Advanced astigmatism-corrected tandem Wadsworth mounting for small-scale spectral broadband imaging spectrometer

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Received 27 August 2012; revised 12 October 2012; accepted 13 October 2012;
posted 15 October 2012 (Doc. ID 175026); published 21 December 2012

Tandem gratings of double-dispersion mount make it possible to design an imaging spectrometer for the weak light observation with high spatial resolution, high spectral resolution, and high optical transmission efficiency. The traditional tandem Wadsworth mounting is originally designed to match the coaxial telescope and large-scale imaging spectrometer. When it is used to connect the off-axis telescope such as off-axis parabolic mirror, it presents lower imaging quality than to connect the coaxial telescope. It may also introduce interference among the detector and the optical elements as it is applied to the short focal length and small-scale spectrometer in a close volume by satellite. An advanced tandem Wadsworth mounting has been investigated to deal with the situation. The Wadsworth astigmatism-corrected mounting condition for which is expressed as the distance between the second concave grating and the imaging plane is calculated. Then the optimum arrangement for the first plane grating and the second concave grating, which make the anterior Wadsworth condition fulfilling each wavelength, is analyzed by the geometric and first order differential calculation. These two arrangements comprise the advanced Wadsworth mounting condition. The spectral resolution has also been calculated by these conditions. An example designed by the optimum theory proves that the advanced tandem Wadsworth mounting performs excellently in spectral broadband. © 2012 Optical Society of America

OCIS codes: 120.4570, 120.6200, 300.6190.

1. Introduction

The important merits of the concave grating spectrometer range from simplicity of the mounting to high efficiency. Scientists in many fields of physics such as astrophysics, planetary atmospheres, plasma phenomena, aeronomy, and Raman and laser-induced spectroscopy, take a particular interest in the development of imaging spectrometers with concave grating [1,2].

The ideal imaging spectrometer possesses the following characteristics: high spatial resolution for excellent imaging quality in all observing waveband, and high spectral resolution for distinguishing spectral lines with extraordinary narrow intervals [3]. A primary problem for the concave grating mounting used for imaging spectrometer is to reduce the astigmatism for all wavelengths [4,5]. The classic toroidal uniform linespace grating and spherical varied linespace grating Rowland spectrometer can make stigmatic imaging at a single wavelength. The modified Rowland mount with toroidal varied linespace grating and type III holographic concave grating, respectively, allows two and three stigmatic points within the observed spectral region [6–8]. Another stigmatic concave-grating spectrometer is the Wadsworth mounting. The anastigmatism and freedom of coma of the arrangement are valid for one special wavelength [9], which is usually the central wavelength.

The common shortcoming of the above designs is that stigmatic images can only be obtained in some individual wavelengths. When these designs are used in spectral broadband, the astigmatism will

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increase rapidly for other wavelengths and it will reduce the imaging ability of the spectrometer. Some special surface diffraction gratings such as parabolic grating are introduced to solute the problem [10]. Another problem is that when these designs are requested for high spectral resolution, the grating ruling density must be manufactured very high. It will also decrease the broadband imaging ability of the spectrometer and improve the difficulty of the grating manufacture, especially for special surface grating.

When the imaging spectrometer is carried by satellite for upper atmosphere observation, it must provide stigmatic imaging over the spectral broadband with a small-scale structure. It should have high spatial resolution to study observation target features with minute spatial extents, and high spectral resolution to distinguish a large variety of close spectral lines simultaneously. The volume and weight of the spectrometer are developing toward close and light.

In Brueckner and Nakada's study [11,12], double-dispersion tandem Wadsworth mounting spectrometer has been studied to provide stigmatism image in spectral broadband with two merits. First, the dispersing properties of the first grating permit the light bundle of each wavelength to enter the feasible portion where it can be diffracted along the local normal of the second grating. The characteristic makes the mounting suitable for broadband stigmatism imaging. Second, two tandem gratings will supply large dispersive power and avoid the stray light.

Brueckner has given one group of appropriate general solutions to the tandem Wadsworth mounting. In his study, the two tandem gratings are symmetric and tangent the same circle with radius R. R is also the radius of curvature of the concave grating. The two principal mountings in the discussion are indicated in Fig. 1.

Figure 1(a) shows a specialized symmetric tandem Wadsworth mounting. O is the vertex of the two concave gratings. The center of the slit and the center of the first grating O are located at the same line. In this situation, the mounting is arranged at normal incidence or as near normal incidence to satisfy the Wadsworth condition as possible. The residual astigmatism difference is very small.

