Liquid-crystal (LC) adaptive optics systems have been widely investigated to solve the spatial resolution limitation of the deformable mirror while it is used on larger aperture telescopes. Furthermore, LC spatial light modulators (SLMs) have other advantages of high precision, low cost, high reliability, and low power consumption. However, the LC SLM also has the disadvantages of low energy utilization and slow response speed. To solve the energy loss caused by the polarization dependence and the dispersion, many kinds of methods were proposed [1–5]. To achieve the high-speed LC SLM, dual-frequency LCs have been considered [6]. However, it needs higher driving voltage, which is not compatible to the very large scale integration (VLSI) technique. Therefore, the active element number is very little and the advantage of the LC SLM is lost. Nematic LCs are compatible to the VLSI and easily manufactured into the SLM. Gauza et al. have acquired a new nematic LC material with a high response speed [7]. Now, Boulder Nonlinear Systems, Inc. has fabricated the high speed LC SLMs with 500 Hz switching frequency. Consequently, in this Letter, we only consider the nematic LC SLM to improve the switching frequency.

The response time of the LC SLM is proportional to the square of the LC thickness [1,8]. Wang et al. have discussed optical response time as a function of the cell gap in detail [9]. Therefore, the LC cell gap should be as slight as possible to obtain higher response speed. However, a thinner LC layer produces lesser phase modulation. A kinoform method is presented to solve this confliction, and larger phase modulation and faster response can be achieved [10]. Using this method, an LC SLM with the phase modulation of 2π can produce several, even tens of micrometer modulation magnitude to compensate the distortion of atmospheric turbulence. Here, the total phase modulation of the LC SLM must exceed 2π in the kinoform mode, then the 2π phase is selected to modulate the wavefront. However, the response time at a specific modulation quantity, e.g., the 2π phase, that is affected by the cell gap does not suit the above principle, and is unresolved. Consequently, it is necessary to optimize the cell gap of the LC device to achieve the fastest response speed of the 2π phase.

The response time of the nematic LC device includes rise time and decay time; normally, the rise time is smaller than the decay time. Therefore, the decay time is studied as the response time of the device in the sections below. In this Letter, hydrodynamics of LCs is utilized to analyze the effect of the cell gap on the response time of the 2π phase. Then, an experiment is performed to validate our analysis.

Figure 1 shows the device configuration of a parallel-aligned LC cell which is regarded as a prototype of the transmissive LC SLM. θ is the rotation angle between LCs director and substrate, and d is the LC cell gap.

In the hydrodynamics of LCs, when backflow and inertial effects are ignored, the director rotation of LCs can be described by the Erickson–Leslie equation [11,12],

\[
\gamma_1 \frac{\partial \theta}{\partial t} = (K_{11}\cos^2 \theta + K_{33}\sin^2 \theta) \frac{\partial^2 \theta}{\partial z^2} + (K_{33} - K_{11}) \sin \theta \cos \theta \left( \frac{\partial \theta}{\partial z} \right)^2 + \epsilon_0 |\Delta e| E^2 \sin \theta \cos \theta, \tag{1}
\]

where \(\gamma_1\) is the rotational viscosity, \(K_{11}\) and \(K_{33}\) are the splay and bend elastic constant, respectively, \(E\) is the

![Fig. 1. Schematics of the parallel-aligned LC cell structure.](image)
electric field strength, and $z$ is the position of the LC layer from one substrate in the perpendicular direction.

When the decay response is studied, $E = 0$, and an approximation of $K_{33} \approx K_{11}$ is assumed; Eq. (1) can be simplified to

$$
\gamma_1 \frac{\partial \theta}{\partial t} = K_{11} \frac{\partial^2 \theta}{\partial z^2}.
$$

(2)

In our analysis, we assume that the surface anchoring energy is strong and the pretilt angle of LCs is 0; the general solution of the differential equation is given as:

$$
\theta(z, t) = \theta_0 \sin(\pi z/d) \exp(-t/\tau_d).
$$

(3)

where $\tau_d = \gamma_1 d^2 / K_{11} \pi^2$ is defined as the device decay time which represents the LC director reorientation time ($10 \rightarrow 1/e\theta$), and $\theta_0$ is the maximum deformation angle, whose position is located as $d/2$ at the $z$ axis [13].

