

# Resonant Magnetic Fields of a Magnetopolaron in a Parabolic Quantum Dot

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## Abstract

*It is shown that for a resonant magnetic field, the two cyclotron masses ( $m_+^*$  and  $m_-^*$ ) in a parabolic quantum dot are split into two cyclotron masses, respectively. The cyclotron mass ( $m_+^*$ ) decreases with decreasing the size of the quantum dot, the stronger the magnetic field strength, the larger the cyclotron mass. The lower branch of the cyclotron mass ( $m_-^*$ ) increases linearly with increasing the magnetic field strength. The upper branch of the cyclotron mass ( $m_-^*$ ) is larger than the bare band mass and enhances as the magnetic field strength increases.*

## 1. Introduction

In recent years, there is a great deal of interest in the investigation of quasi-zero-dimensional electronic systems (quantum dots) derived from originally two-dimensional electronic systems in  $\text{Al}_x\text{Ga}_{1-x}\text{As}/\text{GaAs}$  heterostructure or similar systems<sup>[1-3]</sup>. The electron energy spectrum of such quantum dots is fully quantized. These quantum dots are often referred to as artificial atoms which the atomic potential is in place of the artificially constructed dot potential. Because of the potential of wide device applications and a lot of new physical effects in such structures, understanding the electronic properties of these systems is of particular importance.

The interaction of the electrons with longitudinal-optical (LO) phonons in quantum dots has been investigated by various authors<sup>[4-9]</sup>. Recently, Roussignal *et al.*<sup>[4]</sup> have shown experimentally and explained theoretically that phonon broadening is quite important in very small semiconductor quantum dots. M.C. Klein *et al.*<sup>[6]</sup> studied the size dependence of electron-phonon coupling in semiconductor nanospheres, and derived the expression of the vibrational LO and SO eigenfunctions for a sphere in the continuum approximation. ZHU and GU<sup>[9]</sup> have shown that the polaron effects on quantum dots are larger than those on lateral quantum wires. However, all of them are in the absence of any external magnetic field. In this paper we consider the effect of an external magnetic field  $B$  (applied normal to the plane of the quantum dot) on zero-dimensional polarons in the weak-coupling limit. For the sake of analytic simplicity, we will model the relevant vibrational modes by the corresponding bulk modes, i.e., we will neglect any size quantization of the phonons. This assumption has been used by Schmitt-Rink, Miller, and Chemla<sup>[7]</sup> and Bockelmann and Bastard<sup>[8]</sup> to treat the phonon broadening of optical spectra and the phonon scattering in quantum dots. Taking into account the effect of phonon confinement, one would certainly vary the results in comparison with those of the bulk-phonon model. We will treat this effect in a forthcoming paper.

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## II. Theory

Since the electrons are much more strongly confined in one direction (taken as the  $z$  direction) than in other two directions, we will confine ourselves to consider only the motion of the electrons in the  $x - y$  plane. Kumar *et al.*<sup>[10]</sup> have shown that even if the defining cap layer is square shaped, the confining potential seen by electrons in a quantum dot has nearly circular symmetry. The energy levels are found to be insensitive to the charge in the dot at a fixed gate voltage, and the evolution of energy levels with increasing magnetic field is similar to that for a parabolic potential. These results make the harmonic confining potential model very appealing. In the presence of a magnetic field, this model potential offers exact analytic information on the single-particle energy states. We will use this parabolic confining potential model in the present work. In addition, the effects of a strong modification of the Coulomb forces through the electronic and ionic surface polarization can be significant if there is a large dielectric discontinuity between the quantum dot and the surrounding medium<sup>[11]</sup>. This is not the case for microfabricated dots made with GaAs wells and AlGaAs barriers, therefore we ignore such effects which will be taken into account in another paper.

