

Tripartite entanglement sudden death in Yang-Baxter systems

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Abstract In this paper, we derive unitary Yang-Baxter $\check{R}(\theta, \varphi)$ matrices from the $8 \times 8 \mathbb{M}$ matrix and the $4 \times 4 M$ matrix by Yang-Baxterization approach, where \mathbb{M}/M is the image of the braid group representation. In Yang-Baxter systems, we explore the evolution of tripartite negativity for three qubits Greenberger-Horne-Zeilinger (GHZ)-type states and W-type states and investigate the evolution of the bipartite concurrence for 2 qubits Bell-type states. We show that tripartite entanglement sudden death (ESD) and bipartite ESD all can happen in Yang-Baxter systems and find that ESD all are sensitive to the initial condition. Interestingly, we find that in the Yang-Baxter system, GHZ-type states can have bipartite entanglement and bipartite ESD, and find that in some initial conditions, W-type states have tripartite ESD while they have no bipartite Entanglement. It is worth noting that the meaningful parameter φ has great influence on bipartite ESD for two qubits Bell-type states in the Yang-Baxter system.

Keywords Tripartite entanglement · Entanglement sudden death · Quantum Yang-Baxter equation

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1 Introduction

Entanglement is one of the most intriguing features of quantum mechanics [1], hence to understand its properties and dynamics is of paramount relevance for a number of applications in modern physics. Furthermore, quantum cryptography, quantum information processing, and quantum measurement are examples of branches of physics in which entanglement plays an essential role [2,3]. But despite its fundamental and practical importance and much work in the subject, there are many aspects of entanglement, especially multipartite entanglement, that are in need of further study [4]. Quantum correlations in multipartite systems have a much richer structure than in the bipartite case [5,6], and may be used to implement improved information processing and distributed quantum computing, as well as to reveal higher-nonlocality features of quantum mechanics. In particular, tripartite entanglement is a resource to increase the security of quantum cryptography [7] and it finds applications in quantum secret sharing [8] and quantum cloning [9]. In turn, the generation of tripartite entanglement for qubits has been analyzed for several physical systems including cavity quantum electrodynamics [10], as well as trapped ion quantum computers [11] and magnetic systems in a ring geometry [12].

A major challenge facing experimental implementations of quantum computation, sensing, and communication is decoherence, unwanted interactions between the system and environment. Recently Yu and Eberly [13] investigated the time evolution of entanglement (quantified using the concurrence) of a bipartite qubits system undergoing various modes of decoherence. Remarkably, they find that even when there is no interaction (either directly or through a correlated environment), there are certain states whose entanglement decays exponentially with time, while for other closely related states, the entanglement vanishes completely in a finite time. This phenomenon, called entanglement sudden death (ESD), has been well explored in the case of bipartite systems and there are a number of studies looking at ESD in multipartite systems [14–23].

Yang-Baxter equation (YBE) was originated from solving the δ -function interaction model by Yang [24,25] and statistical models by Baxter [26,27], respectively. It was later introduced to solve many quantum integrable models by Faddeev [28] and Leningrad scholars. Recently, the YBE has been introduced to the field of quantum information and quantum computation. In a series of papers, it has been shown that YBE has a deep connection with topological quantum computation and entanglement swapping [29–37]. Unitary solution of the braided Yang-Baxter (i.e., the braid group relation) and unitary solutions of the quantum Yang-Baxter equation(QYBE) can often be identified with universal quantum gates [38,39]. This provides a novel way to study the quantum entanglement via YBE. In Ref. [40], the authors pointed out that YBE can be tested in terms of quantum optics. In a very recent work [37], it is found that any pure two-qudit entangled state can be achieved by a universal Yang-Baxter Matrix assisted by local unitary transformations.

