An 11-frame phase shifting algorithm in lateral shearing interferometry

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Abstract: In order to eliminate zeroth order effect and to make the phase shifting algorithm insensitive to phase shifting error, an 11-frame phase shifting algorithm is proposed in this paper. The analytical expression of phase-restoration error function is derived. The principle of phase shifting error compensation and the capability of suppressing zeroth order effect are explained, in comparison of existing algorithm. The analytical results show that this algorithm's phase-restoration error is proportional to sine of double shearing phase and to biquadratic of phase shifting error. Finally, we generate the interference patterns of 11-frame algorithm and existing algorithm, restore the shearing phases and calculate the phase-restoration errors by simulations. The simulation results verify the theoretical analyses.

OCIS codes: (120.3180) Interferometry; (120.5050) Phase measurement; (050.5080) Phase shift.

References and links

1. ITRS Organization, "International technology roadmap for semiconductors" (2013), *http://www.itrs.net/*.
2. M. Takeda and S. Kobayashi, "Lateral aberration measurements with a digital Talbot interferometer," Appl. Opt. **23**(11), 1760–1764 (1984).
3. M. Hasegawa, C. Ouchi, T. Hasegawa, S. Kato, A. Ohkubo, A. Suzuki, K. Sugisaki, M. Okada, K. Otaki, K. Murakami, J. Saito, M. Niibe, and M. Takeda, "Recent progress of EUV wave-front metrology in EUVA," Proc. SPIE **5533**, 27–36 (2004).
4. H. Schreiber and J. Schwider, "Lateral shearing interferometer based on two Ronchi phase gratings in series," Appl. Opt. **36**(22), 5321–5324 (1997).
5. J. C. Wyant, "Double frequency grating lateral shear interferometer," Appl. Opt. **12**(9), 2057–2060 (1973).
6. S. Okuda, T. Nomura, K. Kamiya, H. Miyashiro, K. Yoshikawa, and H. Tashiro, "High-precision analysis of a lateral shearing interferogram by use of the integration method and polynomials," Appl. Opt. **39**(28), 5179–5186 (2000).
7. G. W. Leibbrandt, G. Harbers, and P. J. Kunst, "Wave-front analysis with high accuracy by use of a double-grating lateral shearing interferometer," Appl. Opt. **35**(31), 6151–6161 (1996).
8. J. H. Bruning, D. R. Herriott, J. E. Gallagher, D. P. Rosenfeld, A. D. White, and D. J. Brangaccio, "Digital wavefront measuring interferometer for testing optical surfaces and lenses," Appl. Opt. **13**(11), 2693–2703 (1974).
9. P. Hariharan, B. F. Oreb, and T. Eiju, "Digital phase-shifting interferometry: a simple error-compensating phase calculation algorithm," Appl. Opt. **26**(13), 2504–2506 (1987).
10. J. Schmit and K. Creath, "Extended averaging technique for derivation of error-compensating algorithms in phase-shifting interferometry," Appl. Opt. **34**(19), 3610–3619 (1995).
11. P. Groot, "Derivation of algorithms for phase-shifting interferometry using the concept of a data-sampling window," Appl. Opt. **34**(22), 4723–4730 (1995).
12. T. Hasegawa, C. Ouchi, M. Hasegawa, S. Kato, A. Suzuki, K. Sugisaki, K. Murakami, J. Saito, and M. Niibe, "EUV wavefront metrology system in EUVA," Proc. SPIE **5374**, 797–807 (2004).
13. Y. Zhu, K. Sugisaki, M. Okada, K. Otaki, Z. Liu, J. Kawakami, M. Ishii, J. Saito, K. Murakami, M. Hasegawa, C. Ouchi, S. Kato, T. Hasegawa, A. Suzuki, H. Yokota, and M. Niibe, "Wavefront measurement interferometry at the operational wavelength of extreme-ultraviolet lithography," Appl. Opt. **46**(27), 6783–6792 (2007).
14. Y. Zhu, K. Sugisakia, C. Ouchia, K. Ohtaki, Z. Liu, M. Ishii, J. Kawakami, J. Saito, K. Murakami, C. Ouchi, M. Hasegawa, S. Kato, T. Hasegawa, A. Suzuki, and M. Niibe, "Lateral shearing interferometer for EUVL: theoretical analysis and experiment," Proc. SPIE **5374**, 824–832 (2004).

