# Third-order coma-free point in two-mirror telescopes by a vector approach 

Baichuan Ren, ${ }^{12, *}$ Guang Jin, ${ }^{1}$ and Xing Zhong ${ }^{1}$<br>${ }^{1}$ Changchun Institute of Optics, Fine Mechanics and Physics, Chinese Academy of Sciences, Changchun, Jilin 130033, China<br>${ }^{2}$ Graduate School of the Chinese Academy of Sciences, Beijing 100039, China<br>*Corresponding author: renbc@yahoo.cn

Received 6 January 2011; revised 13 April 2011; accepted 20 May 2011; posted 26 May 2011 (Doc. ID 140217); published 13 July 2011


#### Abstract

In this paper, two-mirror telescopes having the secondary mirror decentered and/or tilted are considered. Equations for third-order coma are derived by a vector approach. Coma-free condition to remove misalignment-induced coma was obtained. The coma-free point in two-mirror telescopes is found as a conclusion of our coma-free condition, which is in better agreement with the result solved by Wilson using Schiefspiegler theory. © 2011 Optical Society of America

OCIS codes: $\quad 080.0080,080.1010,110.6770,220.1000$.


## 1. Introduction

The two-mirror telescopes are widely employed in astronomy research. Before any telescope can go into operation, it must be aligned with sufficient accuracy. A perfectly collimated two-mirror telescope will produce no coma on axis. If the secondary mirror is decentered and/or tilted, field-constant coma will be introduced. Freedom from coma is the fundamental condition for the alignment of modern telescopes [1,2]. For any two-mirror telescope system, there will be a point somewhere on the primary mirror axis rotating the secondary mirror about which no coma will be introduced. This is called the "coma-free point" (CFP) [3-5]. This property is used in collimating slow focal ratio two-mirror telescopes purely by tilting the secondary mirror. Although the CFP has been found by some means or other, we concern ourselves with the nature of the decentering aberration function.

The general nature of the decentering aberration function was analyzed in a fundamental paper by Shack and Thompson [6]. Shack established a vector expression for the wave aberration expansion that

[^0]made use of a vector operation to give a concise description of the third-order aberration field performance in a perturbed telescope. This earlier work by Shack has been developed by Thompson, who extended the vector formulation for wavefront aberration dependence on field and aperture up to fifth order $[7,8]$. The misalignment-induced aberration fields of telescopes are discussed using Shack's vector expression by some earlier works [ 9,10$]$. Particular attention is given to astigmatism when misalign-ment-induced coma is removed. Noethe derived the analytical expressions for the CFP of two-mirror systems with stop positions anywhere [11]. Rakich had a paper on this subject using Plate Diagram approach [12]. However, we solve this problem by use of the vector aberration theory conceived by Shack.

Without fast numerical apertures or significant field sizes, the third-order approximation is adequate to describe the aberrations of telescopes. In this work, equations for third-order coma in perturbed two-mirror telescopes are derived via a vector approach conceived by Shack. Two sets of third-order coma, introduced by the base sphere and aspheric departure of the secondary mirror, respectively, are given in Section 2. These two sets can cancel each other as a result of coma-free condition presented in Section 3. Thus, the coma due to translation of
secondary mirror and the coma due to rotation of secondary mirror can cancel out. The CFP derived from our coma-free condition is proved to be identical to the result found by Wilson [4].

## 2. Misalignment-Induced Field-Constant Coma

## A. Vector Approach

The wavefront aberration function can be expanded as a scalar function of two vectors, normalized aperture vector $\vec{\rho}$ in exit pupil plane, which specifies any point in system aperture, and normalized field vector $\vec{H}$, which specifies any point in the system field of view. The wave aberration term for third-order coma of $j$ th surface contributions is [4]

$$
\begin{equation*}
W_{\text {coma }, j}=W_{131, j} H \rho^{3} \cos \phi \tag{1}
\end{equation*}
$$

where $\phi$ is the azimuth angle of the plane containing the ray and the principal ray in the image forming the wavefront [13,14]. The corresponding vector form is

$$
\begin{equation*}
W_{\text {coma }, j}=W_{131, j}(\vec{H} \cdot \vec{\rho})(\vec{\rho} \cdot \vec{\rho}) \tag{2}
\end{equation*}
$$