We have studied the entanglement of three qubits in a new Yang-Baxter system in our recent paper [41]. we have also investigated bipartite Entanglement sudden death in constructed Yang-Baxter systems in another paper [42]. In this paper, We mainly explore tripartite Entanglement sudden death in Yang-Baxter systems where we take

the unitary Yang-Baxter matrix $\check{R}(\theta, \varphi)$ as the evolution operator $U(t)$ directly. In Sect. 2, Yang-Baxter $\check{R}(\theta, \varphi)$ matrices are constructed from the $8 \times 8 \mathbb{M}$ matrix and the $4 \times 4 M$ matrix which both satisfy extraspecial 2-groups algebra relations. In Sect. 3, we study the ESD of three qubits GHZ-type states in the Yang-Baxter system. In Sect. 4, the ESD of three qubits W-type states in the Yang-Baxter system is explored. Bipartite ESD of two qubits Bell-type states is also presented in Sect. 5. It shows that tripartite entanglement sudden death (ESD) and bipartite ESD all can happen in Yang-Baxter systems and shows that ESD all are sensitive to the initial condition. Interestingly, we find that three qubits GHZ-type states have bipartite entanglement and bipartite ESD in the Yang-Baxter system, and find that in some initial conditions, three qubits W-type states have tripartite ESD while they have no bipartite Entanglement after their evolution in the Yang-Baxter system. We also find that the meaningful parameter φ has great influence on bipartite ESD in some initial conditions. The results are summarized in the last section.

2 Yang-Baxter $\check{R}(\theta, \varphi)$ matrices are constructed from the \mathbb{M}/M matrices

We have constructed the new $8 \times 8 \mathbb{M}$ matrix from the $4 \times 4 M$ matrix and have investigated its properties and applications in a recent paper [41]. Here in order to make this paper to be self-contained, we briefly review it as follows:

For 4×4 Yang-Baxter systems of two qubits, we know that the rational solution of the YBE can be expressed as $\check{R}_{i,i+1}(\theta, \varphi) = \sin \theta I_i \otimes I_{i+1} + \cos \theta M_{i,i+1}$ [35], where θ is the spectral parameter and $M_{i,i+1}$ is an operator of braid group which can be expressed in the form of spin operators $S_i^+ = S_i^1 + iS_i^2$ and $S_i^- = S_i^1 - iS_i^2$ as follows

$$M_{i,i+1} = e^{-i\varphi} S_i^+ S_{i+1}^+ - e^{i\varphi} S_i^- S_{i+1}^- + S_i^+ S_{i+1}^- - S_i^- S_{i+1}^+. \quad (1)$$

In terms of the standard basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ of two qubits, the $M_{i,i+1}$ matrix is of the following form:

$$M_{i,i+1} = \begin{pmatrix} 0 & 0 & 0 & e^{-i\varphi} \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -e^{i\varphi} & 0 & 0 & 0 \end{pmatrix}. \quad (2)$$

We constructed $8 \times 8 \mathbb{M}$ matrix for three qubits from the $4 \times 4 M$ matrix [41], \mathbb{M} can be expressed by using spin operators:

$$\mathbb{M} = 2S_{i+1}^3 \left(e^{-i\varphi} S_i^+ S_{i+2}^+ - e^{i\varphi} S_i^- S_{i+2}^- + S_i^+ S_{i+2}^- - S_i^- S_{i+2}^+ \right) \quad (3)$$

Write in terms of the basis of three qubits $\{|000\rangle, |001\rangle, |010\rangle, |011\rangle, |100\rangle, |101\rangle, |110\rangle, |111\rangle\}$, the 8×8 matrix \mathbb{M} is

$$\mathbb{M} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & e^{-i\varphi} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -e^{-i\varphi} \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -e^{i\varphi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{i\varphi} & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \quad (4)$$

Here \mathbb{M}/M is the image of the braid group representation, and the $8 \times 8 \mathbb{M}$ matrix and the $4 \times 4 M$ matrix both satisfy the extraspecial 2-groups relations [43]:

$$\begin{aligned} M_{i,i+1}^2 &= \alpha I \\ M_{i,i+1}M_{i+1,i+2}M_{i,i+1} &= M_{i+1,i+2} \\ M_{i+1,i+2}M_{i,i+1}M_{i+1,i+2} &= M_{i,i+1}. \end{aligned} \quad (5)$$