1. Introduction

With the development of projection lithography technology [1], more attentions to lateral shearing interferometry (LSI) are paid due to its advantages of no need of standard reference wavefront and insensitivity to environmental factor [2–4]. It has been used in surface measurement of optical element and in measurement of optical system's wavefront [5–7]. Phase Shifting Interferometry (PSI) is widely used to calculate the phase of shearing wavefront in LSI, and the Bruning 4-frame algorithm, Schwider–Hariharan 5-frame algorithm, Schmit 6-frame algorithm and de Groot 7-frame algorithm are the common algorithms [8–11], which are not insensitive to phase shifting errors. For the double grating lateral shearing interferometer [12,13], with the order-selection mask for spatial filtering the zeroth order diffracted beam, the ± 1st order diffracted beams interfere with each other. By suppressing the undesired 0th order beam with the spatial filter, there is still a certain percentage of the beam leaking through the 1st order window. The common algorithms can compensate the phase shifting errors well, but they cannot compensate the errors caused by zeroth order effect. Therefore it is significant to design a special phase shifting algorithm to suppress the zeroth order effect and also to compensate the phase shifting error. In 2004, Zhu et al proposed a 9-frame phase shifting algorithm to suppress the 0th order effect [14]. However, as the phase shifting error increases, the accuracy of the algorithm drops considerably. In practice, the grating with small grating constant should be chosen because to set up the micro displacement stage and detector needs enough space. In this case, there is a large amount of phase shifting error caused by the limits of manufactures and installations of the displacement stage and grating. So it is necessary to design an algorithm of phase calculation, which is much more insensitive to phase shifting error and also can suppress zeroth order effect.

An 11-frame phase shifting algorithm to eliminate zeroth order effect and compensate the phase shifting error is proposed in this paper. The analytical expression of phase-restoration error function is derived. The principle of the phase shifting error compensation and the capability of suppressing zeroth order effect are explained. Finally the simulations are given.

2. Lateral shearing interferometer and phase shifting algorithm

The typical double grating lateral shearing interferometer is shown in Fig. 1. The beam diffracted by grating goes through the projection lens. The ± 1st order diffracted beams form the interference pattern, and the 0th and higher order diffracted beams are eliminated by the spatial filter. Although the spatial filter is used, there is still a certain percentage of the 0th order beam reaching CCD detector, which generates interference pattern with ± 1st order diffracted beams.

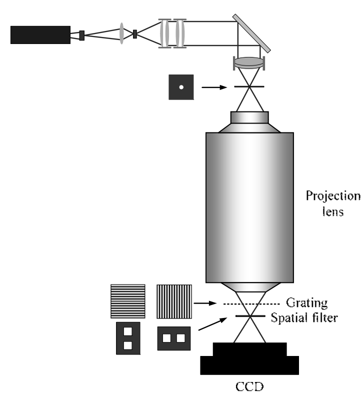

Fig. 1. Block diagram of double grating lateral shearing interferometer.