Figure 1(b) shows another tandem Wadsworth mounting. In the mounting, the first plane grating is placed in the collimating light. The plane grating plays the collimator and the concave grating is the camera. The plane grating is also tangent the circle. O is the center of curvature and is located at the central light of the plane grating. The astigmatic difference of this mounting is zero for all wavelengths in the arrangement. The bundle of entrance light may not be strictly collimating. It will introduce aberrations and decrease the imaging quality. The design can be used as a slitless spectrometer. When it is used for a slit imaging spectrometer, a collimating mirror such as an off-axis parabolic mirror is inserted. It wasn't discussed further in Brueckner's study.

The demerits of these designs happen in two aspects for the close, small-scale, and short focal length spectrometer in the application of upper atmosphere observation. First, when the spectrometer is applied to faint light detection such as extreme or far-ultraviolet ionosphere observation by satellite, it usually adopts the off-axis parabolic mirror as the telescope [13]. The form of the telescope guarantees the required optical energy transmission efficiency. It makes the incidence light on the spectral imaging system off axis, and brings larger off-axis aberrations than normal incidence. In this situation, the symmetrical tandem mounting is no longer suitable for eliminating the astigmatism difference. The imaging quality will decrease as the observation waveband gets wider and the off-axis amount gets larger. Second, in practical engineering applications, these tandem designs are more suitable for long focal length and large-scale spectrometers in general. When these sort of tandem designs are used for the close size spectrometer, the distance between

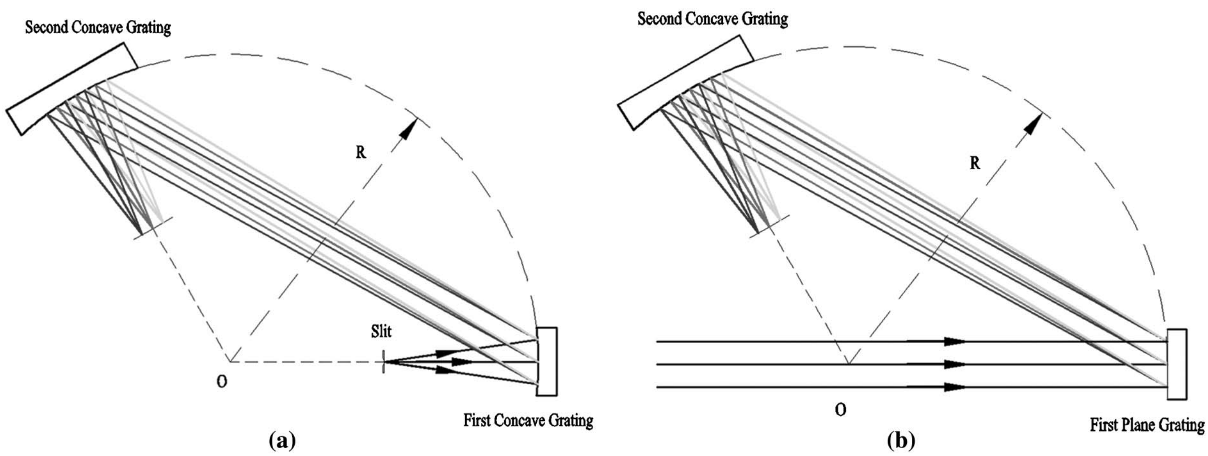

Fig. 1. Two principal tandem Wadsworth mountings. (a) Mounting is composed of two concave gratings of which curvature of radii and the ruling density are equal. (b) Mounting is composed of a plane grating and a concave grating of which the ruling density are equal.

the first grating and the second grating is too long to fulfill the requirement of the volume and the weight, and it will bring interferences among the optical elements, the entrance pupil, and the detector.

To solve the above problems, an advanced tandem Wadsworth mounting is designed in the article. In our design, the two tandem gratings don't tangent the same circle. They locate in a much shorter distance than the traditional tandem mounting. It can save much volume and weight in the spectrometer and eliminate astigmatism in any broadband. In Section 2, we analyze the astigmatism difference in the tandem Wadsworth mounting and obtain the optimum Wadsworth condition. In Section 3, the optimum arrangement of the advanced Wadsworth mounting is calculated by analysis of geometric light path and first order differential calculus to satisfy the spectral broadband imaging conditions. An example for 120–180 nm based on the optimum theory is presented in Section 4. The analysis of the spectral resolution of the design is shown in Section 5. The summary is given in Section 6.