Third-order coma of the system is obtained by summing up the surface contributions:

$$
\begin{equation*}
W_{\text {coma }}=\sum_{j} W_{\text {coma }, j} \tag{3}
\end{equation*}
$$

In a perturbed system, each optical surface introduces aberration contributions of the same form as the surface contributions in a rotationallysymmetric system, but the component field vectors $\vec{H}_{j}$ are measured from centers of symmetry that do not coincide for each surface contribution. Furthermore, two independent sets of aberrations are introduced by each surface: one by the base sphere and the other by the aspheric departure of the surface. As we see in Subsection 2.B, centers of symmetry do not coincide for the spherical and aspheric contributions for each surface. Aspheric contribution to the aberration field associated with a surface must be treated separately from spherical contribution when analyzing the perturbed system. These two sets of aberration terms can be combined vectorially to yield the final aberration contribution for the surface. The system aberration description obtained by adding the surface contributions together must be described as a function of a field vector $\vec{H}$, which is measured from an arbitrarily chosen field center. If the displacement of the surface component center of symmetry from the field center is measured by the vector $\vec{\sigma}_{j}^{o}$ and $\vec{\sigma}_{j}^{*}$, then

$$
\begin{align*}
\vec{H}_{j}^{o} & =\vec{H}-\vec{\sigma}_{j}^{o}  \tag{4a}\\
\vec{H}_{j}^{*} & =\vec{H}-\vec{\sigma}_{j}^{*} \tag{4b}
\end{align*}
$$

as illustrated in Fig. 1. The superscript $o$ denotes the base sphere component and the superscript $*$ denotes the aspheric component in this work.


Fig. 1. Field displacement vectors for surface contribution. $\vec{\sigma}_{j}^{o}$ is for the center of symmetry for the base sphere, and $\vec{\sigma}_{j}^{*}$ is for the center of symmetry for the aspheric departure of the $j$ th surface.

The summation can then be carried out as a vector operation, giving the resultant system aberrations as a function of the field vector $\vec{H}$ and terms involving the displacement vectors $\vec{\sigma}_{j}^{o}$ and $\vec{\sigma}_{j}^{*}$. We can find that

$$
\begin{align*}
W_{\text {coma }, j}^{o} & =W_{131, j}^{o}\left(\vec{H}_{j}^{o} \cdot \vec{\rho}\right)(\vec{\rho} \cdot \vec{\rho}) \\
& =W_{131, j}^{o}\left[\left(\vec{H}-\vec{\sigma}_{j}^{o}\right) \cdot \vec{\rho}\right](\vec{\rho} \cdot \vec{\rho}),  \tag{5a}\\
W_{\text {coma }, j}^{*}= & W_{131, j}^{*}\left(\vec{H}_{j}^{*} \cdot \vec{\rho}\right)(\vec{\rho} \cdot \vec{\rho}) \\
& =W_{131, j}^{*}\left[\left(\vec{H}-\vec{\sigma}_{j}^{*}\right) \cdot \vec{\rho}\right](\vec{\rho} \cdot \vec{\rho}),  \tag{5b}\\
W_{\text {coma }, j}= & W_{\text {coma }, j}^{o}+W_{\text {coma }, j}^{*} \\
= & W_{131, j}^{o}\left[\left(\vec{H}-\vec{\sigma}_{j}^{o}\right) \cdot \vec{\rho}\right](\vec{\rho} \cdot \vec{\rho}) \\
& +W_{131, j}^{*}\left[\left(\vec{H}-\vec{\sigma}_{j}^{*}\right) \cdot \vec{\rho}\right](\vec{\rho} \cdot \vec{\rho}) \\
= & \left(W_{131, j}^{o}+W_{131, j}^{*}\right)(\vec{H} \cdot \vec{\rho})(\vec{\rho} \cdot \vec{\rho}) \\
& -\left[\left(W_{131, j}^{o} \vec{\sigma}_{j}^{o}+W_{131, j}^{*} \vec{\sigma}_{j}^{*}\right) \cdot \vec{\rho}\right](\vec{\rho} \cdot \vec{\rho}) \\
= & W_{131, j}(\vec{H} \cdot \vec{\rho})(\vec{\rho} \cdot \vec{\rho}) \\
& -\left[\left(W_{131, j}^{o} \vec{\sigma}_{j}^{o}+W_{131, j}^{*} \vec{\sigma}_{j}^{*}\right) \cdot \vec{\rho}\right](\vec{\rho} \cdot \vec{\rho}) . \tag{6}
\end{align*}
$$

