here, with complex coefficients β_n^m ($n, m \geq 0$, $n-m$ even) which are suitable for the general pupil of a rotationally symmetric system. According to the relation in the literature [8, 9], the intensity pattern in the focal region is given by:

$$\begin{aligned} I(x,y,f) &= 4\beta_0^0|V_0^0|^2+8{\sum_{n,m}}'\beta_0^0\,\mathrm{Re}(\beta_n^m) \\ &\times \mathrm{Re}[\mathrm{i}^m V_n^m V_0^{0*}]\cos(m\varphi) \\ &- 8{\sum_{n,m}}'\beta_0^0\,\mathrm{Im}(\beta_n^m) \\ &\times \mathrm{Im}[\mathrm{i}^m V_n^m V_0^{0*}]\cos(m\varphi)+C(x,y,f). \end{aligned} \quad (2)$$

The summation with an apostrophe does not include the $m=n=0$ term. Re means the real part, and Im means the imaginary part

$$\beta_n^m = \mathrm{Re}(\beta_n^m)+\mathrm{i}\times\mathrm{Im}(\beta_n^m) \quad (3)$$

$C(x,y)$ is the cross term

$$\begin{aligned} C(x,y,f) &= 4{\sum_{n_1,m_1;n_2,m_2}}''\mathrm{Re}[\beta_{n_1}^{m_1}\beta_{n_2}^{m_2^*}\mathrm{i}^{m_1-m_2}V_{n_1}^{m_1} \\ &\times (x,y,f)V_{n_2}^{m_2^*}(x,y,f)] \\ &\times \cos(m_1\varphi)\cos(m_2\varphi). \end{aligned} \quad (4)$$

The summation with the double apostrophe does not include the $m_1=n_1=m_2=n_2=0$ term. x, y are normalized Cartesian coordinates, and f is a normalized parameter representing defocus

$$\begin{aligned} x &= X\frac{2\pi\mathrm{NA}}{\lambda}, \qquad y=Y\frac{2\pi\mathrm{NA}}{\lambda}, \\ f &= \frac{2\pi}{\lambda}Z\left(1-\sqrt{1-\mathrm{NA}^2}\right). \end{aligned} \quad (5)$$

X, Y and Z are Cartesian coordinates in the imaging region, NA is the numeric aperture, and λ is the wavelength of the illumination. V_n^m is expanded as

$$V_n^m(r,f)=\exp(\mathrm{i}f)\sum_{l=1}^{\infty}(-2\mathrm{i}f)^{l-1}\sum_{j=0}^{p}\upsilon_{lj}\frac{J_{m+l+2j}(\upsilon)}{l\upsilon^l}, \quad (6)$$

$$\upsilon_{lj}=(-1)^p(m+l+2j)\binom{m+j+l-1}{l-1}\binom{j+l-1}{l-1} \times \binom{l-1}{p-j}\Big/\binom{q+l+j}{l} \quad (7)$$

where $l=1,2,3\ldots$ and $j=0,1,2\ldots p$. $\upsilon=2\pi r$, $p=\frac{n-m}{2}$, $q=\frac{n+m}{2}$, and $l>3|f|$. The binomials occurring in equation (7) are given as:

$$\binom{n}{k}=\begin{cases}\dfrac{n!}{k!(n-k)!}, & n\geq k \\ 0, & n<k.\end{cases} \quad (8)$$

Here, the PSF is represented as a Bessel series. Neither discrete numerical integration nor the Fourier method are used to calculate the intensity. If the cross term is deleted, the intensity is given by

$$\begin{aligned} I\approx I_d &= 4\beta_0^0|V_0^0|^2+8{\sum_{n,m}}'\beta_0^0\,\mathrm{Re}(\beta_n^m)\,\mathrm{Re}[\mathrm{i}^m V_n^m V_0^{0*}] \\ &\times \cos(m\varphi)-8{\sum_{n,m}}'\beta_0^0\,\mathrm{Im}(\beta_n^m)\,\mathrm{Im}[\mathrm{i}^m V_n^m V_0^{0*}]\cos(m\varphi). \end{aligned} \quad (9)$$

The matrix function of equation (9) is as follows:

$$I_{mx}=V_{mx}\times A_{mx} \quad (10)$$

where I_{mx} is the column matrix made up of the acquired intensity I,

$$I_{mx}=[I(1,1)I(1,2)\cdots I(1,N)I(2,1)\cdots I(2,N)\cdots I(M,N)]^{\mathrm{T}}, \quad (11)$$

where M and N are the numbers of rows and columns of intensity I, respectively.

