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The dependence of hole width and depth on burning laser duration

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The evolution of power broadening in the hole-burning process is considered. Under a strong field, a deep hole with HWHM less than $0.8K$ (K is the Rabi frequency) can be burnt if a π pulse is applied. The hole width reaches its minimum with a near π pulse if the ratio $T_2/2T_1$ of the two level system is less than 0.78. The minimal width is much narrower than the steady-state one predicted by the optical Bloch equation and comparable to the narrowest that the modified Bloch equation may give.

1. Introduction

A short writing time is requested in data storage applications. Spectral hole-burning may raise the storage density for 3 orders of magnitude over the conventional optical disc. However, only few works have reported on burning detectable holes with a short laser pulse, the duration of which is comparable to that used in the optical disc [1]. Shortening the writing time seems to be difficult especially for materials with small absorption cross-sections.

Increasing the burning power is a possible way to speed up writing. However, power broadening arises in this case, according to the steady-state solution of the optical Bloch equation (OBE) [2]. To understand the evolution of the power broadening and to see what the hole width will be under laser pulse, the transient behaviour is studied in this paper.

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2. Theoretical method

For a two level system, both the conventional and the modified Bloch equations can be written as [3]

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \left[\begin{pmatrix} 0 & -\Delta & 0 \\ \Delta & 0 & K \\ 0 & -K & 0 \end{pmatrix} - \begin{pmatrix} \gamma & 0 & 0 \\ 0 & \gamma & 0 \\ 0 & 0 & 2\gamma \end{pmatrix} - \begin{pmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} \\ \Gamma_{21} & \Gamma_{22} & \Gamma_{23} \\ 0 & 0 & 0 \end{pmatrix}\right]\begin{pmatrix} u \\ v \\ w \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 2\gamma w_{eq} \end{pmatrix}, \quad (1)$$

where Δ and K are the detuning and Rabi frequency respectively, $2\gamma = 1/T_1$, w_{eq} is the inversion at thermal equilibrium and can be approximated by -1 for the optical transition. The first matrix on the right-hand side describes the interaction of the two-level system with the coherent light field, the second accounts for the depopulation, and the third for pure dephasing, the matrix elements $\Gamma_{i,j}$ are related to the model used in modification. With T_2 as the unit of time, all the variables in eq. (1) are dimensionless. From the Laplace transformation of eq. (1) and the initial condition

$$(u, v, w) = (0, 0, -1) \text{ at } t = 0,$$

$w(s, \Delta)$ can be derived. In OBE,

$$\Gamma_{11} = \Gamma_{22} = 1 - \gamma, \quad \Gamma_{12} = \Gamma_{21} = \Gamma_{13} = \Gamma_{23} = 0,$$

$$w(s, \Delta) = -1/s + K^2(s+1)/s[s^3 + (2+2\gamma)s^2 + (1 + 4\gamma + \Delta^2 + K^2)s + (2\gamma + 2\gamma\Delta^2 + K^2)]. \quad (2)$$

If the inhomogeneous line width is much larger than the homogeneous one, the hole shape can be expressed by a convolution

$$\int_{-\infty}^{\infty} [w(t, \Delta) + 1] g_h(\delta - \Delta) \, d\Delta,$$

where g_h is the normalized homogeneous line shape and δ is detuning. The convolution smoothens fine structures in $w(t, \Delta)$, if these structures span a frequency range smaller than 1. In the case where the width of $w(t, \Delta)$ is much greater than 1, the hole shape can be expressed by $w(t, \Delta) + 1$, the hole depth by $w(t, 0) + 1$ and the hole width (HWHM) can be derived from

$$w(t, \Gamma(t)) + 1 = [w(t, 0) + 1]/2. \quad (3)$$

For simplicity, we consider here only this case.

3. Results from the OBE

The value of γ may only be in the interval (0, 1]. If K is greater than $|\frac{1}{2} - \gamma|$, damping oscillation appears in $w(t, 0)$. Figure 1 shows examples of $w(t, 0)$ and $\Gamma(t)$ for $\gamma = 0, 0.05, 0.5$ and 1, $K = 2$.

As t tends to 0, the asymptote of $w(t, \Delta)$ can be derived from the inversion of a Laplace transform which tends to the same limit as $w(s, \Delta)$ when $s \to \infty$. Neglecting terms without s in the numerator and the denominator of eq. (2), we get the asymptote

$$w(t, \Delta) + 1 \to [K^2/(1 + 4r + K^2 + \Delta^2)] \times [1 - \exp[-(1+\gamma)t] \times [\cos bt + (1+\gamma) \sin bt/b]], \quad (4)$$

