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Eliminating alignment error and analyzing Ritchey angle accuracy in Ritchey–Common test

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ABSTRACT

To improve the accuracy of the Ritchey–Common (R–C) test, this study proposes a method that utilizes the relation between system pupil and test flat coordinates to obtain a flat surface and integrate the least square method to eliminate the effect of misalignment. A Ritchey angle between 20° and 50° would be suitable for a simulation test. Testing accuracy is ensured when the error of the Ritchey angle is controlled within $\pm 1°$. To avoid measurement error of the Ritchey angle, the ratio of image size to the pupil plane is used to calculate the value. The accuracy can reach 0.2°. The three Ritchey angles chosen for the experiment are separated into two groups. The residual error between ZYGO and the group of 24.8° and 40.3° is 0.0013 wavelength (λ=632.8 nm). The experimental results confirm that this R–C method is effective and accurate.

1. Introduction

Large optical flats are often used as reference surfaces in auto-collimation optic tests. Thus, a high-quality surface or a suitable and highly precise test method is necessary. The diameter of these flats continues to increase, but no large aperture interferometer is available to cover the entire mirror during tests [1,2]. Instead, several indirect methods are used in large optical flat tests, such as the pentaprism scanning system and the Ritchey–Common (R–C) method. In a pentaprism scanning system [3], the pentaprism may be used to scan in lines to obtain the surface slope of the mirror at each point. The slope data can then be used to integrate the flat surface profile. However, this method is time consuming and unstable during testing [4,5]. The R–C test requires a well sphere with a diameter that is approximately 1.3 times larger than that of the test flat used as a reference [6]. This sphere is easier to fabricate than a large interferometer. This method is common, stable, and effective and can thus be used to test large flat mirrors by using an interferometer with a small diameter.

This study aims to solve the difficulties in the R–C test by proposing a method for obtaining a test flat mirror surface. This method utilizes the relationship between system pupil and flat mirror coordinates to derive the surface from two different test angles. The least square method is used to eliminate the misalignment error to obtain the real surface of the flat mirror. A suitable range of the Ritchey angle is identified, and its error allowance is determined by simulation. The ratio of image size to the pupil plane is used to calculate the Ritchey angle in each test.

2. Theory analysis for rectifying misalignment

Fig. 1 shows the R–C test configuration using a commercial interferometer [7]. In the test, the light beam from the interferometer passing through the transmission sphere diffuses; the focus of the transmission sphere is the center of curvature of the reference sphere. The beam is folded at some degree by the large optical flat to be tested, forming what is called the Ritchey angle θ. The beam is then reflected back off the flat into the interferometer by the return sphere.

2.1. Relationships in coordinate mapping

In the R–C test, when the test beam is inserted through the flat mirror at a certain angle, the image on the pupil plane is compressed in the sagittal direction and forms an oval [8,9]. Thus, the system pupil coordinates no longer correspond to the test flat mirror coordinates, and the angle between these two sets of coordinates becomes the same as the Ritchey angle θ. These two sets of coordinates are illustrated in Fig. 2, where x_s and y_s are the flat mirror coordinates and x_p and y_p are the system pupil coordinates.

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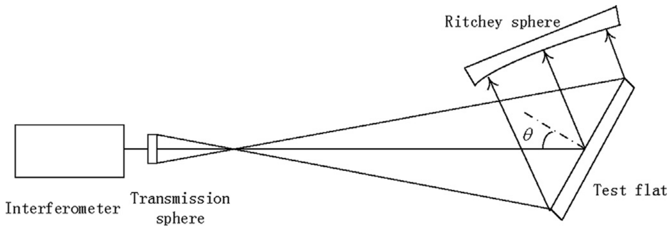

Fig. 1. R–C test set-up.

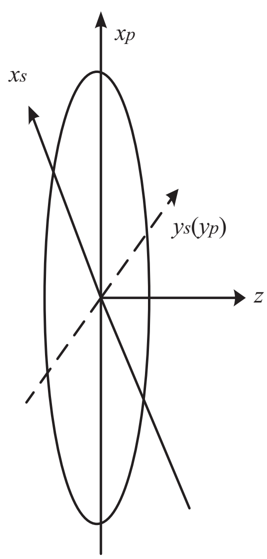

Fig. 2. Relation between the pupil and the flat coordinates.

The geometric relationship between these two sets of coordinates yields two transformation formulations:

$$x_s = \frac{d \times x_p}{d \times \cos\theta - x_p \times \sin\theta} \tag{1}$$

$$y_s = \frac{y_p \times (d + x_p \times \sin\theta)}{d} \tag{2}$$

where d is the distance between the focus of the transmission sphere and the center of the flat mirror. y_s, y_p are in the same direction, their values are approximate but unequal. The incident angle within the flat mirror is represented by

$$\cos\varphi = \frac{d \times \cos\theta - x_p \sin\theta}{\sqrt{d^2 + {x_p}^2 + {y_p}^2}} \tag{3}$$

After two reflections, the wavefront variation caused by the flat surface error is

$$S(x_s, y_s) = T\left[\frac{W(x_p, y_p)}{4 \times \cos\varphi}\right] \tag{4}$$

where $T[\cdot]$ is the function operator that shows the mapping relationship between the system pupil and flat mirror coordinates through formulas (1) and (2).