A_{mx} is the column matrix made up of the imaginary and real parts of the coefficients β_n^m,

$$A_{mx}=[\,\mathrm{Re}(\beta_1^1)\,\mathrm{Im}(\beta_1^1)\cdots\mathrm{Re}(\beta_n^m)\,\mathrm{Im}(\beta_n^m)]^{\mathrm{T}}. \quad (12)$$

V_{mx} is the transfer matrix made up of the base function:

$$V_{mx}=\begin{bmatrix} 4|V_0^0(1,1)|^2, & H_1^1(1,1), & G_1^1(1,1), & \cdots, & H_n^m(1,1), & G_n^m(1,1) \\ 4|V_0^0(1,2)|^2, & H_1^1(1,2), & G_1^1(1,2), & \cdots, & H_n^m(1,2), & G_n^m(1,2) \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 4|V_0^0(1,N)|^2, & H_1^1(1,N), & G_1^1(1,N), & \cdots, & H_n^m(1,N), & G_n^m(1,N) \\ 4|V_0^0(2,1)|^2, & H_1^1(2,1), & G_1^1(2,1), & \cdots, & H_n^m(2,1), & G_n^m(2,1) \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 4|V_0^0(2,N)|^2, & H_1^1(2,N), & G_1^1(2,N), & \cdots, & H_n^m(2,N), & G_n^m(2,N) \\ 4|V_0^0(M,N)|^2, & H_1^1(M,N), & G_1^1(M,N), & \ldots, & H_n^m(M,N), & G_n^m(M,N) \end{bmatrix} \quad (13)$$

while

$$H_n^m(x,y)=8\,\mathrm{Re}[\mathrm{i}^m V_n^m(x,y,f)V_0^{0*}(x,y,f)]\cos[m\varphi(x,y)], \quad (14)$$

$$G_n^m(x,y)=-8\,\mathrm{Im}[\mathrm{i}^m V_n^m(x,y,f)V_0^{0*}(x,y,f)]\cos[m\varphi(x,y)]. \quad (15)$$

Table 1. Retrieved coefficients when different DC-free noise is added.

	SNR = ∞	SNR = 55 dB	SNR = 25 dB
Z_0^0	1.000 00	1.000 00	1.000 46
Z_2^0	0.010 00 + 0.000 00i	0.010 01 + 0.000 00i	0.011 06 + 0.000 05i
Z_2^{-2}	−0.010 00 − 0.022 00i	−0.010 00 − 0.022 00i	−0.009 98 − 0.021 83i
Z_2^2	0.010 00 + 0.002 00i	0.009 99 + 0.002 00i	0.010 80 + 0.002 46i
Z_3^{-1}	0.000 00 + 0.012 00i	0.000 01 + 0.012 01i	0.000 28 + 0.011 78i

n, m are the indices of the Zernike coefficients; x and y are the Cartesian coordinates in the focal region; and f is the normalized defocus parameter, $f \neq 0$. Based on general inverse matrix theory, the solution of equation (10) that minimizes the errors in a least-squares sense is:

$$A_{mx} = (V_{mx}^{\mathrm{T}} V_{mx})^{-1} V_{mx}^{\mathrm{T}} \times I_{mx}. \quad (16)$$

The symbol $^{-1}$ means inverse translation, and $f \neq 0$ means the focus term cannot be used, otherwise this solution will be broken. Solving equation (16) to obtain A_{mx}, the result for the coefficients β_n^m is obtained. This is suitable for the case of small aberrations in optical systems. When medium-to-large aberrations appear, the effect of the cross term $C(x, y)$ will not be negligible.

To eliminate the advanced error introduced by the cross term $C(x, y)$, a predictor–corrector method is used here to increase the dynamic range of the test wavefront [11]. This is an iterative process. The coefficients β_n^m in equation (4) are substituted by the above result to calculate $C(x, y)$; the term $C(x, y)$ is deleted from I; the matrix I_{mx} is replaced by this new I for the next calculation of β_n^m. A similar iterative implementation is carried out, following this process, until the final coefficients β_n^m are obtained. The phase and amplitude in the exit pupil are then derived.

3. Simulation

A simulation is now carried out. The pupil function P is given as $P = Z_0^0 + 0.01Z_2^0 + (-0.01 - 0.022\mathrm{i})Z_2^{-2} + (0.002\mathrm{i} + 0.01)Z_2^2 + 0.012\mathrm{i}Z_3^{-1}$. The numeric aperture (NA) for the optical system is set at 0.16, the sampling radius is 12.3 μm in the focal region, and there are 800 × 800 points acquired in the imaging region. Based on this phase retrieval algorithm, figure 1 shows the absolute errors $|\beta_n^m - \beta_n^m(k)|$ as a function of the number of iterations, with β_n^m being the Zernike coefficients in our simulation. There is rapid convergence.

