Gain-phase grating based on spatial modulation of active Raman gain in cold atoms

Shang-qi Kuang, Chun-shui Jin, and Chun Li
Changchun Institute of Optics, Fine Mechanics and Physics, Chinese Academy of Sciences, Changchun 130033, China
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In order to obtain an atomic grating which can diffract light into the high-order directions more efficiently, a gain-phase grating (GPG) based on the spatial modulation of active Raman gain is theoretically presented. This grating is induced by a pump field and a standing wave in ultracold atoms, and it not only diffracts a weak probe field propagating along a direction normal to the standing wave into the high-order directions, but also amplifies the amplitude of the zero-order diffraction. In contrast with electromagnetically induced grating or electromagnetically induced phase grating, the GPG has larger diffraction efficiencies in the high-order directions. Hence it is more suitable to be utilized as an all-optical router in optical networking and communication.

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By replacing the traveling wave of electromagnetically induced transparency (EIT) [1] with a standing wave, an atomic grating called an electromagnetically induced grating (EIG) is formed [2]. Due to the spatial modulation induced by the standing wave, the amplitude of a resonant probe field changes in a period, and then the probe field is diffracted into the high-order directions. The phenomenon of EIG was experimentally observed in ultracold atoms [3], and it was used to realize optical bistability [4]. For the small diffraction efficiency of EIG, an electromagnetically induced phase grating (EIPG) based on the cross-phase modulation (XPM) of the probe field was demonstrated [5]; the first-order diffraction efficiency of this phase grating can be close to the efficiency of an ideal sinusoidal phase grating. However, the largest diffraction efficiency of EIPG in the first-order direction is still relatively small (≈30%), and the diffracted beam in the second-order direction is too weak to be observed in experiments. Meanwhile, the effect of active Raman gain (ARG) has attracted much attention, and it was used to obtain superluminal [6] or subluminal light propagation [7,8]. Most recently, the manipulation of ARG in a four-level atomic medium has been used to control the group velocity [9] or obtain a large Kerr effect [10,11]. In comparison with the schemes based on the manipulation of the probe absorption of EIT, the schemes based on the modulation of the probe gain of ARG have several advantages [10,11].

In this paper, we theoretically present that a gain-phase grating (GPG) can be formed in cold atoms which are driven by a pump field and a standing wave. Due to the spatial modulation of the standing wave, a weak probe field propagating in a direction normal to the standing wave can be diffracted into the high-order directions effectively, and the amplitude of the zero-order diffraction is amplified. In contrast with the EIPG, the first-order diffraction efficiency of the GPG is larger, and the second-order diffraction efficiency can be comparable with the first-order diffraction efficiency of the EIG. Therefore, we believe that this kind of grating can be utilized as an all-optical routing [12] in the optical networking and communication.

The atomic system under consideration is shown in Fig. 1(a), it can be described by a four-level *N*-type configuration. The transitions $|3\rangle$–$|1\rangle$, $|3\rangle$–$|2\rangle$, and $|4\rangle$–$|2\rangle$ are electric dipole allowed, while the transitions $|2\rangle$–$|1\rangle$ and $|4\rangle$–$|1\rangle$ are electric dipole forbidden. The pump and standing fields drive the transitions $|3\rangle$–$|1\rangle$ and $|4\rangle$–$|2\rangle$, respectively. A weak probe field with a Rabi frequency g interacts with the transition $|3\rangle$–$|2\rangle$, and the Rabi frequencies of the pump field and standing wave are Ω_p and Ω_s, respectively. Without loss of generality, we take these Rabi frequencies to be real, and $2\gamma_i$ ($i = 1,2,3$) represents the rate of spontaneous emission of the corresponding transition. As shown in Fig. 1(b), the standing wave in the x direction is formed by two overlapped fields, and the weak probe and strong pump fields propagate along the z and x directions, respectively. Therefore, the Rabi frequency of the standing wave can be written as $\Omega_s = \Omega \sin(\pi x/\Lambda)$, where Λ is the space period of the standing wave.