From Eq. (6), third-order coma is organized into two subgroups: the first term $W_{131, j}(\vec{H} \cdot \vec{\rho})(\vec{\rho} \cdot \vec{\rho})$ is the linear coma for surface contributions and the second term $-\left[\left(W_{131, j}^{o} \vec{\sigma}_{j}^{o}+W_{131, j}^{*} \vec{\sigma}_{j}^{*}\right) \cdot \vec{\rho}\right](\vec{\rho} \cdot \vec{\rho})$ is the constant coma surface contributions.

## B. Locating the Centers of Symmetry

The lateral misalignment of two-mirror telescope involves a displacement of the secondary mirror optical
axis without any significant change in the mirror spacing and takes the form of tilt and decenter of the secondary mirror with respect to the primary mirror, as shown in Fig. 2.

The aberration field contribution of a spherical surface is centered on the line connecting the center of the pupil, for that surface, with its center of curvature. When an aspheric is placed onto a surface, it provides an additional contribution to the aberration field at the image plane. The center of the aberration contribution due to the aspheric is along the line connecting the vertex of the aspheric cap with the center of the pupil, respectively [6]. Consider the misalignment case illustrated in Fig. 2, where the secondary mirror is tilted and decentered in the meridian plane. Following the descriptions given in [6], the field centers of the spherical and aspheric components of the primary and secondary mirrors are located in Fig. $\underline{2}$. The stop is at the primary mirror; no aspheric contribution of the primary mirror is present. The spherical field center of primary mirror forms at $c_{1}$, where the line through the curvature center of the secondary mirror $\left(C_{2}\right)$ and the image field center of the primary mirror $\left(I_{1}\right)$ intersect with the final image plane $\left(I_{2}\right)$. As the optical axis ray, by definition, passes through the center of its associated object/image plane and the entrance/exit pupil for each surface in the system [7,15], it intersects $I_{2}$ at $c_{1}$, and $c_{1}$ is arbitrarily chosen as the center of field. While coma induced by tilt and decenter is zero, $c_{1}$ is also the "axial point," which does not coincide with the mechanical axis (pointing error in two-mirror telescopes).

The line through $C_{2}$ and the center of entrance pupil center $\left(E_{1}\right)$ intersects $I_{2}$ at the point $c_{2}$, which is the spherical field center symmetry of the secondary mirror. The exit pupil center $E_{2}$ of secondary mirror is also located on this line. The line from $E_{2}$ through the aspheric vertex of the secondary mirror $\left(V_{2}\right)$ locates the point $c_{3}$, which is the field center of symmetry of the aspheric component of secondary mirror.


Fig. 2. Tilted and decentered two-mirror telescope: the stop is at the primary mirror; tilt and decenter are both in the meridian plane.

## C. Third-Order Coma

As described in Subsection 2.B, $c_{1}$ is aberration center of symmetry for the primary mirror and the center of field arbitrarily chosen, i.e., $\vec{\sigma}_{1}^{o}=\vec{\sigma}_{1}^{*}=0$ for the primary mirror, and the third-order coma for primary mirror contribution is then

$$
\begin{align*}
W_{\text {coma }, 1} & =\left(W_{131,1}^{o}+W_{131,1}^{*}\right)(\vec{H} \cdot \vec{\rho})(\vec{\rho} \cdot \vec{\rho}) \\
& =W_{131,1}(\vec{H} \cdot \vec{\rho})(\vec{\rho} \cdot \vec{\rho}) \tag{7a}
\end{align*}
$$

The stop is at primary mirror, so that the primary mirror aspheric coefficient does not affect the aberrations dependence on field, i.e., $W_{131,1}^{*}=0$.