In table 1, the coefficients are displayed for differing noise with signal to noise ratios (SNRs) = ∞, 55, 25 dB. The effect of the noise will be stronger when more noise is added in the phase retrieval process. These results prove that this approach can restrain the effect of noise.

4. Confirmatory experiment

During our confirmatory experiment, an optical imaging system ($F^{\#} = 7$) is tested at a wavelength of 632.8 nm. The intensity pattern is obtained with a 20× objective lens at a defocal distance of −100 μm. The above approach is applied.

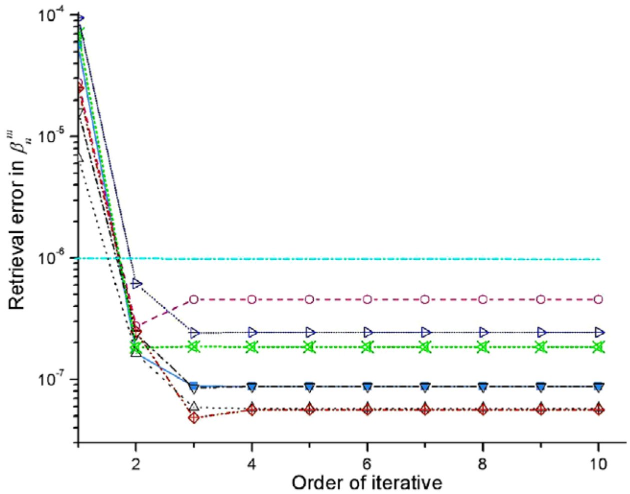

Figure 1. Absolute reconstruction errors $|\beta_n^m - \beta_n^m(k)|$ in each iteration using a predictor–corrector method. β_n^m are the simulated Zernike coefficients representing the exit pupil. k is the iterative order. The largest error is below 10^{-6}.

The retrieval coefficients β_n^m are obtained after 20 iterative processes, and the pupil function is presented. The PSFs for different defocus distances can be reconstructed. Figure 2 shows the reconstructed PSFs contrasting with the captured images. There are still errors in the phase retrieval result, but these errors are small for optical system characterization. In figure 2, the vibration of the platform disturbs the actual intensity patterns coinciding with the simulated PSFs. However, the profiles of the PSFs (dotted lines) and the images (solid lines) are very similar. Noise does not significantly affect the retrieval. These graphic results can prove that the retrieval process is valid and this approach can eliminate the error introduced by vibration.

The wavefront in the exit pupil is reconstructed according to the coefficients β_n^m. Through wavefront-fitting software, the wavefront aberration is analysed in figure 3. The piston, tilt, and defocus are removed. The tested aberrations are 0.0225λ rms (λ = 632.8 nm). Wavefront aberrations are mainly coma −0.063 685λ at 159°, astigmatism 0.069 979λ at 148° and sphere at 0.018 9345λ. These aberrations also could be found in the image patterns in different defocal regions. This lens was also tested with a ZYGO interferometer. As shown in figure 4, the wavefront has 0.015λ rms, and is similar to that in figure 3. This could prove that this approach is valid. The

