An integrated optics technique for measuring the refractive indices of liquids and gases: one and two-sided "leaky mode" methods

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The refractive indices of some liquids and gases were measured by using one and two-sided "leaky mode" methods. The liquid or gas was dropped or filled in the space between the prism and the substrate, so that these three layers formed one or two-sided leaky waveguides. The mode refractive indices of the resonant modes were measured by using a prism coupler, and the refractive index of the liquid or gas was determined from the corresponding mode equations. Measured results showed that the errors of the refractive indices are within $1-3\times10^{-4}$. The measurement precision was analyzed and is discussed theoretically.

I. INTRODUCTION

The prism coupler has been extensively used for measuring the thin dielectric-film parameters. In this measurement, the guided mode method is often used,[1,2] and it is applied for measuring the parameters of the high-refractive index thin film (optical waveguide) deposited on the low refractive index substrate. In addition, the leaky-mode method can also be used for measuring the thin film parameters. That is, the mode indices of the resonant leaky modes are measured by using a prism coupler, and the thin-film parameters are determined from the corresponding mode equations. At first this method has been used only for measuring the parameters of low-index thin films deposited on a high-index substrate or prism,[3,4] and it has been used later for measuring the optical waveguide parameters and hardened-glass surface-layer birefringences by the authors.[5–8] However, in previous measurements thin-film materials measured are usually solids. In this paper, the refractive indices of some liquids and gases were measured by using one and two-side leaky mode methods.

II. ONE AND TWO-SIDED LEAKY WAVEGUIDE

The liquid or gas is dropped or filled in between the prism and the substrate, so that under certain conditions these three-media layers form one or two-sided leaky waveguides, as shown in Fig. 1, where the zigzag model of a light ray is drawn, and n_2, n_1, and n_0 are the refractive indices of the prism, the liquid or gas, and the substrate, respectively, and d is the liquid or gas-layer thickness, θ_1 is the angle between the wave vector and the interface in medium i $(i=0,1,2)$.

When $n_2>n_1>n_0$, the light wave refracted from the prism to the liquid or gas layer is totally reflected at the 1–0 interface, but it is refracted and reflected at the 1–2 interface, and its energy is leaked to one-side for the prism to form a one-sided leaky mode. When n_2, $n_0>n_1$, the light wave is refracted and reflected at both 1–0 and 1–2 interfaces, and its energy is leaked to two sides for the prism and the substrate to form a two-sided leaky model.

Transverse phase shift of the light wave traveling in the liquid or gas layer for one trip back and forth is[9]

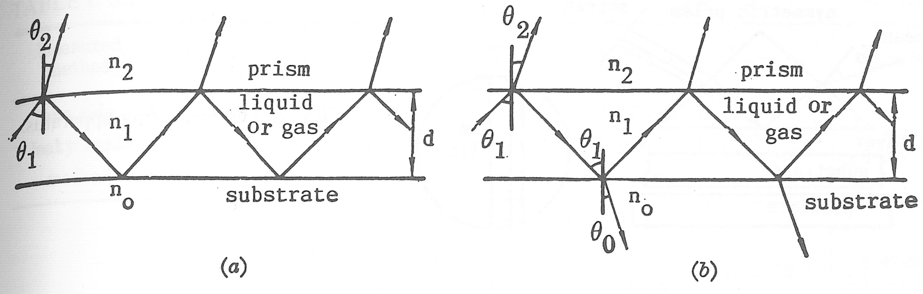

FIG. 1. (a) One-sided leaky waveguide ($n_2 > n_1 > n_0$); (b) two-sided leaky waveguide (n_2, $n_0 > n_1$).