The phase shifting amount of ϕ can be written as

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$$\phi = \frac{2\pi m x}{p}, \tag{1}$$

where m is diffracted order, x is grating displacement and p is grating constant. By controlling the grating displacement of x, the phase shifting amount between ± 1st order beams is as

$$\phi_j = j\frac{4\pi x}{p} = j\frac{\pi}{2}. \tag{2}$$

The ± 1st order beams are signals and the 0th order beam is noise. The complex amplitudes of them can be expressed as

$$E_{-1} = a_{-1}\exp(kW_{-1}), \tag{3}$$

$$E_1 = a_1\exp(kW_1), \tag{4}$$

$$E_0 = a_0\exp(kW_0), \tag{5}$$

where E_{-1},E_1 and E_0 are the complex amplitudes of −1st,1st and 0th order beams respectively, a_{-1}, a_1, a_0 are the amplitudes, W_{-1}, W_1, W_0 are the wavefronts, and k is the wave number. The fringe intensity distribution of I_j for j-th frame is

$$\begin{aligned} I_j &= a_0^2 + a_1^2 + a_{-1}^2 + 2a_0a_1\cos[k(W_1 - W_0) + \phi'_j] \\ &+2a_0a_{-1}\cos[k(W_0 - W_{-1}) + \phi''_j] + 2a_1a_{-1}\cos[k(W_1 - W_{-1}) + \phi_j], \end{aligned} \tag{6}$$

where $\phi'_j = \phi_j / 2$ is the phase shifting amount between 1st order and 0th order, $\phi''_j = \phi_j / 2$ is that between 0th order and −1st order, $k(W_1 - W_{-1})$ is unknown phase needing to be restored, $k(W_1 - W_0)$ and $k(W_0 - W_{-1})$ are undesired phases distorted by 0th order beam. We suppose: $Q = a_0^2 + a_1^2 + a_{-1}^2$, $V_1 = 2a_1a_{-1}$, $V_2 = 2a_0a_1$, $V_3 = 2a_0a_{-1}$,

Equation (6) can be written as following:

$$I_j = Q + V_1\cos(\theta_1 + \phi_j) + V_2\cos(\theta_2 + \phi_j / 2) + V_3\cos(\theta_3 + \phi_j / 2). \tag{7}$$

Using 11 step phase shifting ($j = -5, -4, \cdots 0, \cdots 4, 5$), we have

$$(I_5 - I_{-5}) - 7(I_3 - I_{-3}) + 8(I_1 - I_{-1}) = -32V_1\sin\theta_1, \tag{8}$$

$$-4(I_4 + I_{-4}) + 8(I_2 + I_{-2}) - 8I_0 = -32V_1\cos\theta_1. \tag{9}$$

The phase to be restored is calculated as following:

$$\theta_1 = \tan^{-1}\left[\frac{(I_5 - I_{-5}) - 7(I_3 - I_{-3}) + 8(I_1 - I_{-1})}{-4(I_4 + I_{-4}) + 8(I_2 + I_{-2}) - 8I_0}\right]. \tag{10}$$

From Eq. (10), we conclude that the phase of θ_1 to be restored is not affected by 0th order diffraction under the condition of error free of phase shifting, because θ_1 is unrelated with θ_2 and θ_3.

3. Error analyses and comparisons

Due to the manufacture error and alignment error of micro displacement stage and grating, a mount of phase shifting error always exists in the phase shifting interferometry. So the phase shifting amount is different from Eq. (2), which is as following:

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$$\phi_j = j \cdot \frac{\pi}{2} + j \cdot \varepsilon, \tag{11}$$

where ε is the phase shifting error. It is convenient to divide the numerator part of Eq. (10) into 3 parts, named as A_1, A_2 and A_3. Part A_1 contains terms with θ_1 and constant, and can be expressed as:

$$A_1 = [16\cos(\varepsilon) - 16\cos(\varepsilon)^3 - 32\cos(\varepsilon)^5]\sin\theta_1. \tag{12}$$

We expand Eq. (12) as Taylor expansions for the phase shifting error of ε, and omit the small quantity of high order

$$A_1 = (-32 + 96\varepsilon^2 - 100\varepsilon^4)\sin\theta_1. \tag{13}$$

Part A_2 contains term with θ_2, denoted as:

$$A_2 = -\sqrt{2}\left[8\cos(\varepsilon) + 8\sin(\varepsilon) - 7\cos\left(\frac{3}{2}\varepsilon\right) + 7\sin\left(\frac{3}{2}\varepsilon\right) - \cos\left(\frac{5}{2}\varepsilon\right) - \sin\left(\frac{5}{2}\varepsilon\right)\right]\frac{V_2}{V_1}\cos\theta_2. \tag{14}$$