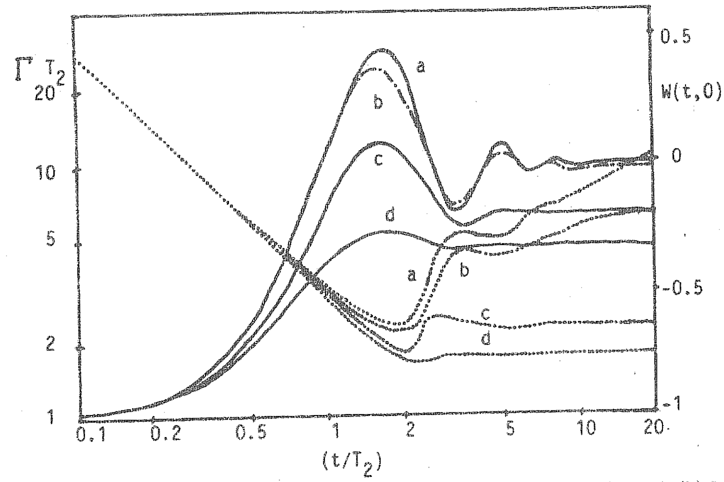

Fig. 1. Calculated $w(t, 0)$ (solid and dashed lines) and $\Gamma(t)$ (dotted lines) for $K = 2$, (a) $\gamma = 0$, (b) 0.05, (c) 0.5 and (d) 1.

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5, (c) 0.5 and (d) 1.

with $b^2 = 4\gamma + \Delta^2 + K^2 - \gamma^2$. If K is much greater than γ, and Kt is much less than 1, $\Gamma(t)$ satisfies the equation

$$\sin^2(\Gamma t/2)/(\Gamma t/2)^2 = \tfrac{1}{2}, \tag{5}$$

hence

$$\Gamma(t) = 2.7831/t, \tag{6}$$

which is the same as the Fourier width of a rectangular pulse of duration t. This is not surprising since both dephasing and depopulation do not influence the system so much in t shorter than their time constants, the spectrum of $w(t, \Delta)$ is therefore mainly determined by the Fourier transformation of the coherent light pulse.

If K is much larger than 1, eq. (4) may be further reduced to

$$w(t, \Delta) + 1 \to [K^2/(\Delta^2 + K^2)] \times \left[1 - \cos(\Delta^2 + K^2)^{1/2} t\right]. \tag{7}$$

Equation (7) holds for t within $[0, 2\pi/K]$, the first damping cycle of $w(t, 0)$. $w(t, 0)$ gets maximum at $t = \pi/K$. From eq. (7), $\Gamma(\pi/K) = 0.8K$. It is taken as the lower limit of the hole width in the region where it decreases with increasing pulse duration. If $\Gamma(\infty)$ is larger than $0.8K$, there must be a minimum at a certain time t_1 in $[\pi/K, \infty)$. From eq. (2),

$$\Gamma(\infty) = (1 + K^2/2\gamma)^{1/2} > K(2\gamma)^{-1/2}.$$

The inequality $K(2\gamma)^{-1/2} > 0.8K$ holds for

$$\gamma < 0.78. \tag{8}$$

Thus under a strong field, $\Gamma(t)$ has a minimum no more than $0.8K$ if γ is smaller than 0.78, according to the OBE.

Consider the dependence of $w(\pi/K, \Delta)$ on Δ. According to eq. (7), the nth peak is at $2[n(n-1)]^{1/2}K$, $w(t, \Delta) + 1$ is $1/(2n-1)^2$ of that at $\Delta = 0$. Δ between neighbouring peaks is $2\sqrt{2}K$ between the first and the second peaks and approaches to $2K$ for large n. Thus, if the burning time is π/K, a deep and considerably narrow hole may be burnt, however, besides the principal hole, there will be series of holes on both sides, the deepest among them is $\frac{1}{9}$ of the central one. It might be eliminated by reducing K or by using an even shorter pulse, as shown in figs. 3(c) and (e).

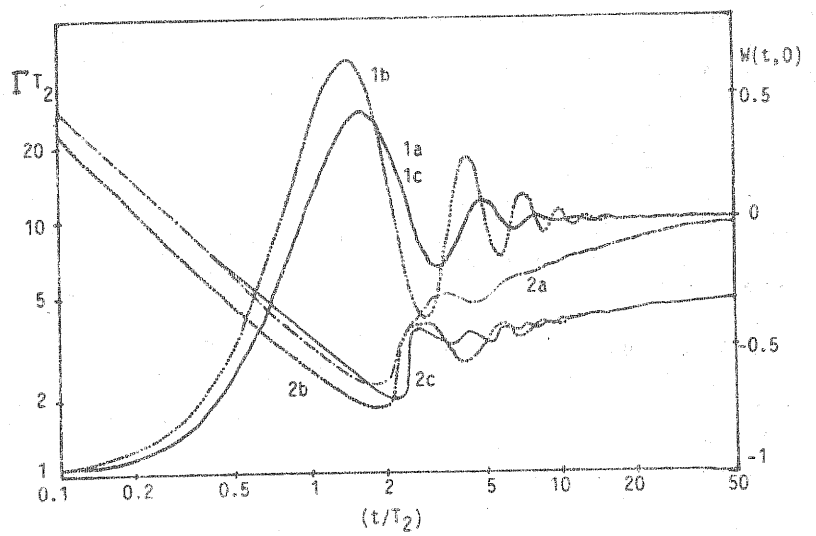

Fig. 2. $w(t, 0)$ (1) and $\Gamma(t)$ (2) for $K = 2$, $\gamma = 0.0217$, calculated with (a) OBE, (b) MG $\tau_c = 0.41475$ and (c) RT $\tau_c = 0.41475$.