2. Analysis of Broadband Astigmatism-corrected Tandem Wadsworth Mounting Condition

According to Brueckner's study, the ruling density and radii of curvature of G_1 and G_2 are assumed to be the same in order to simplify the discussion. The meridian focal distance and the sagittal focal distance of the grating in Wadsworth mounting are given by Beutler [14]:

$$\begin{cases} r'_m = [(\cos i + \cos \theta)/R - \cos^2 i/r]^{-1}\cos^2\theta \\ r'_s = [(\cos i + \cos \theta)/R - 1/r]^{-1} \end{cases}, \quad (1)$$

where r is the source distance, i and θ satisfy the grating equation $d(\sin I + \sin\theta) = m\lambda$, and R is the radius of curvature of the grating. For the first grating, we have $r = r_1$, and $R = R_1$. In our design, the advanced tandem Wadsworth mounting is composed of a plane grating and a concave grating. So $R_1 = \infty$ and $r = \infty$. R_2 is the radius of curvature of the second grating. Taking into account the distances between the two gratings from Fig. 1, and using the corresponding parameters to substitute for the original parameters in Eq. (1), the meridian and sagittal distances at the final image plane can be indicated from Fig. 2.

$$\begin{cases} d_{mCI} = d_{PC} - r'_{1m} = R_2\cos^2\theta_1(\cos i_1 + \cos\theta_1)^{-1} \\ d_{sCI} = d_{PC} - r'_{1s} = R_2(\cos i_1 + \cos\theta_1)^{-1} \end{cases}. \quad (2)$$

The parameters r'_{1m} and r'_{1s} are the meridian and the sagittal focal distances of the first grating, respectively. The relative astigmatic difference can be extinguished by the following formula:

$$|d_{mCI} - d_{sCI}| = R_2(\cos^2\theta_1 - 1)(\cos i_1 + \cos\theta_1)^{-1} = 0. \quad (3)$$

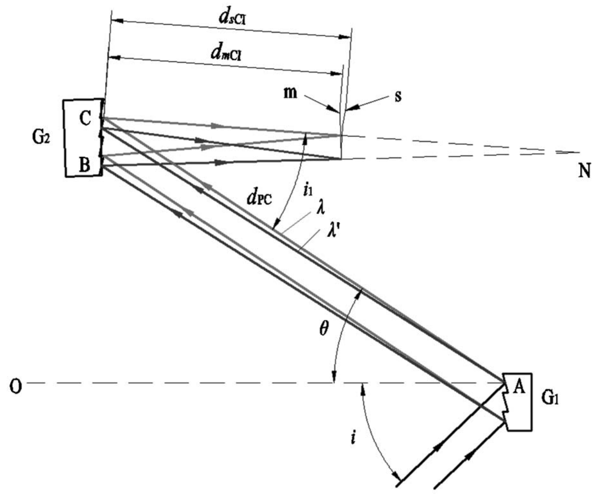

Fig. 2. Tandem Wadsworth mounting. G_1 is the first concave grating with the radius R_1. G_2 is the second concave grating with the radius R_2 centered at N. O is the normal of G_1, and N is the local normal of G_2. The wavelength λ and the wavelength λ' are presented in different lines. i and θ are the incident angle and the diffraction angle of G_1, respectively, which satisfy the grating equation for the wavelength λ. i_1 is the incident angle of G_2 for the same wavelength λ. d_{PC} is the distance between the local vertex A and C. m and s stand for the meridian and sagittal focal planes, respectively. d_{mCI} and d_{sCI} are the meridian and sagittal focal distances from the local vertex of G_2 to the imaging plane, respectively.

When $\theta_1 = 0$, the astigmatic difference for all wavelengths of our mount is zero. It also means that the images of all wavelengths from the first plane grating fall on the local normal of the second grating. We have the optimum Wadsworth condition:

$$d_{mCI} = d_{sCI} = R_2(\cos i_1 + 1)^{-1}. \quad (4)$$

To ensure the light bundle of each wavelength dispersed by the first plane grating to incident on the relative portion of the concave grating to satisfy the condition, the two gratings must be put at the proper position. In the following section, we will discuss the advanced Wadsworth arrangement conditions.

3. Analysis of Advanced Tandem Wadsworth Gratings Arrangement

It should be known that the off-axis aberrations brought out by the telescope and the collimating mirror have been included in the collimating light which was incident onto the plane grating A. These off-axis aberrations increase with the increasing of the off-axis amount. The amounts are reflected by the off-axis angle of the telescope and the collimating mirror. And the influence of these amounts is introduced in our following analysis by the incident angle ζ.