Equation (14) can be expressed as Taylor expansion form as:

$$A_2 = \left[-12\sqrt{2}\varepsilon + O\left(\varepsilon^2\right)\right]\frac{V_2}{V_1}\cos\theta_2. \tag{15}$$

For the grating in the interferometer, the designed ± 1st order diffraction efficiencies are 40%. Actually they are 35%, so the 0th order diffraction efficiency is less than 30%. Calculation shows that the amplitude of 0th order beam going through spatial filter is less than 2% of those of ± 1st order beams. So V_2 and V_3 are much smaller than V_1. And phase shifting error of ε is a small value. A_2 is approximately zero, because A_2 is products of two small quantities (polynomial of ε and V_2/V_1). Similarly, Part A_3 contains terms with θ_3, and it is also approximately zero.

It is convenient to divide the denominator of Eq. (10) into 3 parts, named as B_1, B_2 and B_3. Part B_1 contains terms with θ_1 and constant, and can be expressed as

$$B_1 = \left[32\cos(\varepsilon)^2 - 64\cos(\varepsilon)^4\right]\cos\theta_1. \tag{16}$$

We expand Eq. (16) as Taylor expansions for the phase-shift error of ε, and omit the small quantity of high order. Then we have:

$$B_1 = (-32 + 96\varepsilon^2 - 96\varepsilon^4)\cos\theta_1. \tag{17}$$

Part B_2 contains term with θ_2, denoted as:

$$B_2 = \left[-16\sin(\varepsilon) - 16 + 16\cos(\varepsilon)^2\right]\frac{V_2}{V_1}\cos\theta_2. \tag{18}$$

Equation (18) can be expressed as Taylor expansion form as:

$$B_2 = \left[-16\varepsilon + O\left(\varepsilon^2\right)\right]\frac{V_2}{V_1}\cos\theta_2. \tag{19}$$

By the method similar to analysis of A_2, B_2 is approximately zero. Part B_3 containing terms of θ_3 is also approximately zero.

From Eqs. (15) and (19), it is known that the part containing θ_2 and θ_3 in numerator and denominator is zero. So we can ignore the shearing phase generated by zeroth order beam

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even in the presence of phase shifting error. The algorithm can eliminate the undesired zeroth order effect.

Substituting Eqs. (13), (15), (17) and (19) into Eq. (10), the phase of β_1 with phase shifting error of ε is restored as:

$$\tan\beta_1 = \left(1 + \frac{\varepsilon^4}{8 - 24\varepsilon^2 + 24\varepsilon^4}\right)\tan\theta_1 \approx \left(1 + \frac{\varepsilon^4}{8}\right)\tan\theta_1. \tag{20}$$

Considering the phase-restoration error of $\Delta\theta_1$ is small amount, the phase-restoration error function with 11-frame algorithm can be expressed as

$$\Delta\theta_1 = \beta_1 - \theta_1 \approx \tan\left(\beta_1 - \theta_1\right) = \frac{\varepsilon^4}{16}\sin\left(2\theta_1\right). \tag{21}$$

With 9-frame phase shifting algorithm, the phase can be restored as following expression [14]:

$$\theta'_1 = \tan^{-1}\left[\frac{2\left(I_3 - I_{-3}\right) - 2\left(I_1 - I_{-1}\right)}{\left(I_4 + I_{-4}\right) - 2\left(I_2 + I_{-2}\right) + 2I_0}\right]. \tag{22}$$