Both the centers of symmetry for base sphere and aspheric of secondary mirror contributions do not coincide with that for the primary mirror, which we chose as the field center. The third-order coma for the secondary mirror contribution is then given by

$$
\begin{align*}
W_{\text {coma }, 2}= & W_{131,2}(\vec{H} \cdot \vec{\rho})(\vec{\rho} \cdot \vec{\rho}) \\
& -\left[\left(W_{131,2}^{o} \vec{\sigma}_{2}^{o}+W_{131,2}^{*} \vec{\sigma}_{2}^{*}\right) \cdot \vec{\rho}\right](\vec{\rho} \cdot \vec{\rho}) \tag{7b}
\end{align*}
$$

Here, the $\vec{H}$ and sigma vectors in the image plane use the exit pupil of the two-mirror system and the lines through the middle of this pupil to the center of curvature and surface vertex of the secondary mirror for spherical and aspherical contributions, respectively.

The total third-order coma for a perturbed twomirror telescope is obtained by summing up primary mirror contribution Eq. (7a) and secondary mirror contribution Eq. (7b), as shown in Eq. (8):

$$
\begin{align*}
W_{\mathrm{coma}}= & W_{\mathrm{coma}, 1}+W_{\mathrm{coma}, 2} \\
= & \left(W_{131,1}+W_{131,2}\right)(\vec{H} \cdot \vec{\rho})(\vec{\rho} \cdot \vec{\rho}) \\
& -\left[\left(W_{131,2}^{o} \vec{\sigma}_{2}^{o}+W_{131,2}^{*} \vec{\sigma}_{2}^{*}\right) \cdot \vec{\rho}\right](\vec{\rho} \cdot \vec{\rho}) . \tag{8}
\end{align*}
$$

The first term in Eq. (8) is the same as third-order coma for a centered two-mirror telescope before tilt and decenter are imposed; the second term is the decentering coma, which is constant in both magnitude and direction over the field of view. $-\left(W_{131,2}^{o} \vec{\sigma}_{2}^{o}+\right.$ $\left.W_{131,2}^{*} \vec{\sigma}_{2}^{*}\right)$ is wave aberration coefficient for thirdorder constant coma, which contain base sphere contribution and aspheric contribution. If the coma is corrected in the centered telescope (e.g., RitcheyChrétien, Schwarzschild and Couder forms), then Eq. (8) reduces to a constant value of coma over the field.

The calculation of decentering coma in Eq. (8) is presented in Subsections 2.C. 1 and 2.C.2. Sign conventions will follow the standard optical practice [4], as illustrated in Fig. $\underline{3}$, where $P^{\prime}$ is the image side principal plane, $d_{1}$ is the intermirror distance, $f_{1}^{\prime}$ is the primary focal length, $f^{\prime}$ is the system focal length, $C_{2}$ is the curvature center of secondary mirror, $I_{1}$ is the focus point of primary mirror, $I_{2}$ is the position of the final image, $y_{1}$ is the paraxial aperture ray


Fig. 3. Gaussian optics of a Cassegrain two-mirror telescope.
height, and $s_{2}$ and $L$ are the object and image distances for the secondary mirror, respectively.

In our case, tilt and decenter are coplanar, and $\vec{\sigma}_{2}^{o}$ and $\vec{\sigma}_{2}^{*}$ are collinear. Here we just have to consider the sign of $\vec{\sigma}_{2}^{o}$ and $\vec{\sigma}_{2}^{*}$. The vector equations with $\vec{\sigma}_{2}^{o}$ and $\vec{\sigma}_{2}^{*}$ are expressed in scalar form.

## 1. Base Sphere Contribution

To simplify the calculation, we assume that the aspheric vertex $V_{2}$ of the secondary mirror is fixed, as shown in Fig. 4. The axis of the primary mirror intersects the final image plane at the point $o$.