As in Fig. 3, O is the center of curvature and R_2 is the radius of curvature of the second concave grating. i_i and θ_i satisfy the grating equation. AB_1C_1 is the light path from the first plane grating to the imaging plane for the central wavelength λ, which is indicated as $L_{G1G2} + L_{G2I}$. AB_2C_2 is the light path for different wavelength λ' as $L'_{G1G2} + L'_{G2I}$. γ is the angle between the light diffracted from the second concave grating

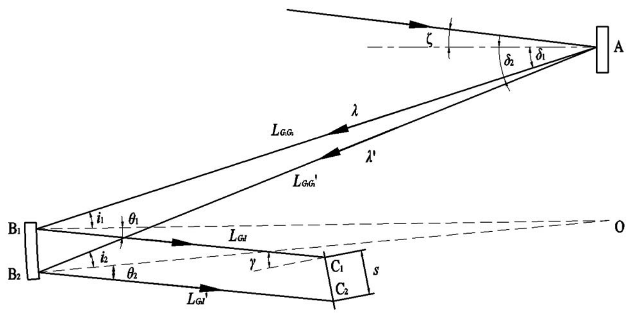

Fig. 3. Diagram of the output of the advanced tandem Wadsworth spectrometer, showing the central ray λ and a ray with slightly different wavelength λ', of which displacement on the imaging plane is s.

and the normal of the imaging plane. The geometric method and the first-order differential calculation will be introduced to obtain the optimum gratings arrangement to satisfy the astigmatism-corrected tandem Wadsworth conditions in Section 2.

The relationships of each vector in two vector triangles AB_1O and AB_2O are obtained in Fig. 3:

$$\overrightarrow{OA} = \overrightarrow{OB_1} - \overrightarrow{AB_1} = \overrightarrow{OB_2} - \overrightarrow{AB_2}, \tag{5}$$

where

$$\begin{cases} \overrightarrow{AB_1} = [-L_{G_1G_2}\cos\delta_1, -L_{G_1G_2}\sin\delta_1] \\ \overrightarrow{AB_2} = [-L'_{G_1G_2}\cos\delta_2, -L'_{G_1G_2}\sin\delta_2] \end{cases}, \tag{6}$$

and

$$\begin{cases} \overrightarrow{OB_1} = [-R_2\cos(\delta_1 - i_1), -R_2\sin(\delta_1 - i_1)] \\ \overrightarrow{OB_2} = [-R_2\cos(\delta_2 - i_2), -R_2\sin(\delta_2 - i_2)] \end{cases}. \tag{7}$$

Then we have

$$\begin{cases} L_{G_1G_2}\cos\delta_1 - R_2\cos(\delta_1 - i_1) = L'_{G_1G_2}\cos\delta_2 - R_2\cos(\delta_2 - i_2) \\ L_{G_1G_2}\sin\delta_1 - R_2\sin(\delta_1 - i_1) = L'_{G_1G_2}\sin\delta_2 - R_2\sin(\delta_2 - i_2) \end{cases}. \tag{8}$$

We differentiate Eq. (8) with respect to δ_1 and evaluate resulting expression at the central wavelength,

$$\frac{di_1}{d\delta_1} = 1 - \frac{L_{G_1G_2}}{R_2\cos i_1}, \tag{9}$$

and

$$\frac{dL_{G_1G_2}}{d\delta_1} = L_{G_1G_2}\tan i_1. \tag{10}$$

The next step is to find $dL_{G2I}/d\delta_1$ by using relationships of the vector triangles AB_1C_1, AB_2C_2, and AC_1C_2. We have

$$\overrightarrow{C_2C_1} = \overrightarrow{AC_1} - \overrightarrow{AC_2} = (\overrightarrow{AB_1} - \overrightarrow{C_1B_1}) - (\overrightarrow{AB_2} - \overrightarrow{C_2B_2}). \tag{11}$$

The coordinate values of vectors are indicated as

$$\overrightarrow{C_2C_1} = [-s\cdot\sin(\gamma + \delta_2 - i_2 - \theta_2), s \cdot\cos(\gamma + \delta_2 - i_2 - \theta_2)], \tag{12}$$

$$\overrightarrow{AC_1} = [-L_{G_1G_2}\cos\delta_1 + L_{G_2I}\cos(i_1 + \theta_1 - \delta_1), -L_{G_1G_2}\sin\delta_1 - L_{G_2I}\sin(i_1 + \theta_1 - \delta_1)], \tag{13}$$

$$\overrightarrow{AC_2} = [-L'_{G_1G_2}\cos\delta_2 + L'_{G_2I}\cos(i_2 + \theta_2 - \delta_2), -L'_{G_1G_2}\sin\delta_2 - L'_{G_2I}\sin(i_2 + \theta_2 - \delta_2)]. \tag{14}$$