With above method Eq. (22) is analyzed, and the phase of β'_1 with phase shifting error of ε, is restored:

$$\tan\beta'_1 = \left(1 + \frac{\varepsilon^2}{2 - 6\varepsilon^2}\right)\tan\theta'_1 \approx \left(1 + \frac{\varepsilon^2}{2}\right)\tan\theta'_1. \tag{23}$$

The phase-restoration error function of $\Delta\theta'_1$ with 9-frame algorithm is expressed as

$$\Delta\theta'_1 = \beta'_1 - \theta'_1 \approx \tan\left(\beta'_1 - \theta'_1\right) = \frac{\varepsilon^2}{4}\sin\left(2\theta'_1\right). \tag{24}$$

From Eqs. (21) and (24), in the case of constant phase shifting error such as $\varepsilon = \pi/6$, the relationships between phase-restoration error and shearing phase for 11-frame and 9-frame algorithm are shown in Fig. 2. The two phase-restoration errors vary as sine of double shifting phases, and the worst condition is at $\pi/4$. With the same shearing phase, the restoration error of 11-frame algorithm is smaller than that of 9-frame algorithm. So the 11-frame algorithm is less sensitive to phase shifting error and has more extensive adaptability.

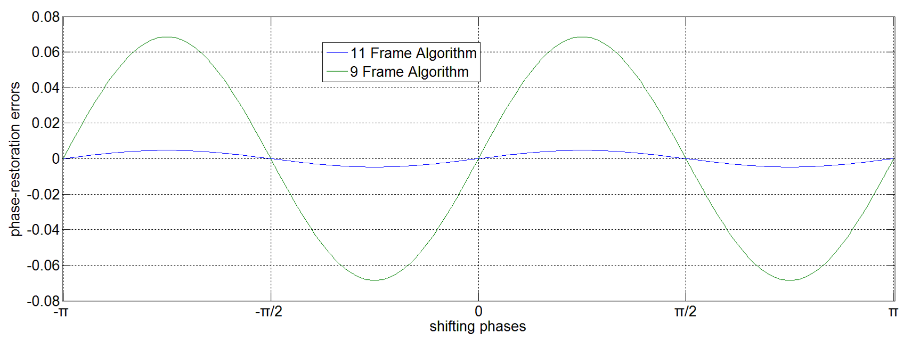

Fig. 2. The relationship between phase-restoration error and shearing phase.

Figure 3 shows the phase-restoration error by 11-frame and 9-frame algorithm in the case of same shifting phase, such as shearing phase of $\pi/4$, according to Eqs. (21) and (24). The restoration error of 11-frame algorithm is proportional to biquadratic of phase shifting error, and that of 9-frame algorithm is proportional to square of phase shifting error. So with the

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same phase shifting error, restoration error of 11-frame algorithm is less than that of 9-frame algorithm. Especially, with large phase shifting error, the accuracy of 11-frame algorithm is much higher than that of 9-frame algorithm.

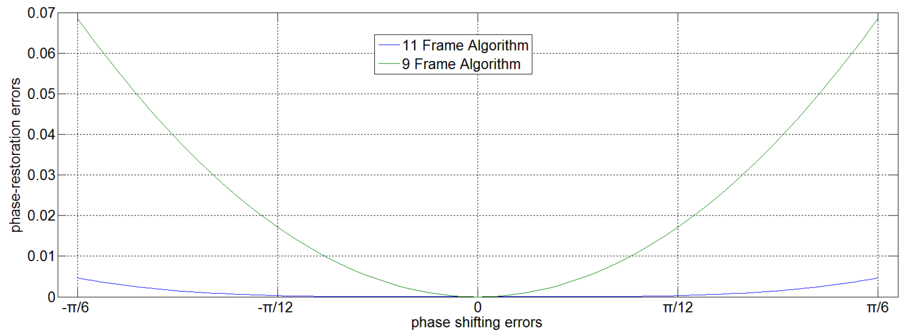

Fig. 3. The relationship between phase-restoration error and phase shearing error.