In the framework of the semiclassical theory, using the dipole approximation and the rotating-wave approximation, the Hamiltonian H_I of the system in the interaction picture is

$$H_I = \hbar[\delta|2\rangle\langle 2| - \Delta_p|3\rangle\langle 3| + (\delta - \Delta_s)|4\rangle\langle 4| - (g|3\rangle\langle 2| + \Omega_p|3\rangle\langle 1| + \Omega_s|4\rangle\langle 2| + \text{H.c.})], \tag{1}$$

where Δ, Δ_p, and Δ_s are the detunings of the probe field, pump field, and standing wave, respectively, and $\delta = \Delta - \Delta_p$. Due to the large detuning of the pump field and the weak intensity of the probe field, we can assume that the majority of the atoms populates at the level $|1\rangle$ ($\rho_{11} \approx 1$), and levels $|2\rangle$, $|3\rangle$, and $|4\rangle$ remain empty [6–8]. Including relaxation terms for the system, the equations of motion for the density matrix of the four-level atomic system can be written as

$$\begin{aligned}
\dot\rho_{13} &= -(i\Delta_p + \gamma_{13})\rho_{13} - i\Omega_p - ig\rho_{12},\\
\dot\rho_{32} &= (i\Delta - \gamma_{32})\rho_{32} - i\Omega_s\rho_{34} + i\Omega_p\rho_{12},\\
\dot\rho_{12} &= (i\delta - \gamma_{12})\rho_{12} - ig\rho_{13} - i\Omega_s\rho_{14} + i\Omega_p\rho_{32},\\
\dot\rho_{34} &= [i(\Delta - \Delta_s) - \gamma_{34}]\rho_{34} - i\Omega_s\rho_{32} + i\Omega_p\rho_{14} + ig\rho_{24},\\
\dot\rho_{24} &= -(i\Delta_s + \gamma_{24})\rho_{24} + ig\rho_{34},\\
\dot\rho_{14} &= [i(\delta - \Delta_s) - \gamma_{14}]\rho_{14} - i\Omega_s\rho_{12} + i\Omega_p\rho_{34},
\end{aligned} \tag{2}$$

where $\gamma_{13} = \gamma_{32} = \gamma_1 + \gamma_2$, $\gamma_{14} = \gamma_{24} = \gamma_3$, $\gamma_{34} = \gamma_1 + \gamma_2 + \gamma_3$, and γ_{12} is the dephasing rate for the transition $|2\rangle$–$|1\rangle$ due to the atomic collisions. For simplicity, we assume $\gamma_1 = \gamma_2 = \gamma_3 = \gamma/2$, and then the steady solution of the element ρ_{32} is

$$\frac{\rho_{32}}{g} = \frac{i\Omega_p^2(\mathcal{M}+\Gamma_{34}\Gamma_{14})}{\Gamma_{13}[(\mathcal{M}+\Gamma_{34}\Gamma_{14})(\Gamma_{12}\Gamma_{32}-\mathcal{M})+\Omega_s^2(\Gamma_{34}+\Gamma_{12})(\Gamma_{32}-\Gamma_{14})]}, \tag{3}$$

where $\mathcal{M}=\Omega_p^2-\Omega_s^2$, $\Gamma_{13}=i\Delta_p+\gamma$, $\Gamma_{14}=i(\Delta_s-\delta)+\gamma/2$, $\Gamma_{34}=-i(\Delta-\Delta_s)+3\gamma/2$, $\Gamma_{32}=i\Delta-\gamma$, and $\Gamma_{12}=-i\delta+\gamma_{12}$.

From Eq. (3), the linear susceptibility χ experienced by the probe field can be written as

$$\chi = 3\pi\mathcal{N}\gamma\frac{\rho_{32}}{g}, \tag{4}$$

where $\mathcal{N}=N_0(\lambda/2\pi)^3$ is the scaled average atomic density, and N_0 and λ represent the atomic density and the wavelength of the probe field. We assume that the interaction length of cold atoms experienced by the probe field in the z direction is L, which is given in units of $\zeta=\lambda/6\pi^2\mathcal{N}$. In the slowly varying envelope approximation and the steady-state regime, the propagation of probe field driven by the atomic polarization can be described by Maxwell's equation as

$$\frac{\partial g}{\partial z} = (-\alpha+i\beta)g, \tag{5}$$

where $\alpha=(2\pi/\lambda)\mathrm{Im}[\chi]$ and $\beta=(2\pi/\lambda)\mathrm{Re}[\chi]$ are the absorption and dispersion coefficients of the probe field. In order to simplify the physical interpretation and focus on the main features of the GPG, the transverse term in Eq. (5) has been neglected [2]. After solving Eq. (5) analytically, we obtain the transmission function of the probe field at $z=L$,

$$T(x) = e^{-\alpha(x)L}e^{i\beta(x)L}. \tag{6}$$

Under the condition that the probe field is a plane wave, the Fraunhofer diffraction pattern is given by the Fourier transform

