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Shallow donors in a harmonic quantum dot in high magnetic fields

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Magnetic field effects on shallow donor impurity states in a harmonic quantum dot are investigated theoretically using the effective mass approximation within the strong confinement regime. The numerical results show that for high magnetic fields, the transition energies for $(0, 0) \rightarrow (0, 1)$ increase linearly with enhancement of the magnetic field strength, while the transition energies for $(0, 0) \rightarrow (0, -1)$ appear to diminish with increasing magnetic field strength.

With recent advances in nanofabrication technology it has become possible to confine electrons in all three spatial dimensions in semiconductors the so-called quantum dots [1–4]. The electron energy spectrum of such quantum dots is fully quantized. These quantum dots are often referred to as artificial atoms with the atomic potential replaced by the artificially constructed dot potential. The novel physical effects and potential device applications with these zero-dimensional structures are a subject of growing experimental [1–4] and theoretical [5–14] interest.

Recent experimental work [2] on quantum dots in a magnetic field has presented the interplay between the competing spatial and magnetic quantization. More recently, Dellow et al. [4] demonstrated experimentally the resonant tunneling through the bound states of a single donor atom in a quantum dot (in the presence of a magnetic field). Several calculations [5–9] have been performed for an electron and an exciton in quantum dots. Merkt et al. [5] have calculated energy spectra of two electrons in a harmonic quantum dot as a function of the dot size and the strength of the magnetic field in the effective mass approximation. Zhu et al. [6] calculated the binding energy of a hydrogenic donor in a spherical quantum dot (in the absence of a magnetic field). In a recent paper, Halonen et al. [7] investigated the properties of an exciton in a parabolic quantum dot in an external magnetic field using the effective mass approximation. The results for the energy and the optical absorption of the ground state and the low-lying excited states were presented. In this Letter, we shall investigate the dot size and the magnetic field effects on shallow donor impurity states in a harmonic quantum dot.

The electrons are much more strongly confined in one direction (taken as the z direction) than in the other two directions. Therefore, we shall confine ourselves to consider only the motion of the electrons in the x–y plane. Kumar et al. [9] have shown that the evolution of energy levels in a quantum dot with increasing magnetic field is similar to that for a harmonic potential. Recently, it was pointed out by Peeters [10] that this is a consequence of the generalized Kohn theorem which is valid for a harmonic confinement potential. These results make the harmonic confining potential model very appealing. We shall use this harmonic confining potential model in the present work.

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We assume that the confining potential in a single quantum dot is harmonic: $V(r)=\frac{1}{2}m^*\omega_0^2r^2$, where m^* is the effective mass of the electron and r is the coordinate vector of a two-dimensional quantity. For the sake of analytic convenience, we consider that a shallow donor is located at the center of the harmonic quantum dot. In the presence of a magnetic field in the z direction, the Hamiltonian for this system in the effective mass approximation is given by

$$H=H_0+H_1\,, \tag{1}$$

$$H_0=\frac{1}{2m^*}(p+eA)^2+\tfrac{1}{2}m^*\omega_0^2r^2\,, \tag{2}$$

$$H_1=-\frac{e^2}{4\pi\epsilon_0\epsilon r}\,, \tag{3}$$

where ϵ is the dielectric constant of the medium.

In the symmetric gauge $A=(-\frac{1}{2}yB,\ \frac{1}{2}xB)$, the energy levels of the unperturbed Hamiltonian H_0 are given by [15,16]

$$E_{nm}^{(0)}=(2n+|m|+1)\hbar\omega_c(\tfrac{1}{4}+\omega_0^2/\omega_c^2)^{1/2}+\tfrac{1}{2}m\hbar\omega_c\,, \tag{4}$$

where $\omega_c=eB/m^*$ is the cyclotron frequency, m is the angular quantum number, $m=0,\ \pm1,\ \pm2,\ldots$, and n is the radial quantum number, $n=0,\ 1,\ 2,\ldots$. The corresponding wave functions are given by

$$|n,m\rangle=\frac{1}{\sqrt{2\pi}}e^{im\theta}\left(\frac{2m^*\omega_c(\frac{1}{4}+\omega_0^2/\omega_c^2)^{1/2}n!}{\hbar(n+|m|)!}\right)^{1/2}\times x^{|m|}L_n^{|m|}(x^2)\exp(-\tfrac{1}{2}x^2)\,, \tag{5}$$

where

$$x=r\left(\frac{m^*\omega_c(\frac{1}{4}+\omega_0^2/\omega_c^2)^{1/2}}{\hbar}\right)^{1/2}\,,$$

$L_n^{|m|}$ are associated Laguerre polynomials.

As in ref. [5], we shall treat the Coulomb interaction H_1 as a perturbative term and use first-order perturbation theory. This approach provides a good description as long as the dot radius is much less than the effective Bohr radius, which is called the strong confinement regime [17]. To describe larger dots of which the dot radius is much larger than the effective Bohr radius (called weak confinement regime), one would numerically diagonalize the exact problem by completely considering the Coulomb interaction. We shall treat this problem in a forthcoming paper.