We assume that the confining potential in a single quantum dot is parabolic:

$$V(\rho) = \frac{1}{2} m^* \omega_0^2 \rho^2, \quad (1)$$

where  $m^*$  is the bare band mass and  $\rho$  is the coordinate vector of a two-dimensional quantity. In the presence of a magnetic field in the  $z$  direction, the Hamiltonian of electron-phonon systems is given by

$$H = H_0 + H_1, \quad (2)$$

$$H_0 = \frac{1}{2m^*} (\mathbf{p} + e\mathbf{A})^2 + \frac{1}{2} m^* \omega_0^2 \rho^2 + \sum_{\mathbf{q}} \hbar \omega_{\text{LO}} b_{\mathbf{q}}^{\dagger} b_{\mathbf{q}}, \quad (3)$$

$$H_1 = \sum_{\mathbf{q}} (V_{\mathbf{q}} e^{i\mathbf{q}\cdot\mathbf{r}} b_{\mathbf{q}} + V_{\mathbf{q}}^{\dagger} e^{-i\mathbf{q}\cdot\mathbf{r}} b_{\mathbf{q}}^{\dagger}), \quad (4)$$

where  $b_{\mathbf{q}}^{\dagger}$  creates a bulk LO phonon of wave vector  $\mathbf{q}$ ,  $\mathbf{q} = (q_{\parallel}, q_z)$  and  $\mathbf{r} = (\rho, z)$  is the coordinate of the electron,

$$V_{\mathbf{q}} = i \frac{\hbar \omega_{\text{LO}}}{q} \left( \frac{\hbar}{2m^* \omega_{\text{LO}}} \right)^{1/4} \left( \frac{4\pi\alpha}{V} \right)^{1/2}. \quad (5)$$

In the symmetric gauge  $\mathbf{A} = (-\frac{1}{2} B y, \frac{1}{2} B x)$ , the energy levels of the unperturbed Hamiltonian  $H_0$  are given by<sup>[12,13]</sup>

$$E_{nm}^{(0)} = (2n + |m| + 1) \hbar \omega_c \left( \frac{1}{4} + \frac{\omega_0^2}{\omega_c^2} \right)^{1/2} + \frac{1}{2} m \hbar \omega_c, \quad (6)$$

where  $\omega_c$  is the cyclotron frequency,  $m$  is the angular quantum number,  $m = 0, \pm 1, \pm 2, \dots$ , and  $n$  is the radial quantum number,  $n = 0, 1, 2, \dots$ . The corresponding wave functions are given by

$$|n, m, 0_{\mathbf{q}}\rangle = \frac{1}{\sqrt{2\pi}} e^{im\theta} \left[ \frac{2m^* \omega_c (1/4 + \omega_0^2/\omega_c^2)^{1/2} n!^{1/2}}{\hbar(n + |m|)!} \right] x^{|m|} L_n^{|m|}(x^2) e^{-x^2/2} |0_{\mathbf{q}}\rangle, \quad (7)$$

where

$$x = \rho \left[ m^* \omega_c \frac{(1/4 + \omega_0^2/\omega_c^2)^{1/2}}{\hbar} \right]^{1/2},$$

$L_n^{[m]}$  are associated Laguerre polynomials, and  $|0_q\rangle$  is a vacuum state of the phonon, which satisfies  $b_q|0_q\rangle = 0$ .

Since the electron-phonon interaction is weak in these systems, in the sense that the Frohlich coupling constant ( $\alpha$ ) is of the order of 0.1, we will use the second-order perturbation theory to obtain the electronic self-energy shift  $\delta E_{nm}$ , which is given by

$$\begin{aligned} \delta E_{nm} &= \sum_{n'} \sum_{m'} \frac{|(n, m, 0_q | H_1 | n', m', 1_q \rangle|^2}{E_{nm}^{(0)} - E_{n'm'}^{(0)} - \hbar\omega_{LO}} \\ &= -\alpha(\hbar\omega_{LO})^2 \left( \frac{\hbar}{2m^* \omega_{LO}} \right)^{1/2} \sum_{n'} \sum_{m'} Q_{nmn'm'} \{ [2(n' - n) + (|m'| \\ &\quad - |m|)] \left( \frac{1}{4} + \frac{\omega_0^2}{\omega_c^2} \right)^{1/2} \hbar\omega_c + \frac{m' - m}{2} \frac{\hbar\omega_c + \hbar\omega_{LO}}{\omega_c} \}^{-1}, \end{aligned} \quad (8)$$