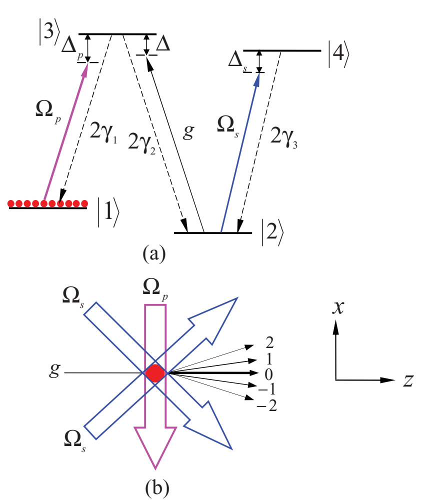

FIG. 1. (Color online) (a) A schematic diagram of the four-level N-type atomic system. The possible levels correspond to the D_2 line of ^{85}Rb atoms: $|1\rangle=|F=3,m_F=-3\rangle$, $|2\rangle=|F=2,m_F=-1\rangle$, $|3\rangle=|F'=2, m_{F'}=-2\rangle$, and $|4\rangle=|F'=1, m_{F'}=0\rangle$. (b) A sketch of a prototype experimental setup.

of the transmission function $T(x)$. Following the results in [2], the diffraction intensity distribution can be written as

$$I(\theta) = |E(\theta)|^2\frac{\sin^2(N\pi\Lambda\sin\theta/\lambda)}{N^2\sin^2(\pi\Lambda\sin\theta/\lambda)}, \tag{7}$$

where N is the number of spatial periods of the atomic grating illuminated by the probe field, and θ is the diffraction angle with respect to the z direction. Here the Fraunhofer diffraction of a single space period is

$$E(\theta) = \int_0^1 T(x)\exp(-2\pi i\Lambda x\sin\theta/\lambda)dx. \tag{8}$$

The diffraction efficiency of the grating is taken to be the ratio of the intensity in the diffracted output to the intensity of the input. Because $I(\theta)$ in our manuscript is normalized such that the intensity of the input probe field is equal to 1, the diffraction efficiency in any diffraction order can be given by the intensity of $I(\theta)$ for that order.

In the following, we demonstrate the calculated results which correspond to transitions of ^{85}Rb atoms, and here $2\gamma/2\pi=6.066$ MHz and $\gamma_{12}/2\pi\approx 1$ kHz are chosen. Figure 2(a) displays the amplitude and phase of the transmission function as a function of x. An investigation of Fig. 2(a) shows that the probe field is amplified, and the probe gain oscillates along the x direction in a period of Λ. The amplification of probe field is attributed to the effect of ARG induced by the pump field, and the periodic intensity pattern of the standing wave changes the probe gain periodically. Therefore, an amplitude grating or gain grating is formed in the cold atomic medium. Due to the Kramer-Kronig relations [13], the dispersion of the probe field also changes periodically in the x direction, and then the phase of the probe field alters in the same period as shown in Fig. 2(a). We can also understand the probe phase modulation by the XPM of the standing field, and at this point, this grating is a phase grating. As a result, we define such a grating as a GPG. In Fig. 2(b), we supply the diffraction pattern according to the transmission function of Fig. 2(a) in the conditions of $\Lambda/\lambda=4$ and $N=5$. In the absence of the phase modulation [$\beta(x)L\equiv 0$], only the zero-order diffraction is significant, which proves that the amplitude grating mainly diffracts the energy into the zero-order direction. Due to the coherent superposition of the probe gains in the spatial periods, the amplitude of the zero-order diffraction in Fig. 2(b) is larger than the probe gain in a single period in Fig. 2(a). When the phase modulation is considered, a fraction of the intensity has been deflected out of the zeroth diffraction into additional side patterns located at $\sin\theta=n\lambda/\Lambda$ ($n=1,2,3$) as shown in Fig. 2(b). The first-order and second-order diffraction efficiencies in Fig. 2(b) are about 50% and 14%, and the diffraction in the third-order direction also has a small amount of energy. In comparison with the EIG and EIPG, the GPG has a larger diffraction intensity in the first-order direction. Because the