Using first-order perturbation theory, we obtain the first-order Coulomb correction,

$$E_{nm}^{(1)}=\langle n,m|-\frac{e^2}{4\pi\epsilon_0\epsilon r}|n,m\rangle\,. \tag{6}$$

For the ground state (0, 0), the correction is given by

$$E_{00}^{(1)}=-\frac{e^2\sqrt{\pi}}{4\pi\epsilon_0\epsilon}\left(\frac{m^*\omega_c(\frac{1}{4}+\omega_0^2/\omega_c^2)^{1/2}}{\hbar}\right)^{1/2}\,. \tag{7}$$

For the excited states $(0,\ \pm1)$, the correction is given by

$$E_{0,\pm1}^{(1)}=-\frac{e^2\sqrt{\pi}}{8\pi\epsilon_0\epsilon}\left(\frac{m^*\omega_c(\frac{1}{4}+\omega_0^2/\omega_c^2)^{1/2}}{\hbar}\right)^{1/2}\,. \tag{8}$$

When only the lowest energy level is occupied, the selection rules allow only two excitations, $(0,0)\to(0,-1)$, and $(0,0)\to(0,1)$ [18,19]. Consequently, the relevant transition energy for the transition $(0,0)\to(0,1)$ is given by

$$\Delta E_{01}=(E_{01}^{(0)}+E_{01}^{(1)})-(E_{00}^{(0)}+E_{00}^{(1)})\,. \tag{9}$$

The relevant transition energy for the transition $(0,0)\to(0,-1)$ is given by

$$\Delta E_{0,-1}=(E_{0,-1}^{(0)}+E_{0,-1}^{(1)})-(E_{00}^{(0)}+E_{00}^{(1)})\,. \tag{10}$$

The numerical results of the transition energy between the ground state and the low-lying excited states in GaAs harmonic quantum dots are presented in figs. 1 and 2. We assume that the Landau radius is always smaller than the size of the quantum dot, which means that the electrons are always confined in the quantum dots. Figure 1 shows the transition energy for $(0,0)\to(0,1)$ in GaAs harmonic quantum dots as a function of the magnetic field strength for three effective confinement lengths ($l_0=60$, 70 and 80 Å). From fig. 1 we can see that for high magnetic fields, the transition energies increase linearly with enhancement of the magnetic field, and the smaller the quantum dot, the larger the transition energy. Figure 2 illustrates the transition energy for $(0,0)\to(0,-1)$ as a function of the magnetic field for three effective confinement lengths ($l_0=50$, 55 and 60 Å). We note that the magnetic fields are stronger than those in fig. 1. It is shown that the transition energies decrease with increasing

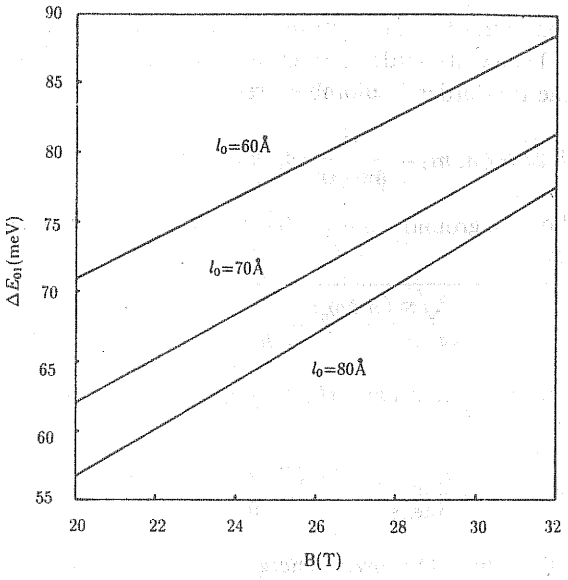

Fig. 1. The transition energy for $(0, 0) \rightarrow (0, 1)$ in GaAs harmonic quantum dots as a function of the magnetic field strength for three effective confinement lengths of the quantum dot $(l_0 = (\hbar/m^* \omega_0)^{1/2})$.

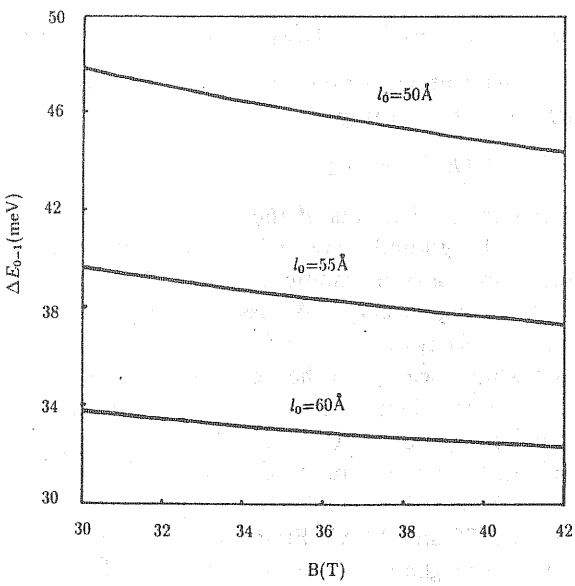

Fig. 2. The transition energy for $(0, 0) \rightarrow (0, -1)$ in GaAs harmonic quantum dots as a function of the magnetic field strength for three effective confinement lengths of the quantum dot $(l_0 = (\hbar/m^* \omega_0)^{1/2})$.

magnetic field strength. The values of the transition energy for $(0, 0) \rightarrow (0, -1)$ are always smaller than those for $(0, 0) \rightarrow (0, 1)$.

In summary, we have investigated the dot size and the magnetic field effects on shallow donor impurity states in a harmonic quantum dot. It is found that within the strong confinement regime, for high magnetic fields the transition energy for $(0, 0) \rightarrow (0, 1)$ increases linearly with enhancement of the magnetic field strength, while the transition energy for $(0, 0) \rightarrow (0, -1)$ appears to diminish with increasing magnetic field strength. The smaller the quantum dot, the larger the transition energy. It should be emphasized that the use of the 2D disk approximation for the electronic wave function is not an essential restriction of this paper, it has been used only for the sake of analytic convenience.

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