where

$$Q_{nmn'm'} = \int_0^\infty dq_{\parallel} [V_{nmn'm'}(q_{\parallel})]^2, \quad (9)$$

$$V_{nmn'm'}(q_{\parallel}) = 2 \left[ \frac{n'n!}{(n' + |m'|)(n + |m|)!} \right]^{1/2} \int_0^\infty dx x^{|m'| + |m| + 1} L_{n'}^{[m']}(x^2) L_n^{[m]}(x^2) J_{m-m'}(-q_{\parallel}x), \quad (10)$$

where

$$l = \left[ \frac{\hbar}{m^*(\omega_0^2 + \omega_c^2/4)^{1/2}} \right]^{1/2}$$

and  $J_n$  are Bessel functions of the first kind.

The electron-phonon coupling lifts the singularity of the electronic self-energy structure while the cyclotron frequencies

$$\omega_{c\pm} = \left[ \left( \frac{\omega_c}{2} \right)^2 + \omega_0^2 \right]^{1/2} \pm \frac{\omega_c}{2}$$

approach the phonon frequency ( $\omega_{LO}$ ). The relevant magnetic field is called the resonant terms. In this paper we only investigate the cyclotron resonance in this resonant region since the features of a magnetopolaron beyond the resonant region have been investigated in our previous paper<sup>[14-15]</sup>. When seeking for the value of  $E_{nm}$  in the resonant region, one is allowed to neglect all the minor off-resonant terms, and consider only the divergent terms in Eq. (8) due to the singularity<sup>[16-18]</sup>. It is obvious that it is not a feature of the actual polaron resonance phenomenon.  $E_{nm}^{(0)} - E_{n'm'}^{(0)}$  in Eq. (8) can be replaced by  $E_{nm} - E_{n'm'}$  so as to obtain Brillouin perturbation theory (IWBPT)<sup>[20]</sup>. Peeters *et al.*<sup>[21]</sup> have discussed the validity of different types of second-order perturbation theories and pointed out that IWBPT gives the correct pinning behavior for the weak-coupling case. This is the case under investigations. Concerning neglect of off-resonant terms, Peeters *et al.*<sup>[21]</sup> and Larsen<sup>[19]</sup> concluded that this treatment cannot obtain quantitative results, but leads to qualitatively correct ones. As a result we only include the resonant terms for simplicity. For the ground state, the energy in the resonant region can be written as

$$E_{00} = \hbar\omega_c \left( \frac{1}{4} + \frac{\omega_0^2}{\omega_c^2} \right)^{1/2} + \alpha(\hbar\omega_{LO})^2 \left( \frac{\hbar}{2m^* \omega_{LO}} \right)^{1/2} \frac{Q_{0000}}{E_{00} - E_{01} + \hbar\omega_{LO}}, \quad (11)$$

and

$$E_{00} = \hbar\omega_c \left( \frac{1}{4} + \frac{\omega_0^2}{\omega_c^2} \right)^{1/2} + \alpha(\hbar\omega_{LO})^2 \left( \frac{\hbar}{2m^*\omega_{LO}} \right)^{1/2} \frac{Q_{0000}}{E_{00} - E_{0-1} + \hbar\omega_{LO}}. \quad (12)$$