The relative retardation $\Delta nd$ of the LC cell at a specific instant can be expressed as

$$
\Delta nd = \int_0^d \frac{n_e n_0}{\sqrt{n_e^2 \cos^2(\theta(z)) + n_0^2 \sin^2(\theta(z))}} dz - n_0 d.
$$

(4)

The average rotation angle $\bar{\theta}$ of all LCs is introduced to simplify the integrating factor of Eq. (4), so the initial phase change ($E$ at an on-state) of the LC cell is approximately described as

$$
\delta_{\theta 0} = \Delta nd \frac{2\pi}{\lambda} \approx \frac{2\pi d}{\lambda} \left[ \frac{n_e n_0}{\sqrt{n_e^2 \cos^2(\theta_0) + n_0^2 \sin^2(\theta_0)}} - n_0 \right].
$$

(5)

$\theta_0$ is the initial average rotation angle at $E$ on-state.

From Eq. (3), the average rotation angle of $t$ instant can be expressed as

$$
\bar{\theta}(t) = \theta_0 \exp(-t/\tau_d).
$$

(6)

Therefore, when the voltage is released instantaneously at $t = 0$, the transmissive phase change of $t$ instant can be derived as

$$
\delta(t) \approx \frac{2\pi d}{\lambda} \left\{ \frac{n_e n_0}{\sqrt{n_e^2 + (n_e^2 - n_0^2) \sin^2(\theta_0 \exp(-t/\tau_d))}} - n_0 \right\}.
$$

(7)

As the phase change of the LC device reaches $2\pi$, we obtain the equation $\delta(t_{2\pi}) = \delta_{\theta 0} + 2\pi$, and substitute Eqs. (5) and (7) to it. The response time of the $2\pi$ phase ($t_{2\pi}$) can be derived as Eq. (8). $t_{2\pi}$ is the longest time of all decay response within the $2\pi$ phase modulation, it represents the response performance of the LC device:

$$
\begin{align*}
\tau_{2\pi}(d) = & -\tau_d \ln \left\{ \frac{1}{\theta_0} \arcsin \left[ \frac{1}{\sqrt{n_e^2 - n_0^2}} \right] \right. \\
& \left. \left( \frac{1}{\sqrt{n_e^2 \cos^2(\theta_0) + n_0^2 \sin^2(\theta_0)}} + \frac{\lambda}{dn_e n_0} \right)^{-2} - \frac{n_0^2}{n_e^2 - n_0^2} \right\}^{1/2} \\
& \cdot \frac{1}{\theta_0}. \\
\end{align*}
$$

(8)

When the value of $t_{2\pi}$ affected with the cell gap is analyzed from Eq. (8), we select parameters of an isothiocyanate LC material to simulate $t_{2\pi}(d)$ which is a fast response LC mixture reported by Gauza et al. [7]. The following are parameters of LCs and the device: $n_e = 1.884$, $n_0 = 1.541$ at incident light $\lambda = 635$ nm, $\gamma_1 / K_{11} = 8.310$ ms $\mu$m$^{-2}$ when the drive voltage is applied to the LC cell, and $\theta_0$ is assumed to 1.065 rad. Figure 2 plots the cell gap-dependent $t_{2\pi}$ value of the parallel-aligned LC cell. The $t_{2\pi}$ first decreases and then slowly increases with the cell gap increasing for 635 nm incident light, and there is an optimal $d$ value at the phase stroke of $2\pi$ to obtain the shortest time. The physical mechanism of this phenomenon could be described as: if the cell gap is thinner than optimal value, the phase change increases acutely in the incipient stage but increases very slowly when the modulation is close to the $2\pi$ phase. Finally, the $t_{2\pi}$ is relatively long. While the cell gap is thicker than the optimal value, the phase increases slowly in all the $2\pi$ ranges, which results in a longer $t_{2\pi}$ than that of the optimal thickness. Moreover, the similar phenomena are obtained at the other incident light wavelength with corresponding parameters, as shown in Fig. 2.