4. Simulations

In this section, we simulate the procedures of lateral shearing interference and shearing phase restoration with zeroth order effect to verify the analyses in Section 3. In the simulations a wavefront of spherical aberration is chosen, which is a common aberration in the measurement of wavefront. The wavefront of 0.5λ spherical aberration located in unit circle is shown in Fig. 4. The wavefront is sheared in x direction, shearing ratio is assumed as 0.1, and the noise is $V_2 = V_3 = 2\%V_1$. When the phase shifting amount is zero, the shearing phase and its interference pattern are shown in Fig. 5.

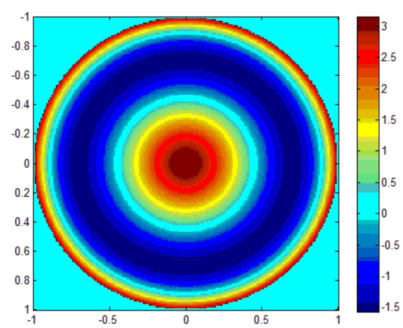

Fig. 4. The wavefront of 0.5λ spherical aberration.

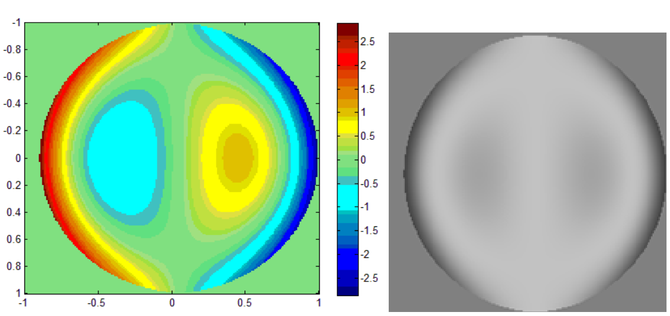

Fig. 5. The shearing phase (a) and interference pattern (b) when shearing ratio is 0.1, $V_2 = V_3 = 0.02V_1$ and phase shearing amount is 0.

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11 interference patterns are obtained with 11 step phase shifting. The phase shifting amount is $\pi/2$ for one step, and the phase shifting error is $\pi/6$. The shearing phase is restored with 11-frame phase shifting algorithm. The restoration phase and its error are shown in Fig. 6.

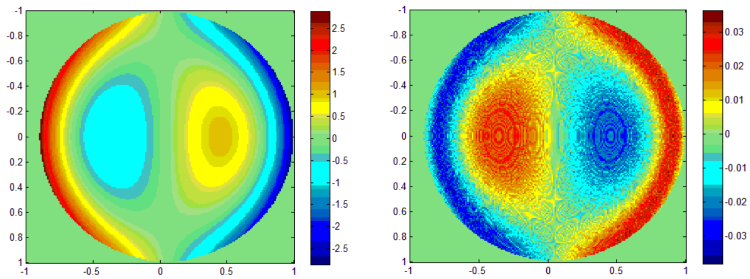

Fig. 6. The shearing phase (a) and phase-restoration error (b) restored with 11-frame algorithm.

4 interference patterns are obtained with 4 step phase shifting. The phase shifting amount is $\pi/2$ for one step, and the phase shifting error is $\pi/6$. The shearing phase is restored with Bruning 4-frame phase shifting algorithm. The restoration phase and its error are shown in Fig. 7.

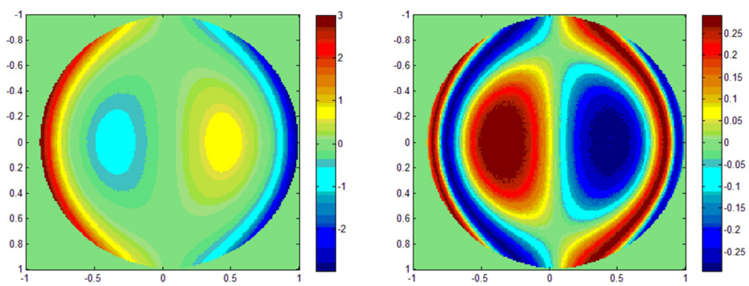

Fig. 7. The shearing phase (a) and phase-restoration error (b) restored with 4-frame algorithm.