2.2. Correction of misalignment

Given that the R–C test requires a sphere for reference, piston misalignment, tilt and defocus cause errors in test data. And power, a common error for large flats, appears as astigmatism in test wavefront. To obtain precise results, multiple tests should be undertaken. Multiple testing is usually done in two ways: by changing the Ritchey angle or by rotating the flat along its normal axis. Considering the conditions in our laboratory, we choose the first method.

We mainly discuss defocus because the influence of piston and tilt can be neglected. In the experiment, the slope of the flat mirror in the tangential direction does not shift the principal ray. Thus, a constant slope in different tests does not introduce a wavefront error. Effect of the transfer and reference spheres can be calibrated and subtracted from the test wavefront. This study uses the relationship between the two sets of coordinates to fit the flat surface and the least square method to eliminate the misalignment error of the optical path. If $W_1(x_p, y_p)$ and $W_2(x_p, y_p)$ are the wavefront deviations in the two tests, the two surface errors $S_1(x_s, y_s)$ and $S_2(x_s, y_s)$ can be obtained by using the relation between the two sets of coordinates; each result contains the alignment error. If the real surface error is $S_0(x_s, y_s)$, the two results can be written as

$$S_1(x_s, y_s) = S_0(x_s, y_s) + a_1 \times D_1(x_s, y_s) \tag{5}$$

$$S_2(x_s, y_s) = S_0(x_s, y_s) + a_2 \times D_2(x_s, y_s) \tag{6}$$

where a_1 and a_2 are the defocus coefficients of the Zernike polynomial in the two tests and D_1 and D_2 are the defocus aberrations indicated

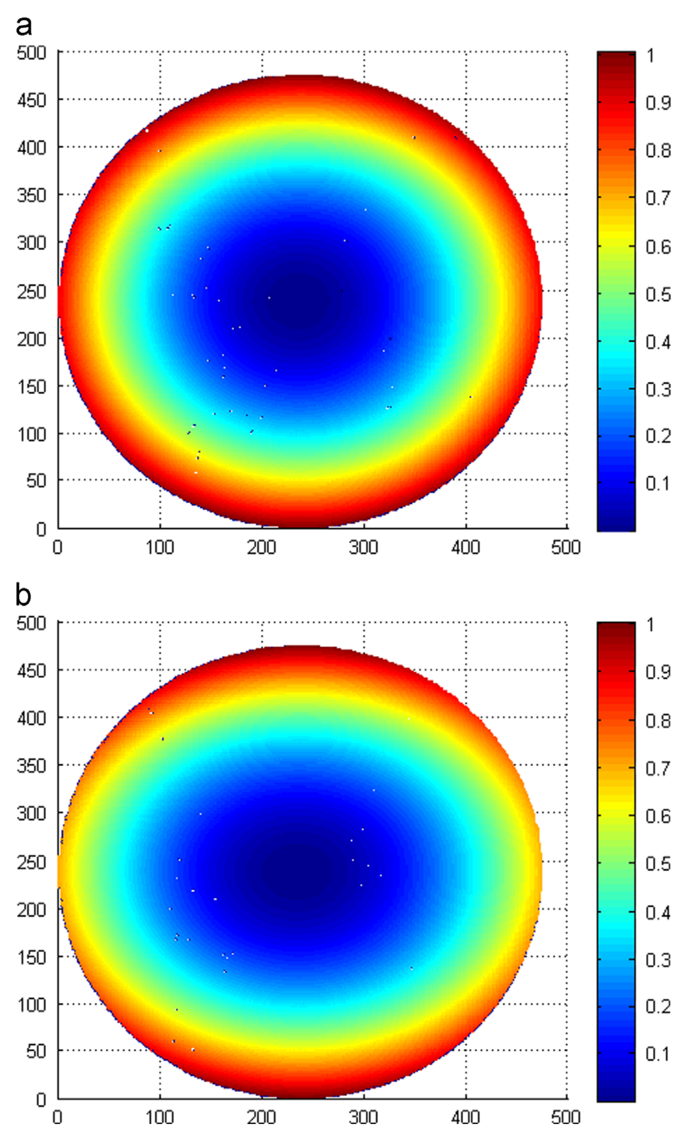

Fig. 3. Defocus shape of different Ritchey angles. (a) 20° surface of defocus and (b) 34° surface of defocus.