In order to calculate the optimal cell gap value, we do the derivation of Eq. (8) and make it equate to zero, as shown in Eq. (9). The solution of Eq. (9), $d_{optimal}$, is the cell gap to obtain the fastest response speed. When the above parameters of isothiocyanate LCs at $\lambda = 635$ nm were used, $d_{optimal}$ was 2.86 $\mu$m.

$$
\frac{\partial t_{2\pi}(d)}{\partial d} = 0.
$$

(9)

Fig. 2. (Color online) Response time of the $2\pi$ phase ($\lambda = 533$ nm, 635 nm, 785 nm) depending on the LC cell gap at $\theta_0 = 1.065$ rad. The filled squares with the error bar are the experimental values at $\lambda = 635$ nm.
The voltage $4.0\, V_{pp}$ was released instantaneously at 0 ms. The solid lines in the figure are theoretical calculated values from Eq. (7).

In the experiment, a series of parallel-aligned LC cells were prepared and injected with the isothiocyanate LC materials. The cell gaps were determined by the interference method. The pretilt of each LC cell was less than 1.0°. The LC cell was sandwiched between two crossed polarizers, and the LC directors without applied voltage is at 45° to the polarizing direction of a polarizer. A semiconductor laser ($\lambda = 635\, \text{nm}$) was used as incident light whose direction is perpendicular to the substrate of the LC device. The voltage $4.0\, V_{pp}$ (squarewave, 2000 Hz) was applied to each LC cell. $\theta_0$ of each cell was equivalent [13] and calculated as 1.065 from Eq. (5). When the voltage was released instantaneously, transient light intensity was recorded by a photoreceiver (New focus, Model 2031) and an oscillograph. The time-dependent transmittance was converted to the time-dependent phase change by the formula $\delta = 2\ \text{arcsin} \sqrt{t}$ [14]. The time between the released instant and instant of the $2\pi$ phase was recorded as $t_{2\pi}$ of the cell. These $t_{2\pi}$ values are also plotted in Fig. 2, and approximately accord with the theoretical values from Eq. (8). Little deviation may originate from theoretical calculation errors in the highly deformed LC region [15] and the error of simplifying average rotation angles.

Figure 3 shows the phase changes of two LC devices depending on response instant. The cell gaps are 2.49 and 2.94 $\mu$m, respectively, and the voltage $4.0\, V_{pp}$ was released at 0 ms. The discrete data points are experimental phase change values, and the solid lines are the theoretical values given by Eq. (7). It is obvious that the theoretical formula can closely describe the phase change of the parallel-aligned LC cell during decay response. In Fig. 3, the experimental response times of $2\pi$ were measured as 8.2 and 7.1 ms, respectively, which were close to the theoretical values (7.93 ms, 7.34 ms). The actual measurement result obviously indicates that the LC cell with about an optimal cell gap can offer shorter response time of $2\pi$ than that of the LC cell with a nonoptimal cell gap.

In summary, the response time depending on the cell gap have been analyzed when the target phase change is $2\pi$. The derived formula can describe real-time phase changes of the parallel-aligned LC cell in decay response. The response time of the $2\pi$ phase change also can be described by the formula with parameters $d$, $n_e$, $n_o$, $K_{11}$, $Z_1$, $\lambda$, and $\theta_0$. The time $t_{2\pi}$ first decreased and then slowly increased with the cell gap increasing; there is an optimal cell gap to obtain the shortest response time of $2\pi$. The optimal cell gap could be obtained via solving the derivative equation. The experimental results also indicate that the LC cell with an optimal cell gap can offer the shortest response time of $2\pi$. Therefore, the method and formula in this Letter can improve the switching frequency of the LC SLMs through the optimal cell gap. Although the transmissive LC cell is employed to investigate as a prototype device of the SLM in this study, the method of optimizing cell gap is not restricted to this type device and can be applied to fabrication of any other LC optical devices with specific modulation to improve the response performance.

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References