It is known that a unitary solution of the YBE can be found via Yang-Baxterization on the solution of the extraspecial 2-groups relations. The Yang-Baxterization of the extraspecial 2-group operator M is [35]:

$$\check{R}(x) = \rho(x)(\mathcal{I} + G(x)M). \quad (6)$$

where $\rho(x)$ and $G(x)$ are some functions of x to be determined, $\mathcal{I} = I_i \otimes I_{i+1} \otimes I_{i+2}$ is identity matrix. One can choose appropriate $\rho(x)$ and $G(x)$ so that $\check{R}(x)$ is unitary. The unitary \check{R} -matrix satisfies the YBE which is of the form,

$$\check{R}_{12}(x)\check{R}_{23}(xy)\check{R}_{12}(y) = \check{R}_{23}(y)\check{R}_{12}(xy)\check{R}_{23}(x), \quad (7)$$

where multiplicative parameters x and y are known as the spectral parameters. In order to make $\check{R}(x)$ a unitary matrix, $\check{R}^\dagger(x)$ should be equal to the inverse $\check{R}^{-1}(x)$. In this way, we obtain that $\check{R}(x) = \frac{x+x^{-1}}{2} \left(\mathcal{I} + \frac{x-x^{-1}}{x+x^{-1}} M \right)$.

In this paper we introduce a new variable parameter θ as $\frac{x-x^{-1}}{2} = \frac{\cosh \theta}{\sqrt{\cosh 2\theta}}$ and $\frac{x+x^{-1}}{2} = \frac{\sinh \theta}{\sqrt{\cosh 2\theta}}$, the matrix $\check{R}(x)$ can be rewritten as $\check{R}(\theta, \varphi) = \frac{\sinh \theta}{\sqrt{\cosh 2\theta}} \mathcal{I} + \frac{\cosh \theta}{\sqrt{\cosh 2\theta}} M$.

We express $\check{R}^{(3)}(\theta, \varphi)$ corresponding to three qubits in terms of the basis of three qubits $\{|000\rangle, |001\rangle, |010\rangle, |011\rangle, |100\rangle, |101\rangle, |110\rangle, |111\rangle\}$ as

$$\check{R}^{(3)}(\theta, \varphi) = \begin{pmatrix} p & 0 & 0 & 0 & 0 & e^{-i\varphi}q & 0 & 0 \\ 0 & p & 0 & 0 & q & 0 & 0 & 0 \\ 0 & 0 & p & 0 & 0 & 0 & 0 & -e^{-i\varphi}q \\ 0 & 0 & 0 & p & 0 & 0 & -q & 0 \\ 0 & -q & 0 & 0 & p & 0 & 0 & 0 \\ -e^{i\varphi}q & 0 & 0 & 0 & 0 & p & 0 & 0 \\ 0 & 0 & 0 & q & 0 & 0 & p & 0 \\ 0 & 0 & e^{i\varphi}q & 0 & 0 & 0 & 0 & p \end{pmatrix} \quad (8)$$

we express $\check{R}^{(2)}(\theta, \varphi)$ corresponding to two qubits in terms of the basis of two qubits $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ as

$$\check{R}^{(2)}(\theta, \varphi) = \frac{1}{\sqrt{\cosh 2\theta}} \begin{pmatrix} p & 0 & 0 & e^{-i\varphi}q \\ 0 & p & q & 0 \\ 0 & -q & p & 0 \\ -e^{i\varphi}q & 0 & 0 & p \end{pmatrix}. \quad (9)$$

where $p = \frac{\cosh \theta}{\sqrt{\cosh 2\theta}}$, $q = \frac{\sinh \theta}{\sqrt{\cosh 2\theta}}$.