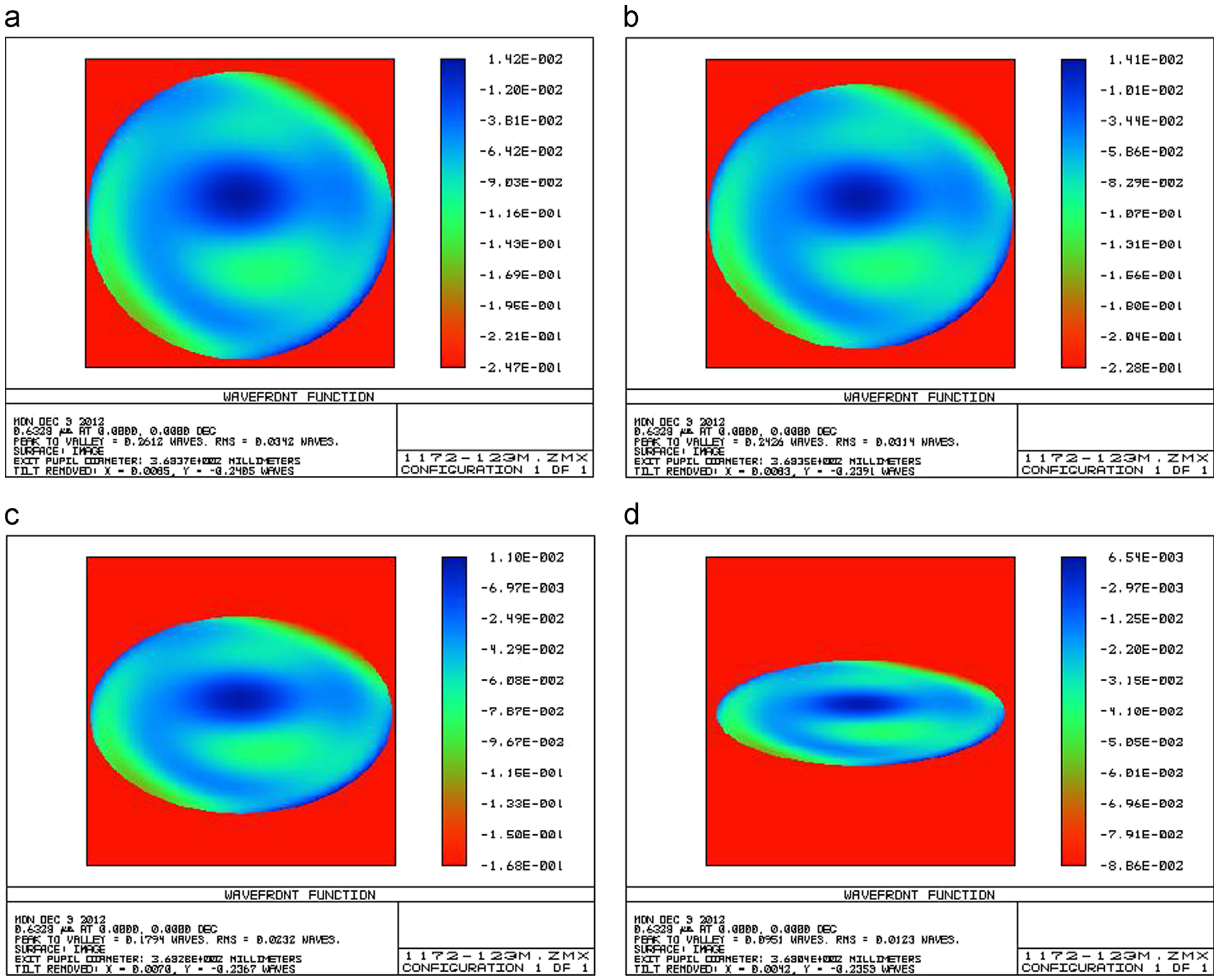

Fig. 4. Wavefront in different Ritchey angles. (a) θ=20° wavefront, (b) θ=40° wavefront, (c) θ=50° wavefront and (d) θ=70° wavefront.

by the Zernike polynomial in the flat mirror coordinates. Fig. 3 shows the defocus shape, where θ is 20° and 34°.

Formulas (5) and (6) yield a polynomial irrelevant to the flat surface:

$$\begin{aligned}\Delta S &= S_1 - S_2\\ &= a_1 D_1 - a_2 D_2\\ &= aD\end{aligned} \tag{7}$$

In formula (7),a and D are vectors for calculation. Fitting all effective data in a unit circle by using least square yields

$$a = \Delta S D^T (DD^T)^{-1} \tag{8}$$

The coefficients a and optimum S_0^* can then be derived through formula (5) or (6).

This method is also suited for flat mirrors with other shapes. The accuracy of the relationship between the two sets of coordinates should always be ensured during the test.

3. Analysis for Ritchey angle

The algorithm mode indicates that the choice of Ritchey angle affects the accuracy of the test result. To derive a true flat surface deviation, we conduct further analyses.

3.1. Influence on aberration

According to the principle of the R–C test, the image on the pupil plane is compressed as an oval shape when the flat mirror is inserted into the test beam at a certain angle, which affects the test wavefront. To derive an appropriate test range of the Ritchey angle, we use Zemax software to simulate this test path.