Based on similar triangles theorem, we can derive:

$$
\begin{gather*}
o c_{1}=\frac{\left(L-s_{2}\right)\left(-\delta C_{2}\right)}{s_{2}-r_{2}},  \tag{9a}\\
o c_{2}=-\frac{\left(L+d_{1}\right)\left(-\delta C_{2}\right)}{d_{1}+r_{2}},  \tag{9b}\\
c_{1} c_{2}=o c_{2}-o c_{1} \\
=\left(\frac{L-s_{2}}{s_{2}-r_{2}}+\frac{L+d_{1}}{d_{1}+r_{2}}\right) \delta C_{2} \\
=\left[\frac{\left(L-r_{2}\right)\left(d_{1}+s_{2}\right)}{\left(s_{2}-r_{2}\right)\left(d_{1}+r_{2}\right)}\right] \delta C_{2} . \tag{10}
\end{gather*}
$$

The maximum field height is $u_{\mathrm{pr} 1} f^{\prime}$ (normalized to 1 ). $u_{\text {pr1 }}$ is the semiangular field in object space, and $f^{\prime}$ is the effective focal length. Normalizing $c_{1} c_{2}$, we can obtain the fractional displacement vector $\vec{\sigma}_{2}^{o}$ by

$$
\begin{equation*}
\vec{\sigma}_{2}^{o}=\frac{c_{1} c_{2}}{u_{\mathrm{pr} 1} f^{\prime}} . \tag{11}
\end{equation*}
$$

The conversion of Seidel coefficient for coma to wavefront aberration for third-order coma is given by

$$
\begin{equation*}
W_{131}^{\prime}=\frac{1}{2}\left(\frac{y_{1}}{y_{m 1}}\right)^{3}\left(\frac{\eta^{\prime}}{\eta_{m}^{\prime}}\right) \sum S_{\mathrm{II}} \cos \phi, \tag{12}
\end{equation*}
$$

where $y_{m 1}$ and $\eta_{m}^{\prime}$ are the normalized paraxial aperture ray height and normalized image height, respectively. $y_{1}$ and $\eta^{\prime}$ are the real paraxial aperture ray
height and real image height [13,14], respectively. $S_{\text {II }}$ is the Seidel coefficient for coma. Comparing Eq. (12) with Eq. (1), we obtain the wavefront aberration coefficients:

$$
\begin{equation*}
W_{131, j}=\frac{1}{2}\left(S_{\mathrm{II}}\right)_{j} . \tag{13}
\end{equation*}
$$

Then, we have

$$
\begin{align*}
W_{131,2}^{o} & =\frac{1}{2}\left(S_{\mathrm{II}}\right)_{2}^{0},  \tag{14a}\\
W_{131,2}^{*} & =\frac{1}{2}\left(S_{\mathrm{II}}\right)_{2}^{*} . \tag{14b}
\end{align*}
$$

Then, wave aberration coefficients for third-order constant coma can be written as

$$
\begin{align*}
& W_{131,2}^{o} \vec{\sigma}_{2}^{o}=\frac{1}{2}\left(S_{\mathrm{II}}\right)_{2}^{0} \vec{\sigma}_{2}^{o},  \tag{15a}\\
& W_{131,2}^{*} \vec{\sigma}_{2}^{*}=\frac{1}{2}\left(S_{\mathrm{II}}\right)_{2}^{*} \vec{\sigma}_{2}^{*}, \tag{15b}
\end{align*}
$$

which recombine existing aberration coefficients into new third-order constant coma coefficients. For small tilt and decenter effects acceptable in practice, the third-order formulae given above are extremely accurate. The Seidel coefficient for third-order coma of the secondary mirror is given by [4]

$$
\begin{equation*}
\left(S_{\mathrm{II}}\right)_{2}=\left(\frac{y_{1}}{f^{\prime}}\right)^{3}\left[-d_{1} \xi+\frac{f^{\prime}}{2}\left(m_{2}^{2}-1\right)-s_{\mathrm{pr} 1} \frac{L}{f^{\prime}} \xi\right] u_{\mathrm{pr} 1} \tag{16}
\end{equation*}
$$

where

$$
\begin{equation*}
\xi=\xi^{0}+\xi^{*}=\frac{\left(m_{2}+1\right)^{3}}{4}\left(\frac{m_{2}-1}{m_{2}+1}\right)^{2}+\frac{\left(m_{2}+1\right)^{3}}{4} b_{s 2} \tag{17}
\end{equation*}
$$