When only the lowest-energy level is occupied, the selection rules allow only two excitations, from the state  $(0, 0)$  to  $(0, -1)$ , and from  $(0, 0)$  to  $(0, 1)$ <sup>[22]</sup>. Consequently, the excited state  $(0, 1)$  energy in the resonant region can be written as

$$E_{01} = 2\hbar\omega_c \left( \frac{1}{4} + \frac{\omega_0^2}{\omega_c^2} \right)^{1/2} + \frac{\hbar\omega_c}{2} + \alpha(\hbar\omega_{LO})^2 \left( \frac{\hbar}{2m^*\omega_{LO}} \right)^{1/2} \frac{Q_{0101}}{E_{01} - E_{00} - \hbar\omega_{LO}}, \quad (13)$$

and the excited state  $(0, -1)$  energy in the resonant region can be written as

$$E_{0-1} = 2\hbar\omega_c \left( \frac{1}{4} + \frac{\omega_0^2}{\omega_c^2} \right)^{1/2} - \frac{\hbar\omega_c}{2} + \alpha(\hbar\omega_{LO})^2 \left( \frac{\hbar}{2m^*\omega_{LO}} \right)^{1/2} \frac{Q_{0-10-1}}{E_{0-1} - E_{00} - \hbar\omega_{LO}}. \quad (14)$$

From Eqs. (11)–(14), we can obtain the four solutions of the cyclotron resonance frequency, which imply four renormalized cyclotron masses (two  $m_+^*$  and two  $m_-^*$ ). That is, the two cyclotron masses ( $m_+^*$  and  $m_-^*$ ) are split into two cyclotron masses, respectively. Since their expressions are too complicated, we only give the numerical results.

### III. Numerical Results

The numerical results of the cyclotron mass in GaAs harmonic quantum dots are presented in Figs. 1–3.

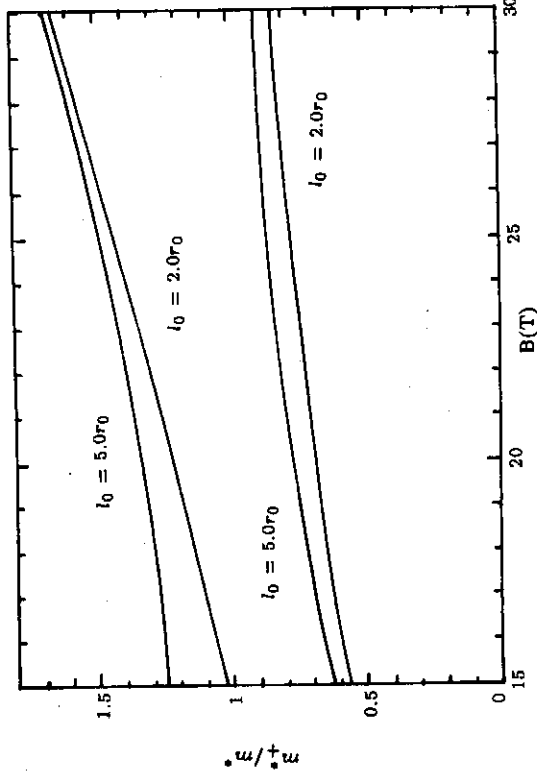


Fig. 1. The two splitting cyclotron resonance masses ( $m_+^*$ ) of GaAs harmonic quantum dots in the resonant magnetic field region as a function of the magnetic field strength ( $B$ ) for two effective confinement lengths, ( $r_0$  is the polaron radius).

Figure 1 shows the two splitting cyclotron resonance masses ( $m_+^*$ ) of GaAs harmonic quantum dots in the resonant magnetic field region as a function of the magnetic field strength ( $B$ ) for two effective confinement lengths. From the figure we can see that in the resonance magnetic field, the cyclotron mass increases with increasing the magnetic field strength. The smaller the quantum dot, the smaller the cyclotron mass. The figure also shows that the upper branch of the cyclotron mass ( $m_+^*$ ) illustrates a significant enhancement of the polaronic effect, which

is very similar to that in the two-dimensional case. And the lower branch of the cyclotron mass ( $m_+^*$ ) has a negative polaronic mass renormalization.