Usually if the flat surface is smooth enough, surface error can be described by Zernike polynomials as formula (9).

$$S = \sum_{n,m} a_i \times Z_{n,m} \tag{9}$$

In formula (9), S is the flat surface error, a_i is the Zernike coefficient of flat mirror and $Z_{n,m}$ represent the series of Zernike polynomials. Zernike polynomials fitting residual error PV can reach to 0.1λ and RMS can reach to 0.01λ. An arbitrary surface is first generated with Zernike polynomials for simulation.

$$\begin{aligned}S = {} & 0.0049Z_{2,0} - 0.0012Z_{2,2} + 0.006Z_{2,-2} - 0.0011Z_{3,1} + 0.0048Z_{3,-1}\\ & - 0.0013Z_{4,0} - 0.0029Z_{3,-3} - 0.0015Z_{3,-3} + 0.0016Z_{4,2} + 0.008Z_{4,-2}\\ & + 0.0014Z_{5,1} - 0.003Z_{5,-1} + 0.0024Z_{6,0} - 0.0025Z_{4,4} - 0.0021Z_{4,-4}\\ & + 0.0017Z_{5,3} + 0.0018Z_{5,-3} - 0.0045Z_{6,2} + 0.0022Z_{6,-2} + 0.0013Z_{7,1}\\ & + 0.005Z_{7,-1} - 0.0035Z_{8,0}\end{aligned} \tag{10}$$

In order to obtain more exact and comprehensive analysis, we choose Zernike polynomial from 4th to 25th to fit flat surface,

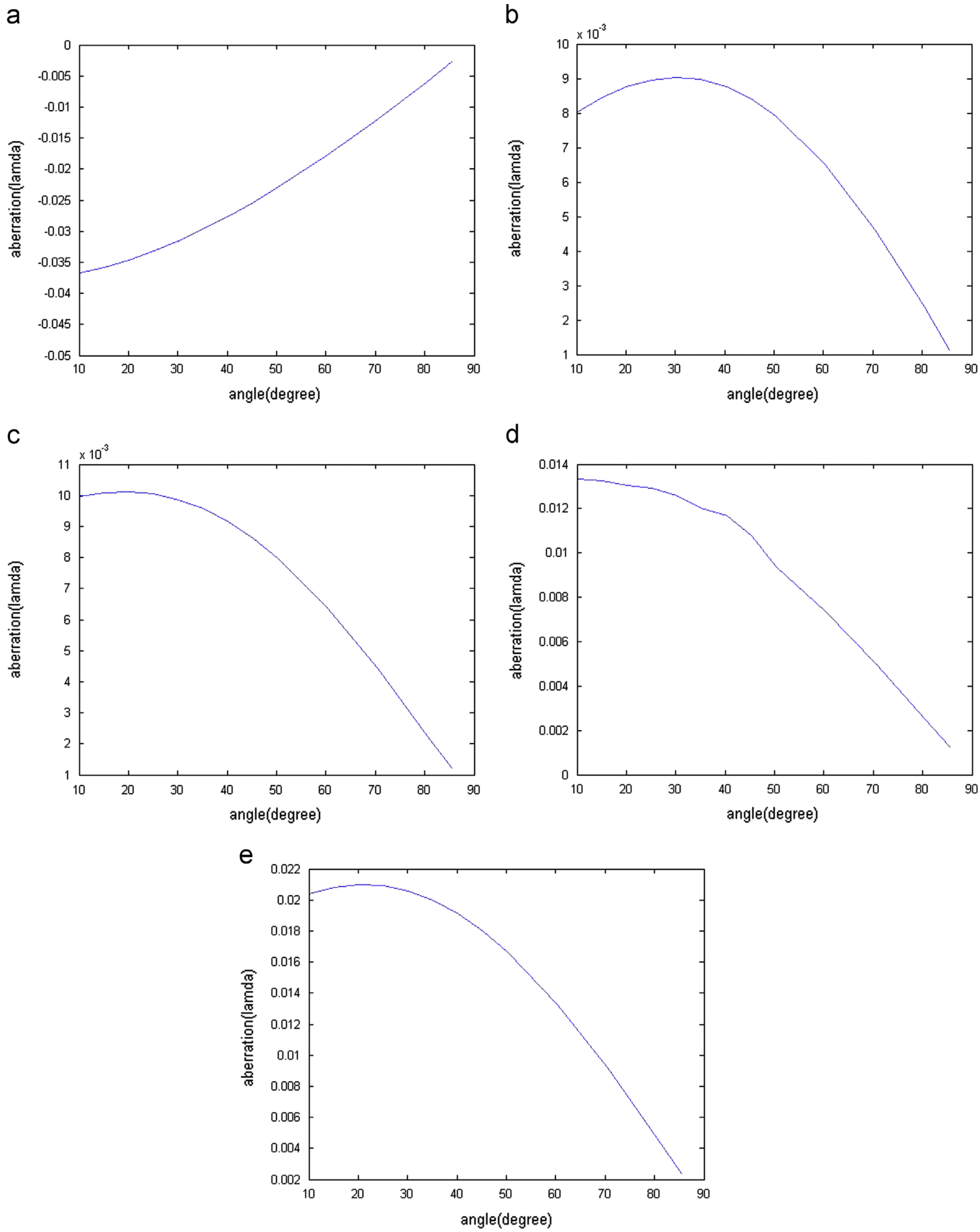

Fig. 5. Influence on aberrations at different Ritchey angles, (a) Relationship between $W_{2,-2}$ and θ, (b) Relationship between $W_{3,1}$ and θ, (c) Relationship between $W_{3,-3}$ and θ, (d) Relationship between $W_{4,-4}$ and θ and (e) Relationship between $W_{5,-1}$ and θ.

which contains low-order, intermediate-order and even high-order aberrations.

