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Measurement of twist angle and surface torsional anchoring strength in a nematic liquid crystal cell

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In this paper a novel method is developed for measuring the twist angle and surface torsional anchoring strength in nematic liquid crystal (NLC) cells. This method is based on a technique developed from the Jones optical propagation matrix. From the measurement, the actual twist angle in NLC cell and the deviation of the LC director on the boundary of the cell from the rubbing direction of the substrate are obtained. A theoretical discussion shows that the surface torsional anchoring strength has a great influence on the value of the deviation, and hence so it can be calculated.

1. Introduction

In preparing experimental samples or practical devices, surface alignment of nematic liquid crystal molecules on a treated substrate is an important and inevitable process. The investigation of the mechanism of the liquid crystal molecular orientation has always fascinated liquid crystal scientists. Usually a rubbing process is used as a conventional way to anchor the surface LC molecules, and it is often considered that the surface anchoring is so strong as to fix the surface LC director in the rubbing direction, and the actual twist angle is just equal to the angle between the rubbing directions on two substrates. Usually the twisted NLC has a special native pitch, and between two glass plates without surface aligning treated layer the twist angle is usually decided by the cell gap and LCs native pitch, expressed as

$$\varphi_0 = 2\pi d/p. \qquad (1)$$

Here, φ_0 is the native twist angle, d is the cell gap, and p is the LC's pitch. For a pure non-chiral NLC material, $p \to \infty$, $\varphi_0 = 0$. When the angle φ_r between the two rubbing directions is not equal to the native twist angle φ_0, the actual twist angle φ_t is influenced by both the bulk elastic force and surface anchoring force of the LC, and the twist angle φ_t will deviate from φ_r. The deviation can be expressed as

$$2\Delta\varphi = \varphi_r - \varphi_t. \qquad (2)$$

Theoretical analysis given by us shows that the anchoring strength has a great influence on the value of this deviation. After the actual twist angle in the NLC cell is known, the deviation and the surface anchoring strength can be calculated. A convenient way to measure the actual twist angle has been proposed [1, 2], and will be developed in this paper. This supplies a new way to study the surface anchoring strength in NLC cells.

There are many methods to measure the polar anchoring energy, such as surface disclination [3, 4], the Freedericksz transition [5, 6], and high field [7, 8]. Information of torsional anchoring energy is relatively scarce, the use of a high magnetic field [9] or the Cano wedge cell [10] being the two main means of measurement. However, for the first method, in order to get a high magnetic field a complex system must be set up and so is very inconvenient; for the second method, a Cano wedge cell must be made which is also a little complex. By using the method introduced in this paper, the torsional

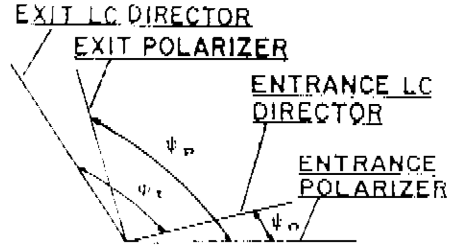

Figure 1. Directions of polarizers and LC cell.

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anchoring strength in NLC cells can be measured directly and simply.

2. Measurement of twist angle

According to the Jones optical propagation matrix, the optical transmission of NLC cells between two polarizers should be described by following formula [1]

$$T=\left\{\frac{1}{\sqrt{(1+u^2)}}\sin\left(\sqrt{(1+u^2)}\varphi_t\right)\sin(\varphi_t-\psi_p)+\cos\left(\sqrt{(1+u^2)}\varphi_t\right)\cos(\varphi_t-\psi_p)\right\}^2+\frac{u^2}{\sqrt{(1+u^2)}}\times\sin^2\left(\sqrt{(1+u^2)}\varphi_t\right)\cos^2(\varphi_t+2\psi_0-\psi_p) \quad (3)$$

where

$$u=\frac{\pi d}{\lambda\varphi_t}\left(\frac{n_e}{\sqrt{(1+w\sin^2\theta)}}-n_o\right)$$

and

$$w=\left(\frac{n_e}{n_o}\right)^2-1;\quad \theta=\frac{1}{d}\int_0^d\theta(z)\,dz.$$

Here, φ_t is the twist angle, ψ_p and ψ_0 are the angles of polarizers and entrance LC director, shown in figure 1. θ is the pretilt angle, n_e and n_o are the extraordinary and ordinary indices of refraction of the LC. λ is the wavelength of incident light. Transmission T can be minimizing with respect to ψ_p and ψ_0:

$$\left.\begin{aligned}\frac{\delta T}{\delta\psi_0}&=0,\quad \frac{\delta^2 T}{\delta\psi_0^2}>0,\\ \frac{\delta T}{\delta\psi_p}&=0,\quad \frac{\delta^2 T}{\delta\psi_p^2}>0.\end{aligned}\right\} \quad (4)$$