From Figs. 6 and 7, for the same wavefront with the same phase shifting error, the maximum absolute value of 11-frame algorithm's restoration error is smaller than 0.04 rad, and that of 4-frame algorithm is smaller than 0.30 rad. The restoration accuracy of 11-frame algorithm is higher than that of 4-frame algorithm.

9 interference patterns are generated with 9 step phase shifting. The phase shifting amount is $\pi/2$ for one step, and the phase shifting error is $\pi/6$. The shearing phase is restored with 9-frame phase shifting algorithm. The restoration phase and its error are shown in Fig. 8.

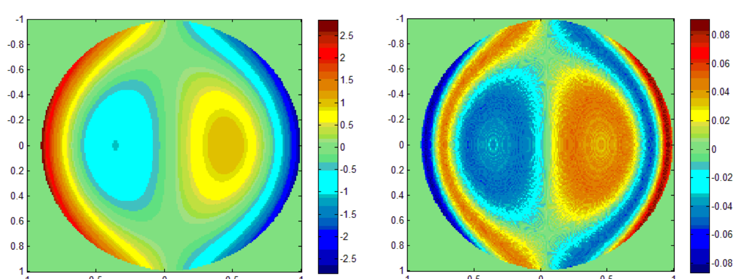

Fig. 8. The shearing phase (a) and phase-restoration error (b) restored with 9-frame algorithm.

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From Figs. 6 and 8, we know that the shearing phase can be restored with both 11-frame and 9-frame algorithm when zeroth order effect exists. For the same wavefront with the same phase shifting error, the maximum absolute value of 11-frame algorithm's restoration error is smaller than 0.04 rad, and that of 9-frame algorithm is smaller than 0.09 rad. Therefore, the restoration accuracy of 11-frame algorithm is higher than that of 9-frame algorithm. This proves the analyses in Section 3. With different phase shifting errors the different interference patterns are generated respectively with 11step and 9 step phase shifting, and the restoration errors are obtained. The restoration rms errors of those algorithms are shown in Fig. 9. The phase-restoration errors increase as the absolute values of phase shifting errors. With the same phase shifting error, restoration accuracy of 11-frame algorithm is higher than that of 9-frame algorithm. Especially, under the condition of large phase shifting error, the accuracy difference is much more remarkable. The restoration rms error with phase shifting error of -$\pi/6$ for 11-frame algorithm is 0.011 rad, and it is 0.056 rad for 9-frame algorithm. 11-frame algorithm's error is 1/5 of 9-frame algorithm's error. The simulations are in good agreement with the tendency of Fig. 3, the analyses results in Section 3.

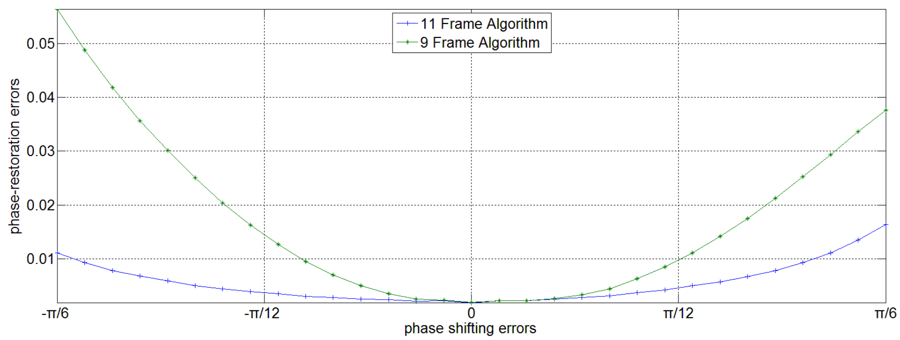

Fig. 9. The relationship between phase-restoration error and phase shearing error given by simulation with 11-frame and 9-frame algorithms under the condition of $V_2 = V_3 = 2\%V_1$.