The simulation wavefronts in four different Ritchey angles after ray tracing in such angles are shown in Fig. 4.

In Fig. 4, the surface shape becomes more compressed when the Ritchey angle becomes larger. Under the same condition, the wavefront has different results with different Ritchey angles. Through simulation, the variations of some aberrations are shown (Fig. 5) when the Ritchey angle is 10° to 80°.

Fig. 5 indicates that the terms of wavefront aberration changes when the Ritchey angle increases. When the test angle changes from 10° to 50°, the curves of the aberrations vary relatively slowly, then the subsequent test data will be processed precisely. When the test angle is larger than 50°, the curves significantly vary rapidly, reducing the accuracy of the result. When the Ritchey angle is 20° to 50°, the deviation is stable and suitable for testing. In an actual test, laboratory conditions, such as equipment structure, should be considered to determine a suitable Ritchey angle for testing.

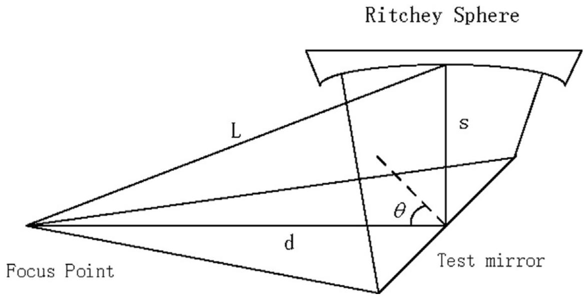

Fig. 6. Triangle method for Ritchey angle measurement.

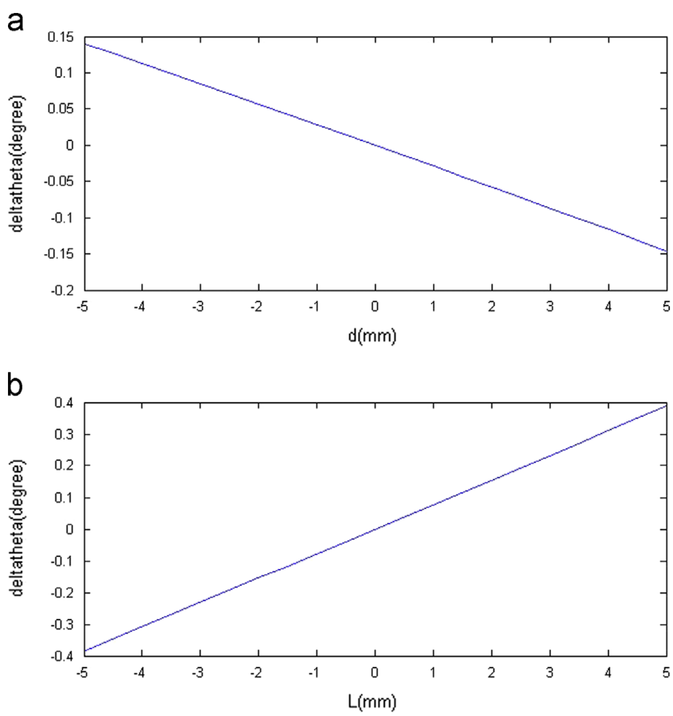

Fig. 7. Influence of measurement error on Ritchey angles. (a) Influence of d on Ritchey angle and (b) Influence of L on Ritchey angle.

Table 1
Residual errors.

($\Delta\theta$)	-0.2°	-0.4°	-0.6°	-0.8°	-1.0°
Residual error (λ)	0.00026	0.00026	0.00026	0.00016	0.00016
($\Delta\theta$)	0.2°	0.4°	0.6°	0.8°	1.0°
Residual error (λ)	0.00026	0.00036	0.00046	0.00046	0.00046

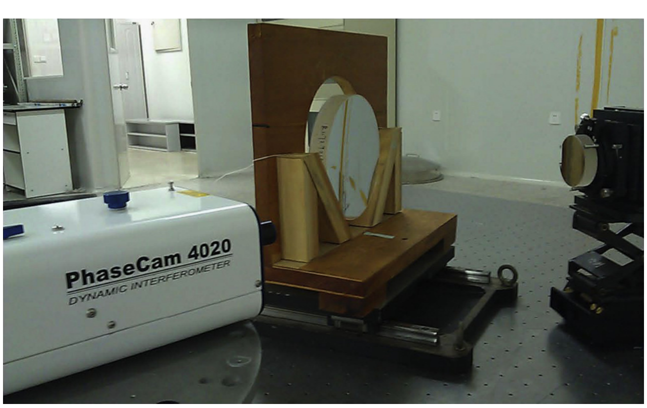

Fig. 8. Experimental layout.

3.2. Analysis of Ritchey angles

The triangle method often used, which measures the three-side lengths (d, s, and L), is shown in Fig. 6. The Ritchey angle is calculated by

$$\cos(2\theta) = \frac{d^2 + s^2 - L^2}{2d \times s} \tag{11}$$

where d is the distance between the focus point and the flat mirror center, s is the distance between the flat mirror center and the reference sphere center, and L is the distance between the focus point and the reference sphere center. The sum of d and s equals the radius of the curvature of the reference sphere.