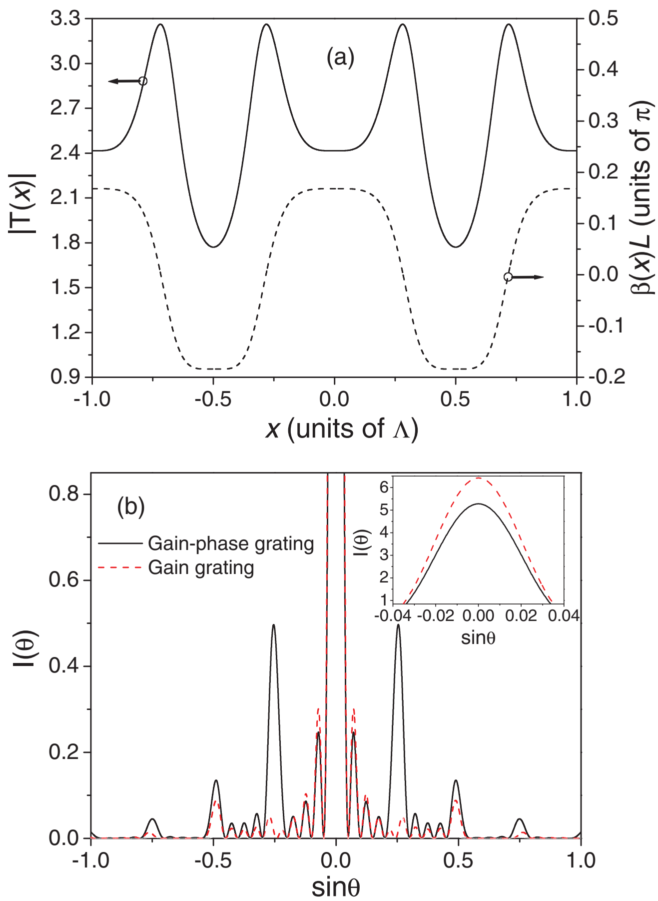

FIG. 2. (Color online) (a) The amplitude (solid line) and phase (dashed line) of the transmission function $T(x)$ as a function of x within two space periods when $\Delta_p = \Delta = -50\gamma$, $\Delta_s = -22\gamma$, $\Omega_p = 0.1\gamma$, $\Omega = 0.11\gamma$, and $L = 100\zeta$. (b) The diffraction intensity as a function of $\sin\theta$ for the corresponding transmission function as shown in (a). The gain-phase grating considers the spatial modulation of the gain and phase of the probe field, while the gain grating ignores the phase modulation in space [$\beta(x)L \equiv 0$]. The inset shows the residual parts of the diffraction patterns.

second-order diffraction efficiency of the GPG is comparable with the first-order diffraction efficiency of the EIG, the second-order diffraction component of the GPG should be observed in experiments.

In order to meet the requirement of ARG that the ground-state population is not depleted, we confirm that the Rabi frequency of the pump field is much smaller than its detuning ($\Omega_p \ll |\Delta_p|$). On this condition, the first-order and second-order diffraction efficiencies are plotted as a function of the intensity or detuning of the pump field in Fig. 3. An investigation of Fig. 3 shows that the diffraction efficiency in the high-order direction increases as the intensity of pump field increases, while the diffraction efficiency decreases as the detuning increases. The reason is that the grating with a larger amplitude and phase modulation can diffract more energy into the high-order directions.

In Fig. 4(a), the diffraction efficiencies in the first-order and second-order directions are given as a function of the intensity of the standing wave for different interaction lengths and $\Delta_s = -22\gamma$. An investigation of Fig. 4(a) shows that diffraction

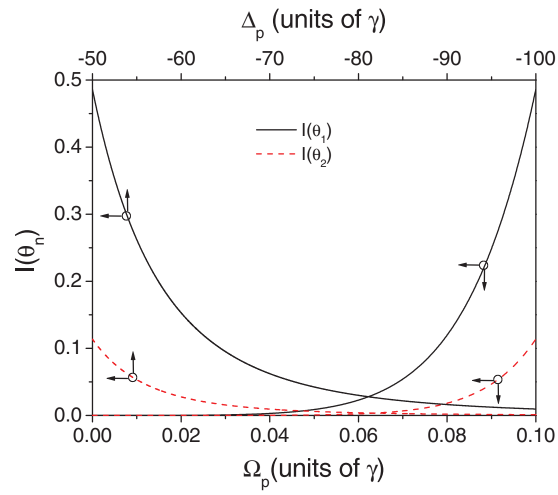

FIG. 3. (Color online) The diffraction intensities in the first-order and second-order directions as a function of the Rabi frequency of the pump field when $\Delta_p = -50\gamma$; the first-order and second-order diffraction intensities as a function of the detuning of the pump field when $\Omega_p = 0.1\gamma$ and $\delta = 0$. Here $\sin\theta_1 = 0.25$, $\sin\theta_2 = 0.5$, and other parameters are the same as in Fig. 2(a).