Then Eq. (15) is obtained:

$$\begin{cases} -s\cdot\sin(\gamma + \delta_1 - i_1 - \theta_1) = L_{G_2I}\cos(i_1 + \theta_1 - \delta_1) - L'_{G_2I}\cos(i_2 + \theta_2 - \delta_2) - L_{G_1G_2}\cos\delta_1 + L'_{G_1G_2}\cos\delta_2 \\ s\cdot\cos(\gamma + \delta_1 - i_1 - \theta_1) = -L_{G_2I}\sin(i_1 + \theta_1 - \delta_1) + L'_{G_2I}\sin(i_2 + \theta_2 - \delta_2) - L_{G_1G_2}\sin\delta_1 + L'_{G_1G_2}\sin\delta_2 \end{cases}. \tag{15}$$

We differentiate Eq. (15) with respect to δ_1 and evaluate the resulting expression,

$$\frac{dL_{G_2I}}{d\delta_1} = \frac{[\sin(\delta_1 - i_1) - \cos(\delta_1 - i_1)]L_{G_1G_2} - L_{G_2I}\left[\frac{L_{G_1G_2}}{R}\frac{(\cos i - \cos\theta_1)}{\cos\theta_1} - \frac{\cos^2 i}{\cos\theta}\right][\sin(i_1 + \theta_1 - \delta_1) + \cos(i_1 + \theta_1 - \delta_1)]}{[\cos(i_1 + \theta_1 - \delta_1) - \sin(i_1 + \theta_1 - \delta_1)]\cos i_1}. \tag{16}$$

In the previous discussion, when the astigmatism-corrected tandem Wadsworth conditions are satisfied, $L_{G_2I} = R_2/(1+\cos i_1)^{-1}$ and the angle θ is zero. It can be differentiated with respect to δ_1:

$$\frac{dL_{G_2I}}{d\delta_1} = \frac{\partial L_{G_2I}}{\partial i_1}\frac{di_1}{d\delta_1} = \frac{R_2 \sin i_1}{(1+\cos i_1)^2}\frac{di_1}{d\delta_1} = \frac{R_2 \sin i_1}{(1+\cos i_1)^2}\left(1 - \frac{L_{G_1G_2}}{R_2 \cos i_1}\right). \quad (17)$$

By combining Eqs. (16) and (17), we obtained the following equation:

$$L_{G_1G_2} = \frac{\tan\gamma \cdot \cos i_1(1+\cos i_1) + \sin i_1}{2\tan\gamma(1+\cos i_1) + \tan i_1}R_2. \quad (18)$$

The optimal grating arrangement in our advanced design to obtain excellent imaging quality in broadband can be expressed as two distances:

$$\begin{cases} L_{G_1G_2} = \frac{\tan\gamma\cdot\cos i_1(1+\cos i_1)+\sin i_1}{2\tan\gamma(1+\cos i_1)+\tan i_1}R_2 \\ L_{G_2I} = R_2/(1+\cos i_1)^{-1} \end{cases}. \quad (19)$$

In general, the angle γ is always very close to 0, so $\tan\gamma \approx 0$. The optimal conditions can be simplified as

$$\begin{cases} L_{G_1G_2} = \cos i_1 R_2 \\ L_{G_2I} = R_2/(1+\cos i_1)^{-1} \end{cases}. \quad (20)$$

The first formula in Eq. (20) implies $di_1/d\delta_1 = 0$ [in Eq. (9)]. It means that the incidence angle i_1 of the first grating is independent of the diffraction angle δ_1 of the second grating, when $\gamma \approx 0$.

4. Example

An example to verify the above design theory is given as follows. The imaging spectrometer is used for ionosphere observation in 120–180 nm. By the requirements of the scientific ionosphere application in an orbit of 830 km, the main parameters of the spectrometer are in Table 1.

The general layout of the imaging spectrometer is shown in Fig. 4.

The main parameters of the design are listed in Table 2. The optical system design program ZEMAX is used to simulate and analyze the optical system [15].