In the simulation, the three-side lengths of the triangle can be given an exact value, but they must have a measurement error in the actual test. The three vertices of the triangle cannot be kept constant while the flat is rotated. Thus, the calculation error is unavoidable.

We analyze the change in the Ritchey angle when d and L have measurement errors, as shown in Fig. 7. Given that L more seriously affects the error than d, we should measure L in a more precise manner.

We also analyze the influence on the flat mirror surface when Ritchey angle measurement error exists. Assuming that the real Ritchey angle is 45°, the root mean square (RMS) of residual errors between the fitting result and the original surface is shown in Table 1.

As shown in Table 1, when the deviation of the Ritchey angle is lower than 0.1°, the accuracy of the residual error between the fitting results and the original surface can be decreased to 0.0005 wavelength. Thus, test accuracy can be ensured.

To obtain a more exact Ritchey angle, the ratio of the width and length to the pupil plane image is used to calculate the angle. Given a 45-degree Ritchey angle in the simulation optical path, we obtain 44.83° with this method, which is a 0.17° difference from the given value. The relative error is 0.37%, calculated from formula (12). Thus, this method can obtain a more precise result than the triangle method. In formula (12), θ is the true Ritchey angle, θ' is the calculated Ritchey angle, and α is the relative error.

$$\alpha = \frac{\theta' - \theta}{\theta} \times 100\% \tag{12}$$

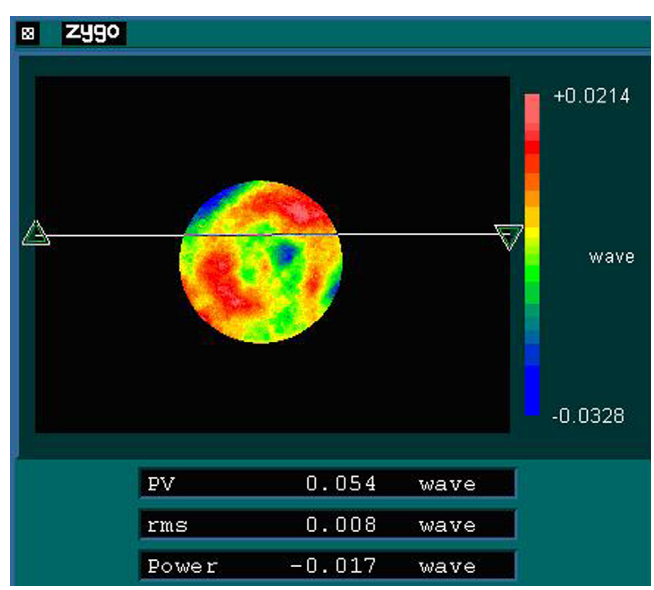

Fig. 9. Reference sphere surface condition.

4. Analysis of experiment and test results

In this experiment, a flat mirror with a diameter of 100 mm is tested. A well sphere with a diameter of 280 mm and a radius of curvature of 1172 mm is used as a reference. The testing system consists of an interferometer, transmission sphere, flat mirror, and reference sphere (Fig. 8). A 4D interferometer is used in R–C test to overcome the effect of vibration, and a ZYGO interfermeter is used for direct test. According to the analysis above, 24.8°, 40.3°, and 53.1° are chosen for the test; 53.1° is beyond the suitable range for the sake of comparison.

The surface of the reference sphere is shown in Fig. 9. We set up an R–C test configuration and make sure that the interferometer center should be coincident with the flat mirror and reference sphere centers. After obtaining wavefront data in 24.8°, we change the Ritchey angle to 40.3° to do for the second test. When the flat mirror is rotated, *d* must be invariant so that the relationship between the two sets of coordinates can stay remain in good agreement, we make use of calibrating crosshair on flat mirror and reference sphere to ensure this. The third test (53.1°) is conducted in the same way.

Wavefront deviations in the three angles are shown in Fig. 10. The width of the sagittal plane of the wavefront profile becomes narrower when the Ritchey angle increases. The three test data sets are divided into two groups: the first group consisting of 24.8° and 40.3° and the second consists of 24.8° and 53.1°. The method proposed above is then used to solve the problem.

The power aberration caused by misalignment in the test must be removed from the flat surface data. We use least square method to remove the power effect in each group. The coefficients of the

Table 2
Coefficients of defocus.