$b_{s 2}$ is the Schwarzschild (conic) constant, $s_{\mathrm{pr} 1}$ is the distance from the first surface to the entrance pupil, and $m_{2}$ is the magnification of the secondary mirror. The entrance pupil was defined as being at the primary mirror, i.e., $s_{\mathrm{pr} 1}=0$. Then, we get

$$
\begin{align*}
\left(S_{\mathrm{II}}\right)_{2}^{0} & =\left(\frac{y_{1}}{f^{\prime}}\right)^{3} \xi^{0}\left[-d_{1}+\frac{2 f^{\prime}}{m_{2}-1}\right] u_{\mathrm{pr} 1} \\
& =-\frac{1}{4}\left(\frac{y_{1}}{f^{\prime}}\right)^{3}\left(1-m_{2}^{2}\right)\left(d_{1}+L+f^{\prime}\right) u_{\mathrm{pr} 1}  \tag{18a}\\
\left(S_{\mathrm{II}}\right)_{2}^{*} & =-\left(\frac{y_{1}}{f^{\prime}}\right)^{3} \xi^{*} d_{1} u_{\mathrm{pr} 1} \\
& =-\frac{1}{4}\left(\frac{y_{1}}{f^{\prime}}\right)^{3}\left(m_{2}+1\right)^{3} b_{s 2} d_{1} u_{\mathrm{pr} 1} \tag{18b}
\end{align*}
$$

$\left(S_{\text {II }}\right)_{2}^{0}$ gives contribution due to a spherical surface, $\left(S_{I I}\right)_{2}^{*}$ gives contribution due to the aspheric form.


Fig. 4. Pure decenter of the center of curvature of the secondary mirror $C_{2}$.

Applying Eqs. (10), (18a), and (11) to Eq. (15a), we have

$$
\begin{align*}
-W_{131,2}^{o} \vec{\sigma}_{2}^{o} & =\frac{1}{8}\left(\frac{y_{1}}{f^{\prime}}\right)^{3}\left(1-m_{2}^{2}\right)\left(d_{1}+L+f^{\prime}\right) u_{\mathrm{pr} 1} \frac{c_{1} c_{2}}{u_{\mathrm{pr} 1} f^{\prime}} \\
& =-\frac{1}{8}\left(\frac{y_{1}}{f^{\prime}}\right)^{3}\left(1-m_{2}^{2}\right)\left(m_{2}+1\right) \delta C_{2} . \tag{19}
\end{align*}
$$

## 2. Aspheric Contribution

Similar to the manner used in Subsection 2.C.1., we assume that the center of curvature of the secondary mirror $C_{2}$ is unchanged, as shown in Fig. $\underline{5}$.

Using similar triangles theorem, we can obtain

$$
\begin{equation*}
\frac{\delta V_{2}}{c_{1} c_{3}}=\frac{-\left(s_{2}^{\prime}\right)_{E}}{L-\left(s_{2}^{\prime}\right)_{E}}, \tag{20}
\end{equation*}
$$

where $-\left(s_{2}^{\prime}\right)_{E}$ is the exit pupil distance from secondary mirror, given as

$$
\begin{equation*}
\left(s_{2}^{\prime}\right)_{E}=\frac{d_{1} f_{2}^{\prime}}{d_{1}+f_{2}^{\prime}} \tag{21}
\end{equation*}
$$

Then, $c_{1} c_{3}$ is derived from Eq. (13):

$$
\begin{equation*}
c_{1} c_{3}=\frac{L-\left(s_{2}^{\prime}\right)_{E}}{-\left(s_{2}^{\prime}\right)_{E}} \delta V_{2}=\left[1-\frac{L\left(d_{1}+f_{2}^{\prime}\right)}{d_{1} f_{2}^{\prime}}\right] \delta V_{2} \tag{22}
\end{equation*}
$$

Normalizing $c_{1} c_{3}$, we obtain the fractional displacement vector $\vec{\sigma}_{2}^{*}$ :


