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Unified Approach for the Kerr Effect, Stark Effect and Doppler Effect in the Atom-Field Interaction

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Abstract *We solve the atom-field interaction model including the Kerr effect, Stark effect and Doppler effect exactly within a unified approach. The detuning in the effective Hamiltonian depends on two conserved operators of the system. Finally, the influence of atomic mass-center-motion on the antibunching effect of photons is discussed.*

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I. Introduction

The Jaynes–Cummings model[1,2] (JCM) that describes a single two-level atom interacting with a single mode quantized electromagnetic field exhibits many new quantum features such as collapse and revival of the atomic population inversion which results from the discreteness of the quantized field. As a basic model in quantum optics, it is not only helpful in understanding the atom-field interaction, but also plays an important role in maser and laser theory.[3,4]

The JCM is extended to many directions. Among them, important generalizations are the JCM including the Kerr effect,[5,6] the Stark effect[7,8] and the Doppler effect[9,10] respectively. We have solved the JCM including the Doppler effect exactly by the unitary transformation technique.[10] In this letter, a unified approach is given for treating not only the Doppler effect, but also the Kerr effect and Stark effect in the meantime and the exact solution is followed. Finally, we will discuss the influence of atomic mass-center-motion on the antibunching effect of photons.

II. The Generalized JCM and Its Solution

Let us consider a two-level atom interacting with a single mode quantized field including the Kerr effect, Stark effect and Doppler effect. In the dipole and rotating wave approximation, the Hamiltonian of the system is[5,7,10] ($\hbar = 1$)

$$H = \frac{\hat{p}^2}{2m} + \frac{1}{2}\omega_0\sigma_3 + \omega a^\dagger a + q(a^\dagger a)^2 + a^\dagger a(\beta_2|\uparrow\rangle\langle\uparrow| + \beta_1|\downarrow\rangle\langle\downarrow|) + g[\sigma_+ a\exp(-\mathrm{i}k\hat{x}) + \sigma_- a^\dagger\exp(\mathrm{i}k\hat{x})]\,, \tag{1}$$

where $\hat{p}$ is the operator of the momentum conjugate to the position $\hat{x}$ along the propagating axis of the cavity field, σ_3 and $\sigma_\pm$ are the usual spin-half operators that describe the internal dynamics of the two-level atom ($|\uparrow\rangle$ denotes excited state and $|\downarrow\rangle$ the ground state) with transition frequency ω_0, a and $a^\dagger$ denote the annihilation and creation operators of the bosonic cavity mode with frequency ω, k is the wave number for the propagating wave, q is the nonlinear parameter of the Kerr medium, β_2, β_1 are the Stark shift parameters, the terms $\exp(\pm\mathrm{i}k\hat{x})$ account for the Doppler effect and g is the atom-field coupling constant.

From the following commutation relations

$$\Big[\frac{1}{2}\sigma_3, \sigma_\pm\Big] = \pm\sigma_\pm\,, \quad \Big[\frac{\hat{p}}{k}, \mathrm{e}^{\pm\mathrm{i}k\hat{x}}\Big] = \pm\,\mathrm{e}^{\pm\mathrm{i}k\hat{x}}\,, \quad [a^\dagger a, a] = -a\,, \quad [a^\dagger a, a^\dagger] = a^\dagger\,, \tag{2}$$

we found that the two operators

$$C_1 = a^\dagger a + \tfrac{1}{2}\sigma_3\,, \qquad C_2 = \hat{p}/k + \tfrac{1}{2}\sigma_3 \tag{3}$$

are conserved operators, i.e.,

$$[C_1, H] = [C_2, H] = 0\,. \tag{4}$$

This is the main point. Using C_1 and C_2, the model can be solved exactly as shown below. Some items in the Hamiltonian can be written in terms of the two conserved operators as

$$\begin{aligned} &\omega a^\dagger a = \omega C_1 - \tfrac{1}{2}\omega\sigma_3\,,\\ &\frac{\hat{p}^2}{2m} = \frac{k^2C_2^2}{2m} + \frac{k^2}{8m} - \frac{k^2C_2}{2m}\sigma_3\,,\\ &q(a^\dagger a)^2 = q(C_1^2 + \tfrac{1}{4}) - qC_1\sigma_3\,,\\ &a^\dagger a(\beta_2|\uparrow\rangle\langle\uparrow| + \beta_1|\downarrow\rangle\langle\downarrow|) = C_1\beta_+ - \tfrac{1}{2}\beta_- + (C_1\beta_- - \tfrac{1}{2}\beta_+)\sigma_3\,, \end{aligned} \tag{5}$$

where $\beta_+ = (\beta_2 + \beta_1)/2$, $\beta_- = (\beta_2 - \beta_1)/2$. So, the Hamiltonian can be rewritten as

$$H = M(C_1, C_2) + \frac{\Delta(C_1, C_2)}{2}\sigma_3 + g[\sigma_+ a\exp(-\mathrm{i}k\hat{x}) + \sigma_- a^\dagger\exp(\mathrm{i}k\hat{x})]\,, \tag{6}$$

where

$$\begin{aligned} M(C_1, C_2) &= \omega C_1 + q\Big(C_1^2 + \frac{1}{4}\Big) + C_1\beta_+ - \frac{1}{2}\beta_- + \frac{k^2C_2^2}{2m} + \frac{k^2}{8m}\,,\\ \Delta(C_1, C_2) &= (\omega_0 - \omega - \beta_+) - \frac{k^2C_2}{8m} + 2(\beta_- - q)C_1\,. \end{aligned} \tag{7}$$