First group	Coefficient (μm)	Second group	Coefficient (μm)
24.8°	0.0154	24.8°	0.0219
40.3°	0.0179	53.1°	0.0306

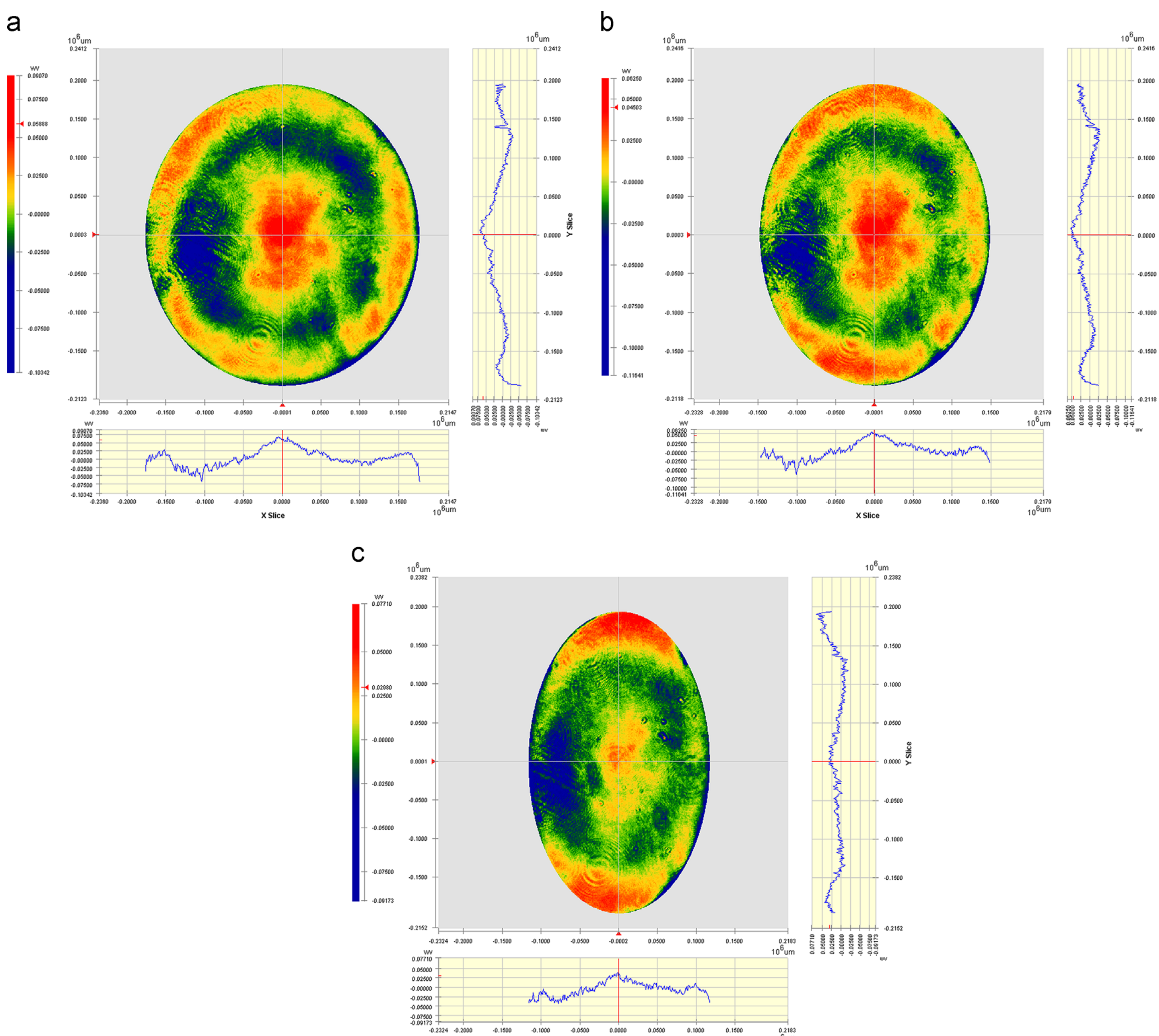

Fig. 10. Wavefront of three different Ritchey angles. (a) 24.8° wavefront, (b) 40.3° wavefront and (c) 53.1° wavefront.