4. Results from the modified OBE

The failure of the OBE to describe FID in $LaF_3:Pr^{3+}$ under a strong field [4] has led to the demand for modification of the OBE. In the modifications, a stochastic process (e.g. Gaussian–Markovian (GM) [5] or random telegraph (RT) [6] process) was used to describe the pure dephasing. Essentially, a new parameter τ_c, coherent time of the stochastic process, was introduced as a fitting parameter. Modification of the OBE also affects the description of hole burning. For the steady state under strong field, it predicts a non-Lorentzian hole shape with an HWHM much narrower than the OBE does [3,7]. The hole shape after a certain excitation pulse would be model-dependent, and it was suggested that the choice of the model could be determined via such a measurement [3].

With the same parameters as in ref. [3] ($\gamma = 0.0217$, $\tau_c = 0.41475$), $w(t, 0)$ and $\Gamma(t)$ are calculated as shown in fig. 2. A minimum of $\Gamma(t)$ exists at $t \approx \pi/K$. Hole shapes predicted by GM, RT and OBE are also calculated for $K = 20$ at $t = \pi/K$ (figs. 3(a)–(c)). No significant difference was found. This is because for $t < T_1, T_2$, the dominant effect is the interaction between the coherent light field and the system. Calculated HWHM are $0.79K$, $0.815K$ and $0.827K$ for GM, RT and OBE respectively, which are a little bit smaller than the steady-state HWHM predicted by GM and RT ($0.85K$ and $0.84K$, respectively). $\Gamma(\infty)$ depends on τ_c. $\tau_c \to 0$ brings the modified equation back to the OBE. In the limit $\tau_c \to \infty$, according to RT, $\Gamma(\infty)$ is $[\gamma^2 + K^2/2]^{1/2}$ and approaches to $\frac{1}{2}\sqrt{2}K$ for large K. It is narrower than $\Gamma(\pi/K)$. The modified optical Bloch equation in this case can be transformed by replacing s, K, Δ, with s/γ, K/γ and Δ/γ to the equation with $\gamma = 1$, hence eq. (8) does not hold.

In summary, with increasing of the laser pulse duration hole width decreases at first, a π pulse can burn a deep hole with HWHM about $0.8K$. Modification of the OBE does not significantly affect the description of hole burning at short time. A minimal hole width can be achieved with a pulse with a duration a little bit larger than π/K in the system where $T_2/2T_1$ is less than 0.78. This width is much narrower than the one OBE predicted, and is comparable to the narrowest predicted by the modified equation.

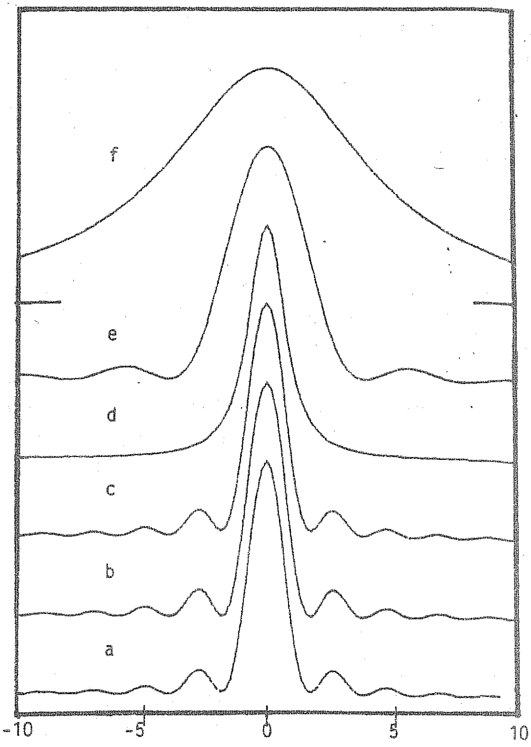

Fig. 3. Normalized hole shapes for $\gamma = 0.0217$, $K = 20$, $t = \pi/K$, calculated with (a) GM $\tau_c = 0.41475$, (b) RT $\tau_c = 0.41475$ and (c) OBE; (d) steady state with GM; (e) $t = \pi/2K$ and (f) steady state with OBE. The steady-state shape calculated with RT is close to (d).

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