The Hamiltonian written in this form is similar to the standard JCM. But the main difference is that the detuning depends on the two conserved operators. Because the operators $M(C_1, C_2)$ and $\Delta(C_1, C_2)$ are conserved operators too, the Hamiltonian can be solved using the standard method of solving JCM.[2]

Let the initial state be

$$|\psi(0)\rangle = \int \mathrm{d}pf(p)|p\rangle \otimes |\uparrow\rangle \otimes \sum_{n=0}^{\infty} C_n|n\rangle = \sum_{n=0}^{\infty}\int \mathrm{d}pC_nf(p)|p\rangle \otimes |\uparrow\rangle \otimes |n\rangle\,, \tag{8}$$

i.e., the atom is in the excited state with momentum distribution $\int \mathrm{d}pf(p)|p\rangle$ and the field is in the superposition of Fock state $|n\rangle$. From Eqs (6), (7) and (8), the wavefunction at time t is obtained

$$\begin{aligned} |\psi(t)\rangle = &\sum_{n=0}^{\infty}\int \mathrm{d}pC_nf(p)F(p,n,t)|p\rangle \otimes |\uparrow\rangle \otimes |n\rangle\\ &+ \sum_{n=0}^{\infty}\int \mathrm{d}pC_nf(p)G(p,n,t)|p+k\rangle \otimes |\downarrow\rangle \otimes |n+1\rangle\,, \end{aligned} \tag{9}$$

where

$$\begin{aligned} F(p,n,t) &= \exp[-\mathrm{i}M(p,n)t]\Big\{\cos[A(p,n)t] - \mathrm{i}\frac{\Delta(p,n)}{2}\frac{\sin[A(p,n)t]}{A(p,n)}\Big\},\\ G(p,n,t) &= \exp[-\mathrm{i}M(p,n)t]\Big\{-\mathrm{i}g\sqrt{n+1}\,\frac{\sin[A(p,n)t]}{A(p,n)}\Big\},\\ A(p,n) &= \sqrt{\frac{\Delta^2(p,n)}{4} + g^2(n+1)}\,,\\ \Delta(p,n) &= (\omega_0 - \omega - \beta_+) + 2(\beta_- - q)\Big(n + \frac{1}{2}\Big) - \frac{k^2}{2m} - \frac{kp}{m}\,,\\ M(p,n) &= (\omega + \beta_+)\Big(n + \frac{1}{2}\Big) + q\Big(n^2 + n + \frac{1}{2}\Big) - \frac{1}{2}\beta_- + \frac{k^2}{4m} + \frac{p^2}{2m} + \frac{kp}{2m}\,. \end{aligned} \tag{10}$$

In the expression of $\Delta(p,n)$, the term $-kp/m$ is Doppler shift and $-k^2/2m$ associates with recoil of the atom with mass m.

So far, the exact solution of the model is obtained by introducing the two conserved operators. Next, we proceed to discuss the influence of atomic mass-center-motion on antibunching effect of photons.

III. Antibunching Effect of Photons

The statistical properties of photon distribution can be studied by evaluating the second-order correlation function defined as

$$g^{(2)}(t) = \frac{\langle a^{\dagger 2} a^2 \rangle}{\langle a^\dagger a \rangle^2} - 1 = \frac{\langle (a^\dagger a)^2 \rangle - \langle a^\dagger a \rangle}{\langle a^\dagger a \rangle^2} - 1\,, \tag{11}$$

Note that $g^{(2)}(t) < 0$ implies a sub-Poissonian statistics or antibunching, which is a non-classical characteristic. If $g^{(2)}(t) > 0$, the field exhibits super-Poissonian statistics or bunching, if $g^{(2)}(t) = 0$, the field is of Poissonian type. From Eq. (9), it is easy to get the expectation values of operators $a^\dagger a$ and $(a^\dagger a)^2$ on the state $|\psi(t)\rangle$

$$\langle a^\dagger a \rangle = \sum_{n=0}^{\infty} \int \mathrm{d}p |C_n|^2 |f(p)|^2 [n|F(p,n,t)|^2 + (n+1)|G(p,n,t)|^2]\,,$$

$$\langle (a^\dagger a)^2 \rangle = \sum_{n=0}^{\infty} \int \mathrm{d}p |C_n|^2 |f(p)|^2 [n^2|F(p,n,t)|^2 + (n+1)^2|G(p,n,t)|^2]\,. \tag{12}$$

Let us suppose that the momentum distribution of atom is of Gaussian type

$$|f(p)|^2 = 1/\sqrt{2\pi\Delta p}\exp[-(p-p_0)^2/2(\Delta p)^2] \tag{13}$$

and the initial field is in the coherent state

$$|C_n|^2 = \exp(-\bar{n})\bar{n}^n/n!\,, \tag{14}$$

where p_0 and Δp are the momentum center and width of atomic wavepacket respectively, $\bar{n}$ is the mean photon number of the coherent field.