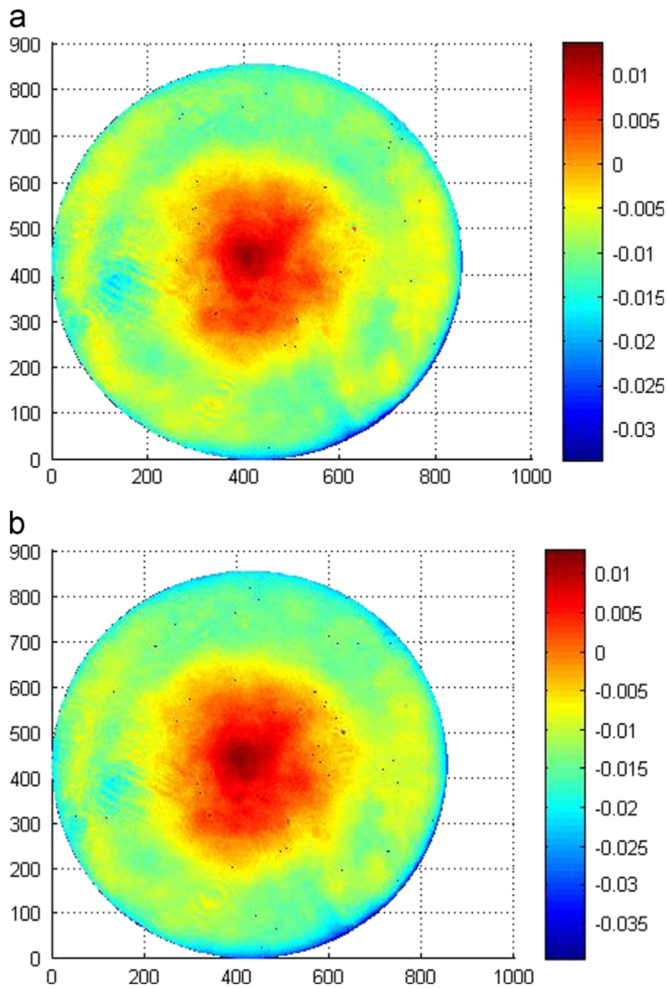

Fig. 11. (Two groups of surface results. (a) First group surface result: PV=0.149 λ, RMS=0.0177 λ and (b) Second group surface result: PV=0.166 λ, RMS=0.0221 λ.

power aberration are shown in Table 2. Finally, the flat surface is shown in Fig. 11.

The PV of the first group is 0.149 wave, and its RMS is 0.0177 waves; the PV of the second group is 0.166 wave, and its RMS is 0.0221 wave. For the diameter of 4D is limited, the direct result measured by Zygo which PV is 0.145 wave and the RMS is 0.019 wave, as shown in Fig. 12 [10]. The residual error is about 0.0013 and 0.0031 waves, respectively. Thus, the accuracy of this method can reach 0.01 waves, thus fulfilling the requirements of high-accuracy surface tests. The residual error is larger in the second group for a test angle that exceeds the suitable range. Therefore, a suitable angle can help improve test accuracy.

The experimental results demonstrate the effectivity and accuracy of this R–C method, but some problems must be solved. First, the high-frequency errors of the flat mirror are not resolved when test data are interpolated from an elliptical region into a round region. Second, a high-accuracy measurement equipment is necessary to reduce position error because rotating the flat mirror

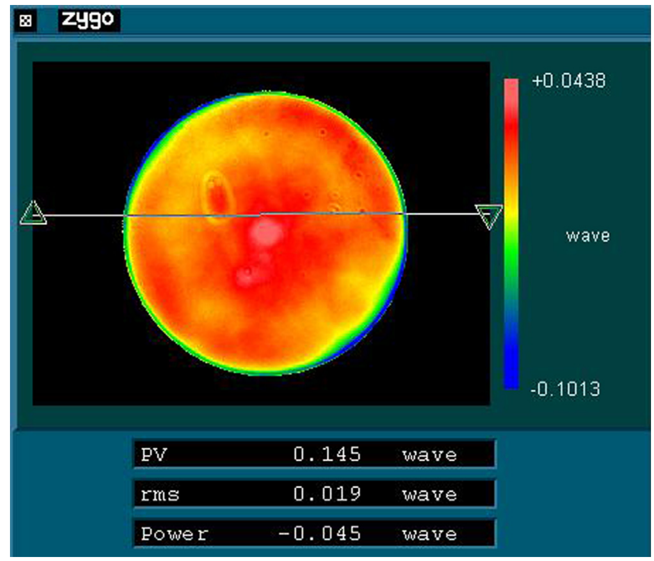

Fig. 12. Flat surface measured with Zygo.

inevitably varies *d*. Third, vibration often occurs during testing, and multiple tests should be taken to avoid incidental error.

5. Conclusion

To improve the accuracy of the R–C test, this study introduces a method that eliminates the misalignment effect and obtains a real surface configuration that utilizes the relationship between system pupil and test flat mirror coordinates. Based on the simulation, a Ritchey angle between 20° and 50° is suitable for testing. The allowance for the Ritchey angle error is $\pm 1°$; this allowance protects the result from significant influences. To reduce calculation errors, we use the ratio of image size to the pupil plane in calculating the Ritchey angle; this approach is more effective than the trigonometric approach. Finally, we test a flat mirror with a diameter of 100 mm in three different Ritchey angles. The results of the R–C test and the direct measurement with Zygo are very close; their accuracy can reach 0.01 wave. In the first group, PV and RMS can reach 0.004 and 0.0013 wave, respectively, which are better than the values in the second group. Therefore, two Ritchey angles in the suitable range improve test accuracy.

References

[1] P. Mallik, C. Zhao, J.H. Burge, Optical Engineering 46 (2007) 023602.
[2] J. Yellowhair, J.H. Burge, Applied Optics 46 (2007) 8466.
[3] J. Zhang, X. Zhang, Z. Zhang, Optics and Precision Engineering 20 (2012) 492.
[4] J. Shao, D. Ma, Z. Nie, Optics and Precision Engineering 19 (2011) 2583.
[5] J. Wang, Z. Wang, B. Wang, Optics and Precision Engineering 19 (2011) 1165.
[6] D. Malacara, Optical Shop Testing, 1992.
[7] K.L. SHU, Applied Optics 22 (1983) 1879.
[8] S. Han, E. Novak, M. Schuring, Proceedings of SPIE 4399 (2001) 131.
[9] S. Han, E. Novak, M. Schuring, Proceedings of SPIE 4842 (2003) 270.
[10] ZYGO Corporation, ZYGO Corporation User Manual (2004) 0429A.