3 ESD of the GHZ-type states in the Yang-Baxter system

We use the Wootters concurrence [44] to measure bipartite entanglement, defined as

$$C_{AB}(t) = \max \left\{ 0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4} \right\} \quad (10)$$

where $\{\lambda_i\}$ are the eigenvalues of the matrix $\rho_{AB} \left(\sigma_y^A \otimes \sigma_y^B \right) \rho_{AB}^* \left(\sigma_y^A \otimes \sigma_y^B \right)$ in decreasing order, with ρ^* denoting the complex conjugate of ρ and $\sigma_y^{A/B}$ are the Pauli matrices for atoms A and B. For three qubits system, we can also use the concurrence C_{AB} to measure bipartite entanglement between qubits A and B after partial trace over another qubit.

When we consider mixed states, the complete classification of tripartite entanglement is still an open problem. In this paper, in order to measure the tripartite entanglement, according to the classification of entanglement in three-qubit systems in [45], we employ the tripartite negativity of a state ρ :

$$N_{ABC}(\rho) = (N_{A-BC} N_{B-AC} N_{C-AB})^{\frac{1}{3}} \quad (11)$$

where $N_{A-BC} = -2\sum_i \sigma_i (\rho^{TI})$, $\sigma_i (\rho^{TI})$ being negative eigenvalues of ρ^{TI} , the partial transpose of ρ with respect to subsystem I, $\langle k_I, j_{JK} | \rho^{TI} | i_I l_{JK} \rangle = \langle i_I, j_{JK} | \rho | k_I l_{JK} \rangle$, with I = A, B, C, and JK = BC, AC, AB, respectively.

As Sect. 2, we have got the unitary Yang-Baxter $\check{R}^{(3)}(\theta, \varphi)$ matrix for three qubits system. Here we let the parameter φ be time-independent while the spectral parameter θ is in a time-dependent fashion $\theta = \kappa t$ (κ is a constant coefficient). Next we will

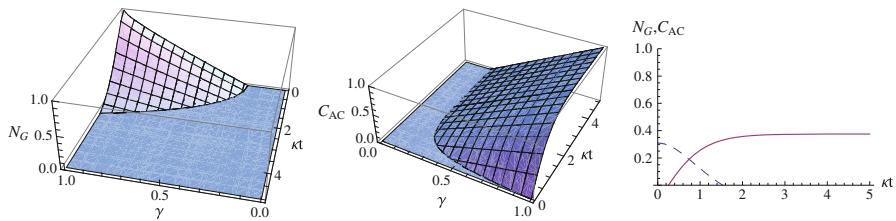


Fig. 1 Demonstration of tripartite entanglement sudden death and bipartite entanglement sudden death of GHZ-type states. *Left* The tripartite negativity N_G as a function of the initial state parametrized by γ , and κt . *Center* The bipartite entanglement C_{AC} versus the time κt and the parameter γ . *Right* While we let $\gamma = \frac{1}{2}$, the bipartite entanglement C_{AC} versus the time κt (solid line), the tripartite negativity N_G versus the time κt (dashed line)

explore the effect of the unitary $8 \times 8 \check{R}^{(3)}(\theta, \varphi)$ matrix on the entanglement of the following three qubits GHZ-type states,

$$\rho = \frac{1-\gamma}{8} \mathcal{I} + \gamma \rho_G, \quad (12)$$

where $\rho_G = (|000\rangle + |111\rangle)(\langle 000| + \langle 111|)$ and $(0 < \gamma < 1)$. we take the unitary Yang-Baxter matrix $\check{R}(\theta, \varphi)$ as the evolution operator $U(t)$ directly. Then for initial GHZ-type states as Eq. 12, we get the density matrix,

$$\rho(t) = U(t)\rho U(t)^\dagger = \check{R}(\theta, \varphi)\rho \check{R}(\theta, \varphi)^\dagger. \quad (13)$$