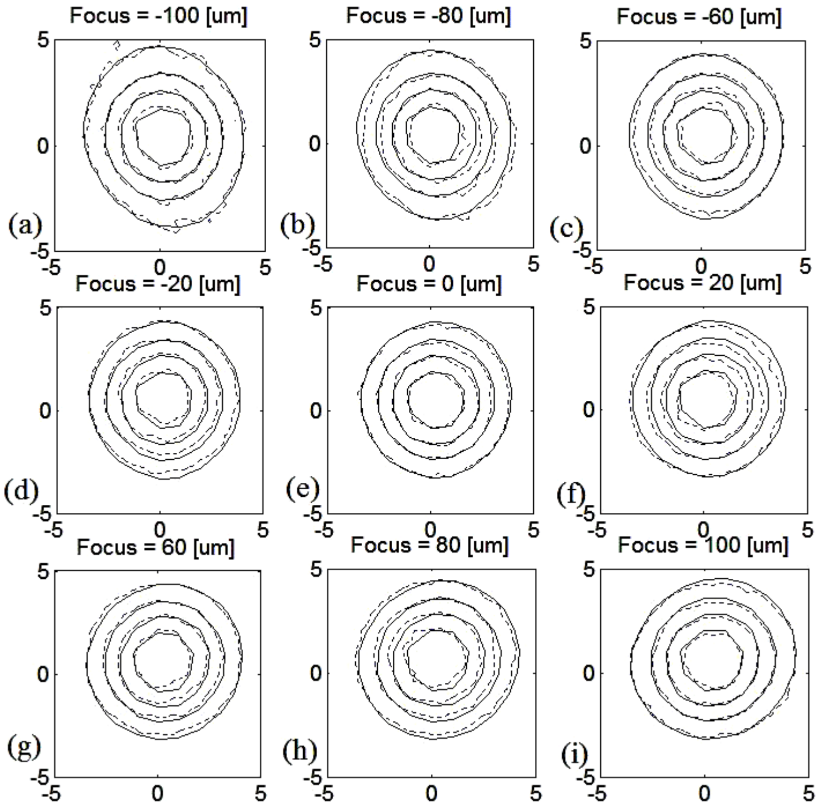

Figure 2. Profiles of PSFs and images at different defocus distances. The solid lines are the images obtained in the charge-coupled device with differing amounts of defocus: from (a) to (i) −100, −80, −60, −20, 0, 20, 60, 80, 100 μm, respectively. The PSFs (dotted lines) are reconstructed from the retrieval coefficients. Vibration disturbs the profile of the image coinciding with the PSF.

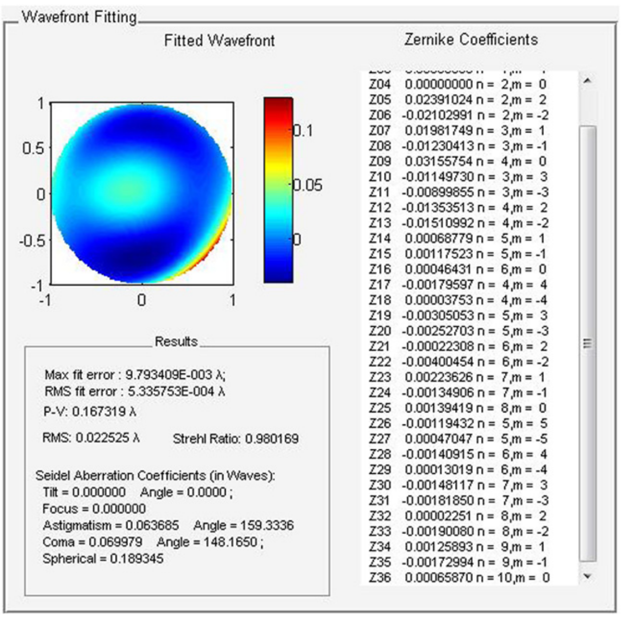

Figure 3. Wavefront aberration analysed through wavefront-fitting software. In the above left, the wavefront is shown when the piston, tilt and defocus are removed. The Seidel and Zernike coefficients are also listed.

difference between this result and the interferometer result is less than 0.01λ. The errors could come from aberrations of the objective lens.

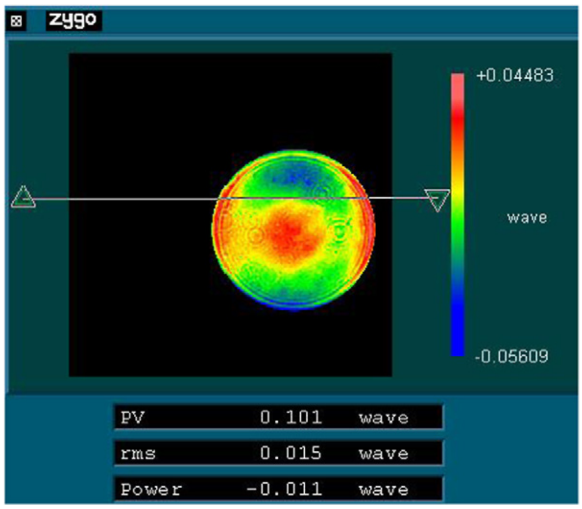

Figure 4. ZYGO testing result for the camera lens is 0.015λ rms, when the piston, tilt and defocus aberrations are removed.

5. Conclusions

In conclusion, we have proposed an algorithm for phase retrieval through a single defocused intensity pattern using general inverse matrix theory. It is based on the scalar extended Nijboer–Zernike theory. The algorithm aims at excluding errors brought by multi-intensity pattern capturing.

In our confirmatory tests, the wavefront aberration obtained is 0.0225λ rms. The errors in our tests are estimated to be less than 0.01λ. This algorithm is valid and has good convergence against noise and our approach for phase retrieval has good accuracy.

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