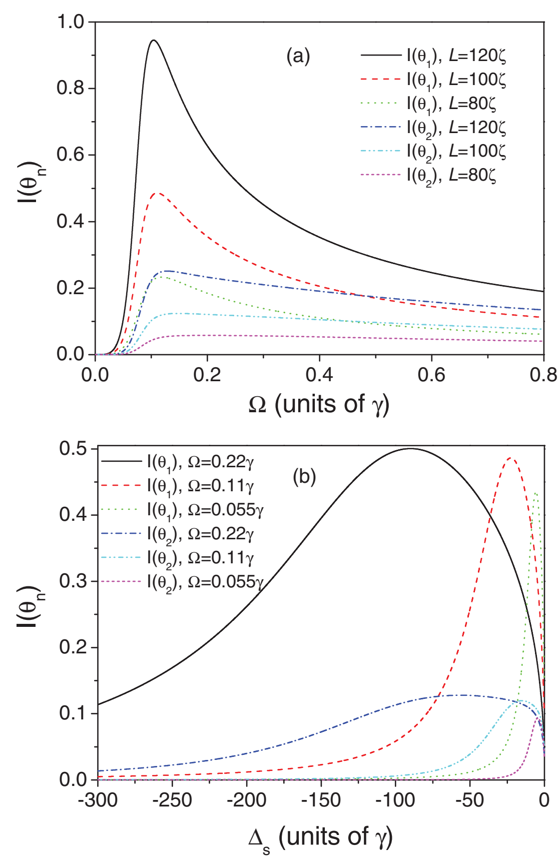

FIG. 4. (Color online) (a) The first-order and second-order diffraction intensities as a function of the intensity of the standing wave for different interaction lengths and $\Delta_s = -22\gamma$. (b) The first-order and second-order diffraction intensities as a function of the detuning of the standing field for different standing-wave intensities. In both cases, $\sin\theta_1 = 0.25$, $\sin\theta_2 = 0.5$, and other parameters are the same as in Fig. 2(a).

efficiencies increase as the interaction length increases, and at $L = 120\zeta$, for a Rabi frequency of standing wave as 0.11γ, the diffraction efficiencies in the first-order and second-order directions can reach 94% and 25%. Increasing the interaction length, the gain and XPM of the probe field increase, and then the high-order diffraction efficiency increases. In order to ensure the weak-field approximation of the probe field in our calculations, the largest interaction length under consideration is $L = 120\zeta$, which leads to an amplification of the probe field by a factor of four. In Fig. 4(b), for the different standing-wave intensities, we depict the first-order and second-order diffraction intensities as a function of the detuning of the standing wave. It is seen from Figs. 4(a) and 4(b) that there are optimum parameters of the standing wave, such as $\Omega = 0.11\gamma$ and $\Delta_s = -22\gamma$; this result proves that there is an optimum phase modulation of the probe field. Due to the ac Stark shift of the state $|2\rangle$ induced by the standing wave, there is a change in the probe detuning, and the largest one induced by a certain standing wave is $\delta_s = \Omega^2/\Delta_s$ [14,15]. At the frequency of the half-height of the gain profile, the probe field has the largest dispersion coefficient. Hence the largest phase modulation occurs when the largest change of the probe detuning equals to the half width at half maximum of the gain profile. Because the full width at half maximum of the gain profile is about $2\gamma_{12}$ [6], the change of the probe detuning δ_s which leads to an optimum phase modulation, can be derived as

$$\delta_s = -\delta_p - \gamma_{12}, \tag{9}$$

where $\delta_p = -\Omega_p^2/\Delta_p$ is the ac Stark shift of the state $|3\rangle$ induced by the pump field. Inserting the optimum parameters of the standing wave in Fig. 4 into Eq. (9), one can obtain good agreement. Owing to the high diffraction efficiency in the high-order direction, the GPG can be used as an all-optical router [12] which has a great potential application in optical networking and communication.

In conclusion, we theoretically demonstrate that a GPG can be induced by the periodic modulation of ARG in an ultracold atomic medium. This grating is induced not only by the amplitude modulation of the probe gain, but also by the modulation of the probe phase. In the GPG, the gain grating induces an amplification of the diffracted beam in the zero-order direction, and the phase grating deflects the energy into high-order directions effectively. In comparison with the EIG and the EIPG, the GPG has larger diffraction intensities in the high-order directions, thus it may be utilized as an all-optical router which is an important optical device in optical networking and communication.

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