According to Eqs. 8, 11–13, by calculation, we get tripartite negativity of the state $\rho(t)$,

$$N_G(\rho(t)) = \sqrt[3]{\frac{1}{16}(5\gamma - 1)^2 \left(\frac{\gamma}{\cosh 2\kappa t} - \frac{1-\gamma}{4} \right)} \quad (14)$$

we find that only when $\gamma > \frac{1}{5}$, the tripartite negativity exists. As shown in Fig. 1, tripartite ESD occurs after the evolution of the GHZ-type states in the Yang-Baxter system. It shows that tripartite ESD is sensitive to the initial condition, only when $\frac{1}{5} < \gamma < 1$, the ESD for tripartite negativity occurs. According to Eqs. 8, 10, 12, 13 we can measure bipartite entanglement C between any two qubits of ρ after partial trace over the third qubit. We get $C_{AB} = C_{BC} = 0$ and $C_{AC} = \gamma \tanh 2\kappa t - \frac{1-\gamma}{4}$. It is worth noting that the initial GHZ-type states have no bipartite entanglement, but after their evolution in the Yang-Baxter system, we get bipartite entanglement between qubit A and qubit C for GHZ-type states. We also plot bipartite entanglement C_{AC} in the center figure of the Fig. 1, we can see that bipartite entanglement C_{AC} also exists only when $\frac{1}{5} < \gamma < 1$. Interestingly, from the right figure of the Fig. 1, we can see that after a certain time there is no longer tripartite entanglement, but bipartite entanglement for GHZ-type states.

4 ESD of the W-type states in the Yang-Baxter system

Unlike GHZ state, the W state $|W\rangle = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle)$, retains a high degree of (bipartite) entanglement upon partial trace over one qubit. Thus, W-type states allow some comparison between tripartite and bipartite entanglement evolution. Then we explore the effect of the unitary $8 \times 8 \check{R}^{(3)}(\theta, \varphi)$ matrix on the entanglement of the following three qubits W-type states,

$$\rho = \frac{1-\gamma}{8}\mathbb{I} + \gamma\rho_W = \frac{1-\gamma}{8}\mathbb{I} + \gamma|W\rangle\langle W|, \quad (15)$$

According to Eqs. 8, 11, 13, 15, by calculation, we get tripartite negativity of the state $\rho(t)$

$$N_W(\rho(t)) = \frac{1}{12}\sqrt[3]{\left((3+8\sqrt{2})\gamma - 3\right)n_+n_-} \quad (16)$$

where $n_{\pm} = 3\gamma + 4\gamma\sqrt{\frac{4}{\cosh^2 2\kappa t} + 4 \pm 4\frac{\tanh 2\kappa t}{\cosh 2\kappa t} + \tanh^2 2\kappa t} - 3$, we find that only when $\gamma > \frac{3}{3+8\sqrt{2}} \approx 0.21$, the tripartite negativity exist. As shown in Fig. 2, tripartite ESD also occurs after the evolution of W-type states in the Yang-Baxter system, but the ESD of W-type states is very sensitive to the initial condition. From the left figure of the Fig. 2, we can clearly see that tripartite ESD occurs only when the parameter γ changes in a very small range (from 0.21 to 0.25), and the value of tripartite negativity N_W is very small. According to Eqs. 8, 10, 13, 15 we also measure bipartite entanglement C between any two qubits of ρ after partial trace over the third qubit. We get

$$C_{AB} = \frac{\gamma}{3 \cosh 2\kappa t} + \frac{\gamma}{3}(1 - \tanh 2\kappa t) - 2\sqrt{\left(\frac{3+\gamma}{12} + \frac{\gamma}{3} \tanh 2\kappa t\right)\left(\frac{3-\gamma}{12} - \frac{\gamma}{6 \tanh 2\kappa t}\right)} \quad (17)$$

$$C_{BC} = \frac{\gamma}{3} + \frac{\gamma}{3 \cosh 2\kappa t} - 2\sqrt{\left(\frac{3+\gamma}{12} - \frac{\gamma}{3} \tanh 2\kappa t\right)\left(\frac{3-\gamma}{12} - \frac{\gamma}{6 \tanh 2\kappa t}\right)} \quad (18)$$

$$C_{AC} = \frac{2\gamma}{3 \cosh 2\kappa t} - 2\sqrt{\frac{(3-\gamma)^2}{144} - \frac{\gamma^2}{36 \cosh^2 2\kappa t}} \quad (19)$$