To show the importance of the best distance between the plane grating and the concave grating

Table 1. Parameters of the FUV Imaging Spectrometer

Parameters	Value
Spectral range	120–180 nm
Spatial resolution/mrad	0.4
IFOV/°	4 × 0.12
Pixel/μm	25 × 25

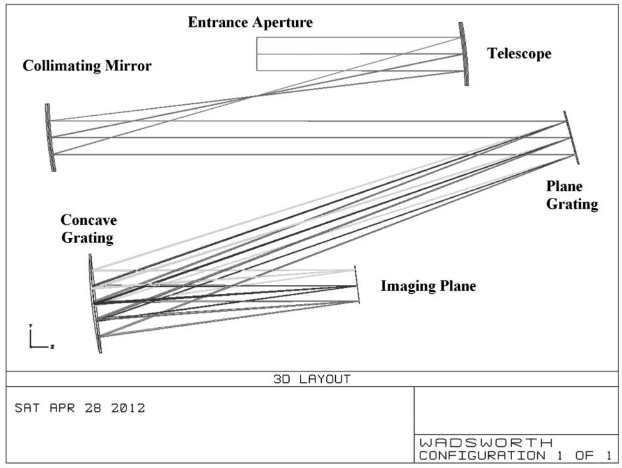

Fig. 4. Layout of advanced tandem Wadsworth mounting FUV imaging spectrometer.

Table 2. General Characteristics of the FUV Imaging Spectrometer

Parameters	Values
Telescope	
Radius of curvature	200 mm
Entrance diameter	16 mm
Off-axis amount	20 mm
Collimating mirror	
Radius of curvature	200 mm
Off-axis amount	20 mm
First grating	
Ruling density	1200 line/mm
Incidence angle	15°
L_{G1G2}	236 mm
Second grating	
Ruling density	1200 line/mm
Radius of curvature	250 mm
Incidence angle	10.8°
L_{G21}	126 mm

for obtaining excellent manifestation over the spectral broadband, we compare the variation of RMS spot radii with changes of the wavelengths in different arrangement. According to the comparison in Fig. 5, it is obviously that when L_{G1G2} is adjacent to $\cos i_2 R$, outstanding imaging quality will be obtained over the whole wavelength region. However, if $L_{G1G2} \gg \cos i_2 R$ or $L_{G1G2} \ll \cos i_2 R$, excellent imaging quality will only be achieved in the vicinity of the central wavelength, and the best imaging quality of different fields happens in different position of each wavelength.

In Fig. 6, the modulation transfer function (MTF) of the margin and central wavelengths in each field is larger than 0.65 at Nyquist frequency (20 lp/mm). It indicates that our optimization will obtain excellent imaging quality at the imaging plane.

5. Analysis of Spectral Resolution

According to Eq. (15), by differentiating with respect to δ_1, another expression can be obtained:

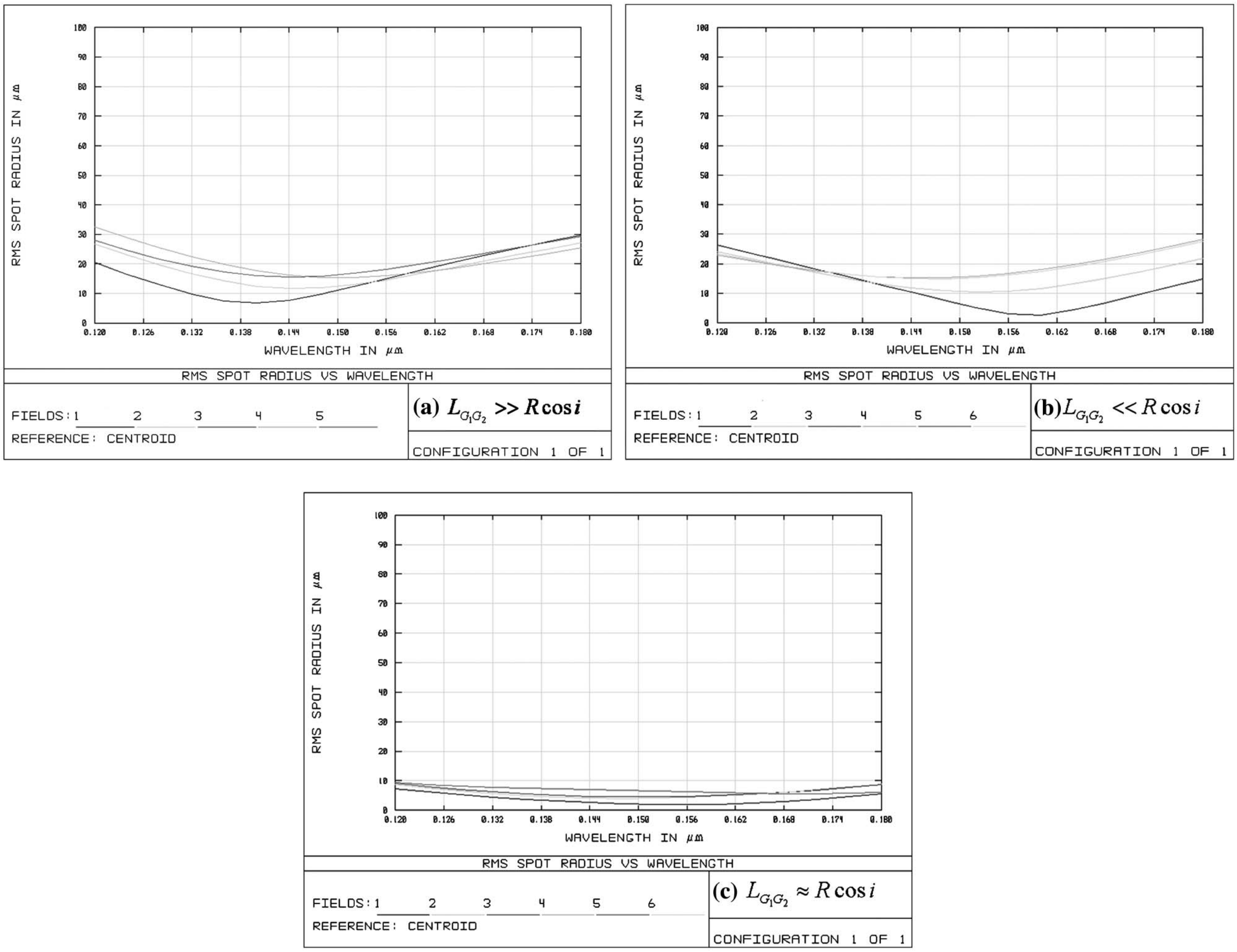

Fig. 5. RMS spot radius versus wavelength in different values of L_{G1G2}.

$$\frac{ds}{d\delta_1} = \left[1 + \frac{\cos\gamma}{\cos i_1(1+\cos i_1)} + \frac{\sin\gamma \sin i_1}{\cos i_1(1+\cos i_1)^2}\right] L_{G_1G_2} - \frac{R_2 \sin i_1 \sin\gamma}{(1+\cos i_1)^2}. \quad (21)$$

Equation (21) can be simplified with the optimal condition L_{G1G2} when $\gamma \approx 0$:

$$\frac{ds}{d\delta_1} = \frac{R(1+\cos i_1 + \cos^2 i_1)}{(1+\cos i_1)}. \quad (22)$$

Here we have

$$\frac{d\delta_1}{ds} = \frac{d\lambda}{ds} \cdot \frac{d\delta_1}{d\lambda}. \quad (23)$$

In Eq. (23), $d\delta_1/d\lambda$ is the angular dispersion which is decided by the first order differential grating equation:

$$\frac{d\delta_1}{d\lambda} = \frac{m}{d \cos \delta_1}, \quad (24)$$

where d is the ruling density of the grating. With Eqs. (22), (23), and (24), we have

$$\frac{d\lambda}{ds} = \frac{d\delta_1}{ds} \cdot \frac{d\lambda}{d\delta_1} = \frac{(1+\cos i_1)}{R(1+\cos i_1 + \cos^2 i_1)} \cdot \frac{d \cos \delta_1}{m}. \quad (25)$$

In Eq. (25), $d\lambda$ is the minimal spectral interval which the spectrometer can recognize. It can be considered as the spectral resolution of the spectrometer. ds is the width of the slit imaging on the imaging plane corresponding to $d\lambda$. It is represented as Eq. (26) by the analysis of the optical system in Fig. 3:

$$ds = \frac{f_2 \cos\zeta}{f_1 \cos\delta_1} b. \quad (26)$$

In Eq. (26), f_1 and f_2 are the focal lengths of the collimating mirror and the focusing mirror. b is the width of the slit. ζ and δ_1 satisfy the grating equation. The spectral resolution of the advanced imaging spectrometer is

$$d\lambda = \frac{(1+\cos i_1)}{R_2(1+\cos i_1 + \cos^2 i_1)} \cdot \frac{d \cos \delta_1}{m} \cdot ds = \frac{(1+\cos i_1)\cos\zeta}{(1+\cos i_1 + \cos^2 i_1)} \cdot \frac{d f_2}{m R_2 f_1} b. \quad (27)$$