Fig. 5. Pure decenter of the aspheric vertex $V_{2}$ of the secondary mirror.

$$
\begin{equation*}
\vec{\sigma}_{2}^{*}=\frac{c_{1} c_{3}}{u_{\mathrm{pr} 1} f^{\prime}} . \tag{23}
\end{equation*}
$$

Applying Eqs. (22), (18b), and (23) to Eq. (15b), we obtain

$$
\begin{align*}
-W_{131,2}^{*} \vec{\sigma}_{2}^{*} & =\frac{1}{8}\left(\frac{y_{1}}{f^{\prime}}\right)^{3}\left(m_{2}+1\right)^{3} b_{s 2} d_{1} u_{\mathrm{pr} 1} \frac{c_{1} c_{3}}{u_{\mathrm{pr} 1} f^{\prime}} \\
& =-\frac{1}{8}\left(\frac{y_{1}}{f^{\prime}}\right)^{3}\left(m_{2}+1\right)^{3} b_{s 2} \delta V_{2} . \tag{24}
\end{align*}
$$

## 3. Coma-Free Condition

Now, considering the case of Fig. 1, lateral misalignment of the secondary mirror can be represented by the separation of the two optical axes at the center of curvature $C_{2}$ and the aspheric vertex $V_{2}$. From Eqs. (19) and (24), we know that misalignment coma is proportional to the separation $\delta C_{2}$ and $\delta V_{2}$, respectively, for base sphere and aspheric departure contributions. The total third-order constant coma is obtained by summing up Eqs. (19) and (24):

$$
\begin{align*}
-\left(W_{131,2}^{o} \vec{\sigma}_{2}^{o}+W_{131,2}^{*} \vec{\sigma}_{2}^{*}\right)= & -\frac{1}{8}\left(\frac{y_{1}}{f^{\prime}}\right)^{3}\left(1-m_{2}^{2}\right) \\
& \times\left(m_{2}+1\right) \delta C_{2}-\frac{1}{8}\left(\frac{y_{1}}{f^{\prime}}\right)^{3} \\
& \times\left(m_{2}+1\right)^{3} b_{s 2} \delta V_{2} \\
= & -\frac{1}{8}\left(\frac{y_{1}}{f^{\prime}}\right)^{3}\left(m_{2}+1\right)^{2} \\
& \times\left[\left(1-m_{2}\right) \delta C_{2}\right. \\
& \left.+\left(m_{2}+1\right) b_{s 2} \delta V_{2}\right] . \tag{25}
\end{align*}
$$

There is a pivot point rotating the secondary mirror about which the base sphere contribution and aspheric contribution in Eq. (25) will cancel out. The total system third-order coma will remain unchanged across the field.
The coma-free condition is then given by solving for the zero of Eq. (25):

$$
\begin{equation*}
\left(1-m_{2}\right) \delta C_{2}+\left(m_{2}+1\right) b_{s 2} \delta V_{2}=0 \tag{26}
\end{equation*}
$$

We can derive the CFP from Eq. (26), which can also be written as

$$
\begin{equation*}
\frac{\delta C_{2}}{\delta V_{2}}=\frac{m_{2}+1}{m_{2}-1} b_{s 2}=\frac{r_{2}-z_{\mathrm{CFP}}}{-z_{\mathrm{CFP}}} \tag{27}
\end{equation*}
$$

$z_{\text {CFP }}$ is the distance from secondary mirror to the CFP, which is derived from Eq. (27), as shown in Fig. 2:

$$
\begin{align*}
z_{\mathrm{CFP}} & =r_{2} /\left[1-\left(\frac{m_{2}+1}{m_{2}-1}\right) b_{s 2}\right] \\
& =s_{2} /\left[\left(\frac{m_{2}+1}{2 m_{2}}\right)\left\{1-\left(\frac{m_{2}+1}{m_{2}-1}\right) b_{s 2}\right\}\right] . \tag{28}
\end{align*}
$$

From Eq. (28), we can know that the position of the CFP is dependent only on the secondary mirror
characteristics. In a classical Cassegrain telescope $\left(b_{s 2, \mathrm{cl}}=-\left(\frac{m_{2}-1}{m_{2}+1}\right)^{2}\right)$, the CFP is located at the prime focus. In an RC telescope $\left(b_{s 2, \mathrm{RC}}=-\left[\left(\frac{m_{2}-1}{m_{2}+1}\right)^{2}+\right.\right.$ $\left.\frac{2 f^{\prime}}{d_{1}\left(m_{2}+1\right)^{3}}\right]$, the CFP is located between the secondary mirror vertex and the prime focus.