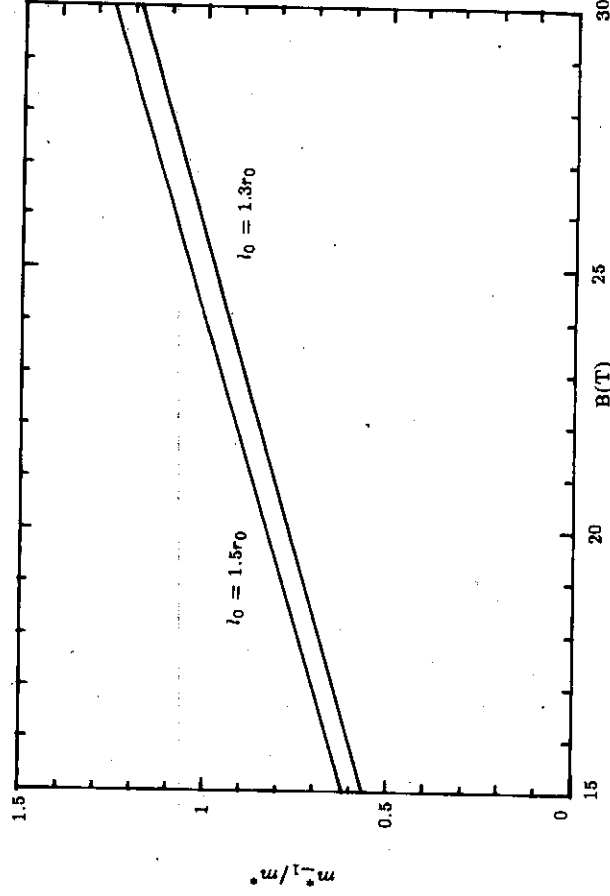


Fig. 2. The lower branch ( $m_{-1}^*$ ) of the two splitting cyclotron resonance masses ( $m_{-}^*$ ) of GaAs harmonic quantum dots in the resonant magnetic field region as a function of the magnetic field strength ( $B$ ) for two effective confinement lengths ( $r_0$  is the polaron radius).

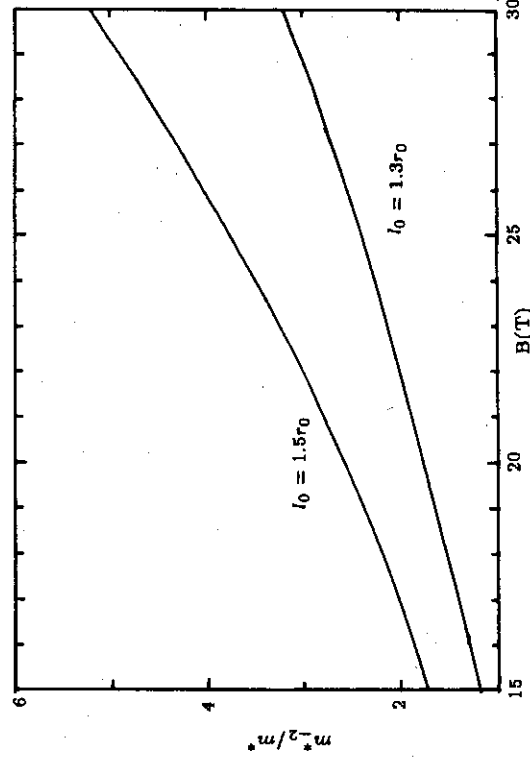


Fig. 3. The upper branch ( $m_{-2}^*$ ) of the two splitting cyclotron resonance masses ( $m_{-}^*$ ) of GaAs harmonic quantum dots in the resonant magnetic field region as a function of the magnetic field strength ( $B$ ) for two effective confinement lengths ( $r_0$  is the polaron radius).

Figures 2 and 3 demonstrate the two splitting cyclotron resonance masses ( $m_{-}^*$ ) of GaAs harmonic quantum dots in the resonant magnetic field region as a function of the magnetic field strength ( $B$ ) for two effective confinement lengths. It is shown that the lower branch ( $m_{-1}^*$ ) of the cyclotron mass ( $m_{-}^*$ ) increases linearly with enhancing the magnetic field strength. The

upper branch ( $m_{-2}^*$ ) also increases monotonically with increasing the magnetic field strength and is much larger than the bare band mass in small quantum dots.