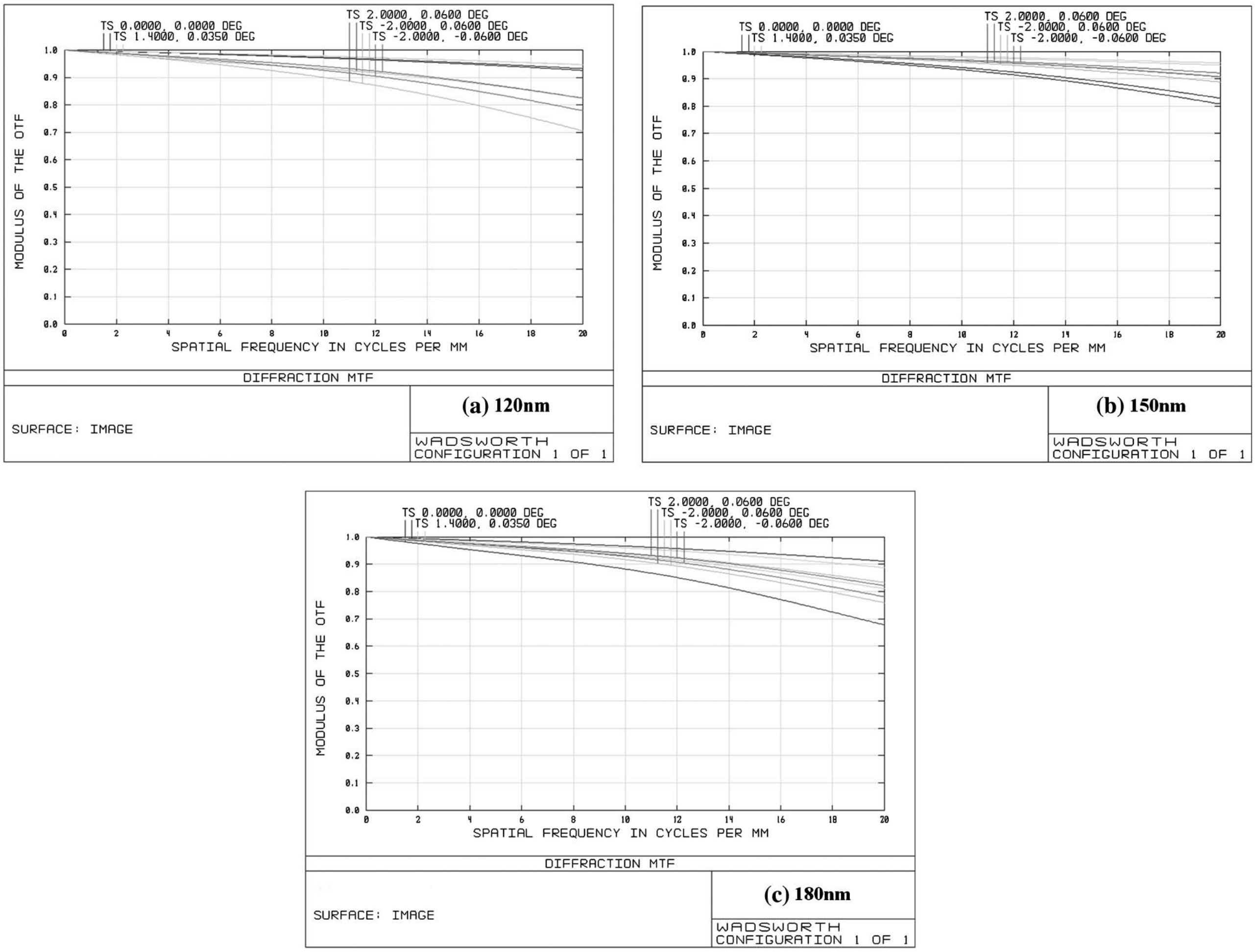

Fig. 6. MTF of the advanced tandem Wadsworth imaging spectrometer under central and marginal wavelengths in each field of view.

According to the given values of each parameter of the design, the spectral resolution of the advanced tandem Wadsworth mounting is 0.53 nm.

6. Summary

An astigmatism-corrected imaging spectrometer with advanced tandem Wadsworth mounting has been designed. The design is suitable for small-scale and off-axis incidence imaging spectrometer, especially for weak light observation by satellite. The optimum geometric locations of two tandem gratings and the imaging plane have been obtained by astigmatism analysis, the geometric and first order differential calculation, for excellent broadband imaging. The analysis indicates that the imaging quality in broadband is mainly related to the two optimal distances between the first plane grating and the second concave grating, and the second concave grating and the imaging plane, when the ruling density and radii of curvature of gratings are decided. An example for a 120–180 nm imaging spectrometer is designed based on the optimum theory. We also analyze the spectral resolution of the sort of the advanced tandem Wadsworth mounting imaging spectrometer. The design results present that the advanced tandem Wadsworth mounting has high spatial resolution and spectral resolution in broadband. The design concept and theory can be adapted to a wide range of relative applications in many other wavebands.

The research was supported by the National Natural Science Foundation of China (NSFC) under grant 41074126.

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