This result is identical to the one derived by Wilson using the Schiefspiegler approach. In general, the position of the CFP is of great importance in the design of systems for active correction of decentering coma [4].

In the general case, $\vec{\sigma}_{2}^{o}$ and $\vec{\sigma}_{2}^{*}$ due to misalignment perturbations of secondary mirror are noncollinear and Eq. (25) must be combined by vector addition. Equation (25) can also be reduced to coincide with the result solved by Wilson. The reduction will use treatment in which decenter of the spherical surface is equivalent to tilt; as a result, a spherical surface does not have a unique vertex. Here we do not give more representation.

## 4. Summary

It is common practice to align two-mirror telescopes by tilting and/or decentering the secondary mirror until the coma is removed. In this paper, we explore the coma correction in a fully quantitative form. Derived by a vector approach, the coma-free condition to remove the misalignment-induced coma is presented. The analytic description presented in Section 2 reveals how third-order coma is induced in misaligned two-mirror telescopes. If necessary, the third-order astigmatic field can also be calculated with this approach [6].

The conclusions in this work can be applied to design unobscured reflective telescopes, where there always are decentered and/or tilted elements. We will do this work in the future.

## References

1. B. McLeod, "Collimation of fast wide-field telescopes," Publ. Astron. Soc. Pac. 108, 217-219 (1996).
2. R. N. Wilson and B. Delabre, "Concerning the alignment of modern telescopes: theory, practice, and tolerances illustrated by the ESO NTT," Publ. Astron. Soc. Pac. 109, 53-60 (1997).
3. W. B. Wetherell and M. P. Rimmer, "General analysis of aplanatic Cassegrain, Gregorian, and Schwarzschild telescopes," Appl. Opt. 11, 2817-2832 (1972).
4. R. N. Wilson, Reflecting Telescope Optics I (Springer, 1996).
5. D. J. Schroeder, Astronomical Optics (Academic, 1987).
6. R. V. Shack and K. Thompson, "Influence of alignment errors of a telescope system on its aberration field," Proc. SPIE 251, 146-153 (1980).
7. K. P. Thompson, "Description of the third-order optical aberrations of near-circular pupil optical systems without symmetry," J. Opt. Soc. Am. A 22, 1389-1401 (2005).
8. K. P. Thompson, "Multinodal fifth-order optical aberrations of optical systems without rotational symmetry; spherical aberration," J. Opt. Soc. Am. A 26, 1090-1100 (2009).
9. T. Schmid, K. P. Thompson, and J. P. Rolland, "Alignment induced aberration fields of next generation telescopes," Proc. SPIE 7068, 70680E (2008).
10. T. Schmid, K. P. Thompson, and J. P. Rolland, "A unique astigmatic nodal property in misaligned Ritchey-Chrétien telescopes with misalignment coma removed," Opt. Express 18, 5282-5288 (2010).
11. L. Noethe and S. Guisard, "Analytical expressions for field astigmatism in decentered two mirror telescopes and application to the collimation of the ESO VLT," Astron. Astrophys. Suppl. Ser. 144, 157-167 (2000).
12. A. Rakich, "Calculation of third-order misalignment aberrations with the optical plate diagram," in International Optical Design Conference, OSA Technical Digest (CD) (Optical Society of America, 2010), paper IThB7P.
13. H. H. Hopkins, Wave Theory of Aberrations (Oxford, 1950).
14. W. Welford, Aberration of the Symmetrical Optical System (Academic, 1974).
15. J. M. Sasian, "How to approach the design of bilateral symmetric optical system," Opt. Eng. 33, 2045-2061 (1994).

[^0]:    0003-6935/11/213918-06\$15.00/0
    © 2011 Optical Society of America