#### IV. Conclusion

In conclusion, we have investigated the cyclotron resonance of a magnetic topolaron in a harmonic quantum dot in the resonance magnetic fields. We find that for resonant magnetic fields, the two cyclotron masses ( $m_+^*$ ) and ( $m_-^*$ ) in a harmonic quantum dot are split into two cyclotron masses, respectively. The cyclotron mass ( $m_+^*$ ) decreases with decreasing the size of the quantum dot, the stronger the magnetic field strength, the larger the cyclotron mass. The lower branch of the cyclotron mass ( $m_-^*$ ) increases linearly with increasing the magnetic field strength. The upper branch of the cyclotron mass ( $m_+^*$ ) is larger than the bare band mass and enhances as the magnetic field strength increases. The results indicate that the polaronic effects can be shown up in experiments in quantum dots rather than in two-dimensional quantum wells. It should be emphasized that the use of 2D disk approximation for the electronic wave function is not an essential restriction of this paper, it has been done only for the sake of analytic convenience and clarity of the final results. Introduction of the finite width of the electronic wave function in  $z$  direction into the above formalism is straightforward and it will reduce the effective electron-phonon coupling. But the qualitative features in this paper are independent of this approximation. Finally, it is hoped that this paper would stimulate more experimental work which will be helpful in an understanding of the role of electron-LO-phonon interaction in quasi-zero dimensional systems.

#### References

- [1] C. Sikorski and U. Merkt, *Phys. Rev. Lett.* **62**(1989)2164.
- [2] T. Demel, D. Heitmann, P. Grmbow and K. Ploog, *Phys. Rev. Lett.* **64**(1990)788.
- [3] A. Lorke, J.P. Kotthaus and K. Ploog, *Phys. Rev. Lett.* **64**(1990)2559.
- [4] P. Roussignol, D. Ricard and C. Flytzanis, *Phys. Rev. Lett.* **62**(1989)312.
- [5] M.G. Bawendi, W.L. Wilson, L. Rothberg, P.J. Carrol, T.M. Jedju, M.L. Steigerwald and L.E. Brus, *Phys. Rev. Lett.* **65**(1990)1623.
- [6] M.C. Klein, F. Hache, D. Ricard and C. Flytzanis, *Phys. Rev.* **B42**(1990)11123.
- [7] S. Schmitt-Rink, D.A.B. Miller and D.S. Chemla, *Phys. Rev.* **B35**(1987)8113.
- [8] U. Bockelmann and G. Bastard, *Phys. Rev.* **B42**(1990)8947.
- [9] K.D. ZHU and S.W. GU, *J. Phys. C: Condens. Matter.* **4**(1992)1291.
- [10] A. Kumar, S.E. Laux, and F. Stern, *Phys. Rev.* **B42**(1990)5166.
- [11] L.E. Brus, *J. Chem. Phys.* **80**(1984)4403.
- [12] V. Fock, *Z. Phys.* **47**(1928)446.
- [13] C.G. Darwin, *Proc. Cambridge Philos. Soc.* **27**(1930)86.
- [14] K.D. ZHU and S.W. GU, preprint CCAST-91-36.
- [15] K.D. Zhu and S.W. Gu, preprint CCAST-91-44.
- [16] S.D. Sarma, *Phys. Rev. Lett.* **52**(1984)859; 1570(E).
- [17] S.D. Sarma and A. Madhukar, *Phys. Rev.* **B22**(1980)2823.
- [18] X.J. Kong, C.W. Wei and S.W. Gu, *Phys. Rev.* **B39**(1989)3230.
- [19] D.M. Larsen, *Phys. Rev.* **B30**(1984)4595; 4807.
- [20] G. Lindemann, R. Lassnig, W. Seidenbusch and E. Gornik, *Phys. Rev.* **B28**(1983)4693.
- [21] F.M. Peeters and J.T. Devreese, *Phys. Rev.* **B31**(1985)3689.
- [22] W. Que, G. Kirzenow and E. Castano, *Phys. Rev.* **B43**(1991)14079.