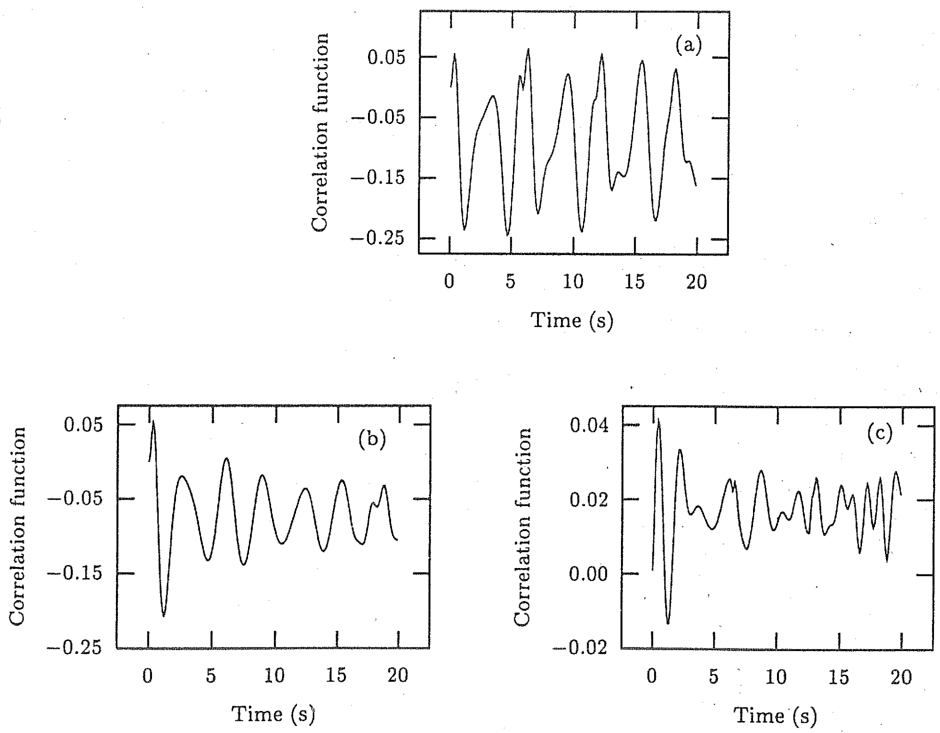

Fig. 1. The field second-order correlation function as a function of time, (a) $\Delta p = 0.1$, (b) $\Delta p = 1.0$, (c) $\Delta p = 15.0$.

The second-order correlation function for the extended JCM is plotted numerically in Fig. 1. Figures 1a, 1b and 1c correspond to the atomic wavepacket width $\Delta p = 0.1$, 1.0, 15.0 respectively. The other parameters are chosen as $m = 1.0$, $k = 1.0$, $p_0 = 0.0$, $\bar{n} = 1.44$, $\beta_1 = \beta_2 = 0.0$, $g = 1.0$, $q = 0.5$, $\omega = \omega_0 = 1.0$.

In Fig. 1a, the field exhibits bunching at the initial short part of time, and then antibunching. The field oscillates between bunching and antibunching. The behavior is similar to the case of original JCM because the width ($\Delta p = 0.1$) of atomic wavepacket is narrow and has little effect on the evolution of second-order correlation function. When the width increases ($\Delta p = 1.0$), antibunching exists at most of the time because of the effect of atomic momentum distribution as shown in Fig. 1b. When the width increases to $\Delta p = 15.0$, the field shows bunching effect at most of the time in Fig. 1c. In this case, the momentum distribution of atomic wavepacket has great influences on the behavior of $g^{(2)}(t)$.

It can be seen that the width of atomic wavepacket, i.e., the atomic mass-center-motion has great effects on the statistical properties of photon distribution. The field tends to show bunching or antibunching other than oscillating between them when the widths of atomic wavepacket are wide enough.

IV. Conclusion

A simple unified approach is found and applied to solving the extended JCM including the Kerr effect, Stark effect and Doppler effect and the exact solution is obtained. The main points are to find the two conserved operators and rewrite the Hamiltonian of the system considered in terms of them. When the width of atomic wavepacket is large and the atomic motion along the cavity axis is not negligible, the effect of atomic mass-center-motion should be considered. It is shown that the atomic mass-center-motion has great influence on the behavior of field. When the width of atomic wavepacket is large enough, the field inclines to display antibunching or bunching other than oscillating between them as in the original JCM.

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