In Fig. 3, we can see that bipartite ESD for any pair of qubits in W-type states occurs. It also shows that all bipartite ESD are sensitive to the initial condition. For bipartite entanglement C_{BC} , only when $0.4 < \gamma < 0.5$, C_{BC} exists and bipartite ESD occurs. For bipartite entanglement C_{AC} , only when $0.55 < \gamma < 1$, C_{AC} exists and bipartite ESD occurs. For bipartite entanglement C_{AB} , only when $0.55 < \gamma < 1$, C_{AB} exists

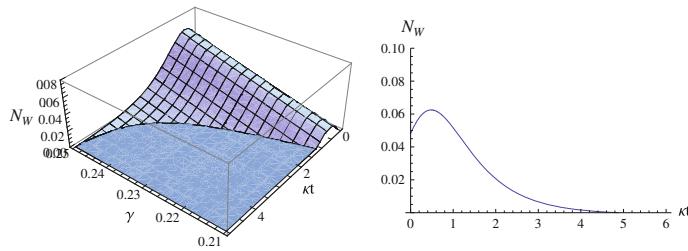


Fig. 2 Demonstration of tripartite entanglement sudden death of W-type states. *Left* The tripartite negativity N_W as a function of the initial state parametrized by γ , and κt . *Right* The tripartite negativity N_W versus the time κt while $\gamma = 0.25$

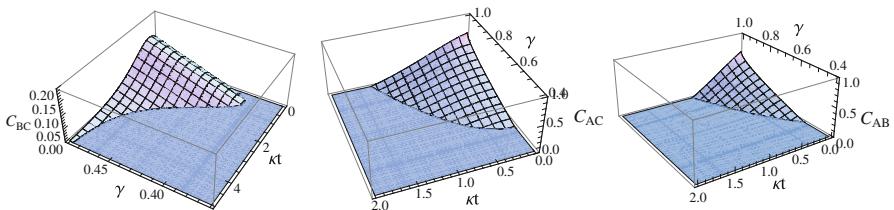


Fig. 3 Demonstration of bipartite entanglement sudden death of W-type states. The *Left*, *Center* and *Right* corresponds to bipartite entanglement C_{BC} , C_{AC} and C_{AB} versus parameters γ , κt , respectively

and bipartite ESD occurs. So it is interesting that when $0.21 < \gamma < 0.4$, tripartite W-type entanglement exists, even when there is no longer any bipartite entanglement in the system, and tripartite W-type ESD occurs when $0.21 < \gamma < 0.25$. In addition, when $0.4 < \gamma < 1$, ESD occurs for bipartite entanglement, but not for tripartite entanglement.

5 ESD of the 2 qubits Bell states in the Yang-Baxter system

As Sect. 2, we have got the unitary Yang-Baxter $\check{R}^{(2)}(\theta, \varphi)$ matrix for two qubits system. Then we also explore the evolution of two qubits states in the 4×4 Yang-Baxter system. As is known, Bell state $|B\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ is maximally entangled two qubits state. Next we start with Bell-type states as follows,

$$\rho = \frac{1-\gamma}{8}\mathbb{I} + \gamma\rho_B = \frac{1-\gamma}{8}\mathbb{I} + \gamma|B\rangle\langle B|, \quad (20)$$

According to Eqs. 9, 10, 13, 20, by calculation, we get bipartite concurrence of the state $\rho(t)$

$$C_B(\rho(t)) = \gamma \sqrt{\frac{1}{2}(1 - \cos \varphi) \tanh^2 2\kappa t + \frac{1}{\cosh^2 2\kappa t}} - \frac{1-\gamma}{2} \quad (21)$$