From equations (3) and (4), the minimal transmission conditions are given as

$$(1+u^2)\cos^2\left(\sqrt{(1+u^2)}\varphi_t\right)\sin(2\varphi_t-2\psi_p)-\sin^2\left(\sqrt{(1+u^2)}\varphi_t\right)\sin(2\varphi_t-2\varphi_p)-\sqrt{(1+u^2)}\sin\left(2\sqrt{(1+u^2)}\varphi_t\right)\cos(2\varphi_t-2\psi_p)=0. \quad (5)$$

The optical phase difference from NLC cell can be related by [2]

$$\mathrm{tg}(\delta_x-\delta_y)=A/B, \quad (6)$$

where

$$A=-u\sin\left(2\sqrt{(1+u^2)}\varphi_t\right)$$

and

$$A=\sqrt{(1+u^2)}\cos(2\varphi_t)\cos\left(2\sqrt{(1+u^2)}\varphi_t\right)+\sin(2\varphi_t)\sin\left(2\sqrt{(1+u^2)}\varphi_t\right).$$

Here, δ_x and δ_y are the optical phases of X axis and Y axis.

ψ_p under minimal transmission can be measured easily by rotating the LC cell and polarizers. The optical phase shift $(\delta_x-\delta_y)$ can be obtained by using a compensator.

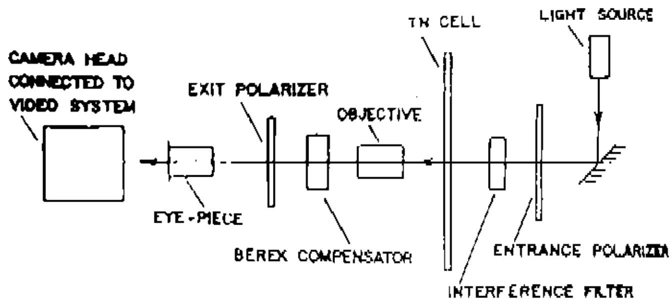

Figure 2. Arrangement of measuring equipment.

The arrangement of measuring equipment is shown in figure 2. Once ψ_p and $(\delta_x-\delta_y)$ are known, the actual twist angle φ_t can be calculated, by solving equations (5) and (6).

Using this method, the twist angles of several NLC cells were measured and the results are shown in the table, where φ_0 is the native twist angle, φ_r is the rubbing angle and φ_t is the twist angle. From the table, it is seen that the actual twist angles of LC cells have different deviations from φ_r. A theoretical analysis of the deviation is given later, which supplies a novel method to measure the surface torsional anchoring energy.

5. Theoretical analysis of surface torsional anchoring

According to continuum theory, the free energy per unit area is obtained as the sum of the bulk elastic energy F_b and the surface anchoring energy F_s as follows [10]:

$$F=F_b+F_s \quad (7)$$

where

$$F_b=\frac{K_2}{2d}(2\pi d/p-\varphi_t)^2$$

and

$$F_a=\tfrac{1}{2}A\sin^2\Delta\varphi.$$

Here, K_2 is the twist elastic constant, p is the LC's native pitch, d is the cell gap, A is the surface anchoring strength, $\Delta\varphi$ is the deviation of the surface LC director from the

Results of measurement. Here $p\rightarrow\infty$ means the LC material is a pure nematic LC.

	Sample				
	1	2	3	4	5
Pitch/μm	15	15	15	125	∞
$K_2/10^{-12}$ N m^{-1}	3·9	3·9	3·9	3·1	4·1
Gap/μm	8·9	9·1	8·1	9·6	10·1
ϕ_0/°	214	218	194	27·5	0
ϕ_r/°	270	270	225	90	90
ϕ_t/°	250	259	223	87	89
$2\Delta\phi$/°	20	11	2·0	3·0	1·0
$A/10^{-6}$ J m^{-2}	1·60	3·21	14·0	12·8	71·7

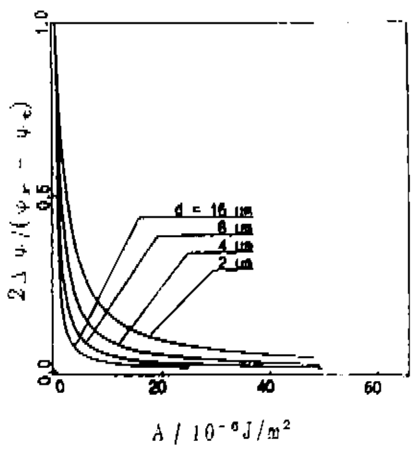

Figure 3. Theoretical relationship between A and $\Delta\phi$ at different values of cell gap. $K = 1 \times 10^{-12}\,\mathrm{N\,m^{-1}}$.

rubbing direction. φ_t is the actual twist angle which is related to φ_r and $\Delta\varphi$ by the following formula:

$$\varphi_t = \varphi_r - 2\Delta\varphi. \tag{8}$$

By minimizing F or $\delta F/\delta\Delta\varphi$, the torque balance equation can be given as

$$\frac{2K_2}{d}(\varphi_t - \varphi_0) - A\sin(2\Delta\varphi) = 0. \tag{9}$$

From this equation the surface anchoring strength A is expressed as

$$\begin{aligned}\frac{Ad}{K_2} &= \frac{2(\varphi_r - \varphi_0 - 2\Delta\varphi)}{\sin(2\Delta\varphi)}\\ &= \frac{2(\varphi_t - 2\pi d/p)}{\sin(2\Delta\varphi)}.\end{aligned} \tag{10}$$

In equation (10), φ_r, the cell gap d, the LCs twist elastic constant K_2 and pitch p are easy to obtain. Clearly from equation (10) A can be calculated when the twist angle φ_t is determined.