With the simulation conditions of $V_2 = V_3 = 0.5\%V_1$ and $V_2 = V_3 = 1\%V_1$, the restoration rms errors of 9-frame algorithm and 11-frame algorithm are shown in Figs. 9 and 10 respectively. From Figs. 9–11, for different degrees of zeroth order effect with large amount of phase shifting error, phase-restoration error of 11-frame algorithm is much smaller than that of 9-frame algorithm.

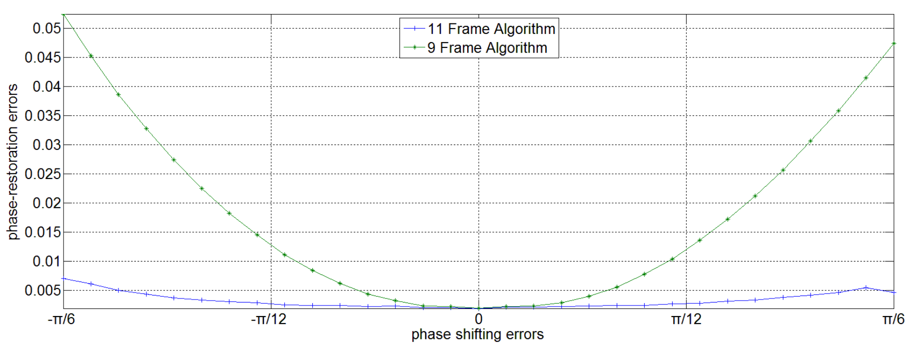

Fig. 10. The relationship between phase-restoration error and phase shearing error given by simulation with 11-frame and 9-frame algorithms under the condition of $V_2 = V_3 = 0.5\%V_1$.

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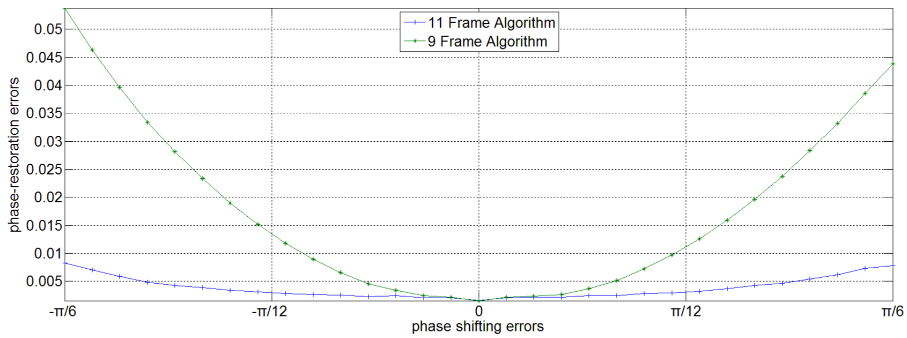

Fig. 11. The relationship between phase-restoration error and phase shearing error given by simulation with 11-frame and 9-frame algorithms under the condition of $V_2 = V_3 = 1\%V_1$.

5. Conclusions

The 11-frame phase shifting algorithm is proposed, and the analytical expression of phase-restoration error function is derived. The principle of phase shifting error compensation and the capability of suppressing zeroth order effect are explained. The algorithm is compared with existing algorithm. From analysis results, we conclude that it is better to use 11-frame algorithm to restore shearing phase with zeroth order effect and to compensate phase shifting error. This algorithm's phase-restoration error is proportional to sine of double shearing phase and to biquadratic of phase shifting error. With the same phase shifting error, phase-restoration error of 11-frame algorithm is smaller than that of existing algorithm. Finally, we simulate the interference patterns of 11-frame algorithm and existing algorithm, restore the shearing phases and calculate the phase-restoration errors. The simulations verify our theoretical analyses.

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