As shown in Fig. 4, bipartite ESD also occurs after the evolution of Bell-type states in the Yang-Baxter system. Interestingly, it shows that bipartite ESD is sensitive not

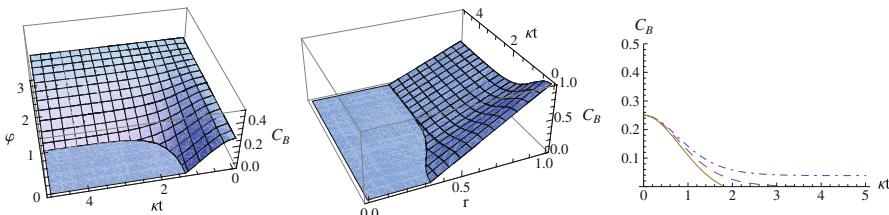


Fig. 4 Demonstration of bipartite entanglement sudden death of Bell-type states in the Yang-Baxter system. *Left* The bipartite entanglement C_B as a function of parameters φ , and κt while we let $\gamma = \frac{1}{2}$. *Center* The bipartite entanglement C_B as a function of parameters γ , and κt while we let $\cos \varphi = \frac{1}{2}$. *Right* While we let $\gamma = \frac{1}{2}$, the bipartite entanglement C_B versus κt for different parameters φ : $\cos \varphi = \frac{1}{3}$ (dotdashed line), $\cos \varphi = \frac{1}{2}$ (dashed line), $\cos \varphi = \frac{2}{3}$ (solid line)

only to the initial condition, but also to the parameter φ . It is known that the parameter φ originates from the q-deformation of the braiding operator U with $q = e^{-i\varphi}$ [46, 47], and φ may have a physical significance of magnetic flux [48, 49]. In some papers, it has been shown that the parameter φ has a deep connection with the Berry phase [35, 39]. From the right figure and the left figure of the Fig. 4, we can clearly see that the parameter φ has a great influence on bipartite ESD when the initial condition is determinate. It is obvious that bipartite ESD occurs only when the parameter φ changes in a certain range (ranges from $\frac{1}{3}$ to 1). It shows that bipartite ESD occurs earlier when the value of $\cos \varphi$ is bigger.

6 Summary

In summary, we derive unitary Yang-Baxter $\check{R}(\theta, \varphi)$ matrices from the $8 \times 8 M$ matrix and the $4 \times 4 M$ matrix by Yang-Baxterization approach, where the parameter φ is time-independent and the parameter θ is in a time-dependent fashion $\theta = \kappa t$. Then we explore the evolution of three qubits Greenberger-Horne-Zeilinger (GHZ)-type states and W-type states in the Yang-Baxter system. We show that tripartite ESD can happen in the Yang-Baxter system and show that the ESD is sensitive to the initial condition. Interestingly, we find that GHZ-type states also have bipartite entanglement in the Yang-Baxter system, and find that in some initial conditions, after a certain time there is no longer tripartite entanglement, but bipartite entanglement. It is worth noting that in some initial conditions, W-type states have tripartite ESD while they have no bipartite Entanglement after their evolution in the Yang-Baxter system. We also study the evolution of two qubits Bell-type states in the 4×4 Yang-Baxter system. Specially, it shows that bipartite ESD is sensitive not only to the initial condition, but also to the parameter φ . The meaningful parameter φ has great influence on bipartite ESD in some initial conditions.

In this paper, we extend the study of our previous work [42] to tripartite Yang-Baxter systems. The extension to the case of tripartite systems is not trivial due to the higher degree of complexity of the problem and also for the peculiar structure of tripartite entanglement. These studies of this paper and the previous paper can be considered

as the onset of ESD in Yang-Baxter systems. It is also interesting and significant to investigate ESD in multipartite($N > 3$) Yang-Baxter systems, which we shall investigate subsequently.

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