4. Results and discussion

The twist angles of various cells were measured and listed in the table, and the values of the surface torsional anchoring strength were calculated from equation (10). The results listed in the table show that the smaller the value of A, the greater is the deviation $\Delta\varphi$. In the limiting case of $A = 0$, from equation (9) we can get $\varphi_t = \varphi_0$, where φ_0 is the native twist angle. In the other limiting case, when the surface torsional anchoring energy $A \to \infty$, from equation (9) we can get $\Delta\varphi = 0$. This means no deviation of the LC director. Of course these are unavailable, but we can conclude from these cases that the stronger surface anchoring, the smaller will be the deviation. This trend is clearly shown in figure 3, of the theoretical curves of $\Delta\varphi$, versus A, which are obtained from equation (9).

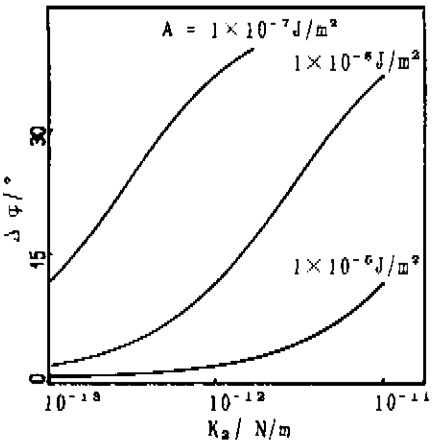

Figure 4. The relationship of $\Delta\phi$ and K_2. Here $\phi_r = 90°$, $\phi_0 = 0°$ and $d = 6\,\mu$m.

From the curve in figure 3 it can also be seen that the thicker the cell gap, the smaller the deviation $\Delta\varphi$, and this means that the bulk effect becomes weaker. In equation (10) it is shown that the cell gap has the same influence on the deviation as the surface anchoring energy.

The LC twist elastic constant K_2 still has great influence on the value of $\Delta\varphi$. From equation (10), we can obtain another form of equation (9)

$$\varphi_r - \varphi_0 = 2\Delta\varphi + C\sin(2\Delta\varphi). \tag{11}$$

Here

$$C = \frac{Ad}{2K_2}.$$

From the above equation, it is seen that the smaller the twist elastic constant K_2, the bigger is C, and the smaller the deviation $\Delta\varphi$. The relationship of K_2 and $\Delta\varphi$ is shown in figure 4.

From equation (11) we obtain figure 5, the theoretical relationship between $\Delta\varphi$ and $(\varphi_r - \varphi_0)$. In figure 5 it is

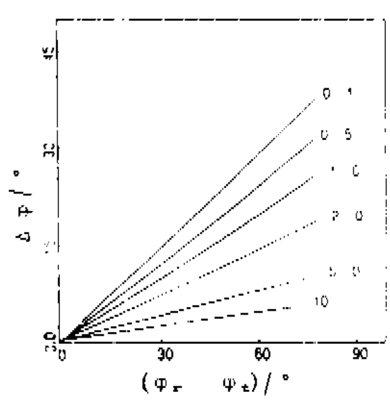

Figure 5. $\Delta\phi$ versus $(\phi_r - \phi_0)$. Cell gap = 6 μm; pitch = 15 μm; $K_2 = 5 \times 10^{-12}\,\mathrm{N\,m^{-1}}$. A values ($10^{-6}\,\mathrm{J\,m^{-2}}$) are given next to each curve.

shown that $\Delta\varphi$ is almost linearly proportional to the value of $(\varphi_r - \varphi_0)$.

As we know, the deviation of twist angle will reduce the contrast ratio of LCDs. In the table we find that even though the surface anchoring energy is almost $10^{-4}\,\mathrm{J\,m^{-2}}$, the difference of φ_r and φ_t is still equal to $1°$. However, by rubbing technology the surface torsional anchoring energy is usually about $10^{-5} \sim 10^{-7}\,\mathrm{J\,m^{-2}}$. In order to avoid the deviation, from figure 5 we can reduce the value of $(\varphi_r - \varphi_0)$. Another way is to calculate the value of the deviation theoretically, and then add the rubbing angle φ_r with an additional value of $\Delta\varphi_r$ to compensate for the deviation of the actual twist angle. But it is hard to know accurately the value of the surface torsional anchoring energy because it changes with the rubbing condition.

5. Conclusions

A new simple method for measuring the actual twist angle and surface torsional anchoring strength in NLC cells is introduced in this paper. From the measurement, the deviation of the LC director at the surface from the rubbing direction can also be obtained. Experimental and theoretical results show that the value of the deviation $\Delta\varphi$ cannot be neglected, and some ways to reduce the deviation have been proposed.

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