$$\psi = 2k_1 d - 2\phi_{10} - 2\phi_{12}, \tag{1}$$

where m is the mode number, and $-2\phi 10$ and $-2\phi_{12}$ are the phase shifts of light waves reflected at 1–0 and 1–2 interfaces, respectively. When $\psi = 2m\pi$ ($m = 1,2,3,...$) (i.e., transverse resonant), we obtain the mode equation of the resonant leaky mode[9]:

$$k_1 d = m\pi + \phi_{10} + \phi_{12}, \tag{2}$$

$$\phi_{12} = (\pi/2) \quad (r_{12} < 0), \tag{3}$$

$$\phi_{10} = \begin{cases} \tan^{-1}\left[\left(\frac{n_1}{n_0}\right)^{2\alpha}\left(\frac{p_0}{k_1}\right)\right] & \text{one-sided leaky mode,} \\ (\pi/2) \quad (r_{10} < 0) & \text{two-sided leaky mode,} \end{cases} \tag{4}$$

$$\left.\begin{aligned} &k_1 = n_1 k \cos\theta_1 = (n_1^2 - N^2)^{1/2} k, \quad p_0 = (N^2 - n_0^2)^{1/2} k, \\ &N = n_i \sin\theta_i = n_1 \sin\theta_1, \end{aligned}\right\} \tag{5}$$

where N is the mode index, and r_{10} and r_{12} are the reflection coefficients of light waves reflected at 1–0 and 1–2 interfaces, respectively, and $k = (2\pi/\lambda)$ and λ are the wave number and the wavelength in vacuum, respectively.

From Eqs. (1)–(5), we can show that[9]

$$\frac{\partial\psi}{\partial N} = -2 d_f k \tan\theta_1, \tag{6}$$

$$d_f = \begin{cases} d + (J_0^\alpha p_0)^{-1}, & \text{one-sided leaky mode} \\ d & \text{two-sided leaky mode} \end{cases} \tag{7}$$

$$J_0 = \left(\frac{N}{n_0}\right)^2 + \left(\frac{N}{n_1}\right)^2 - 1. \tag{8}$$

Equation (6) is derived from the mode equation, where d can be regarded as the effective thin-film thickness (or the effective mode energy thickness) for the guided mode, and it can be simply regarded as the effective thin-film thickness for the leaky mode.

Equation (2) is used for determining the refractive index of the liquid and gas, and Eq. (6) is used for analyzing the measurement precision.

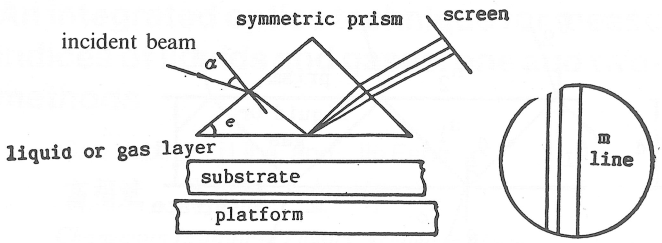

FIG. 2. The measuring apparatus.

III. MEASUREMENT

The measurement apparatus is shown schematically in Fig. 2, where the light source is a parallel optical beam with $\lambda = 0.6328\ \mu$m and 1 mm aperture, and the symmetric prism is made of ZF_6 glass, and the prism parameters are: $n_2 = 1.75132$, and $\epsilon = 45°33'46''$. For the one and two-sided leaky mode methods, quartz glass ($n_0 = 1.4571$) and K_9 glass ($n_0 = 1.51369$) is used as the substrate, respectively.

A symmetric prism was placed on the substrate surface, and after clamping the prism and the substrate together, it was placed on the two-dimensional translation stage of the rotating gonimeter table. The laser beam was directed onto the center of the prism base by adjusting the prism position. The liquid or gas was dropped or filled in between the prism and the substrate, so that the light wave was refracted from the prism to the liquid or gas layer to form one or two-sided leaky modes. The leaky-mode light wave emerging from the prism is projected on the observation screen, and the pressure of the prism fixture was adjusted to change the thickness of the liquid or gas layer; the incident angle α between the laser beam and the prism hypotenuse face was changed by turning the rotating table. When some m-lines appear and are brightest on the screen, one simultaneously reads the incident angle α_m, corresponding to the m-order resonant mode. the measured value of the mode index can be expressed as[1]

$$\overline{N}_m = \sin\alpha_m \cos\epsilon + (n_2^2 - \sin^2\alpha_m)^{1/2} \sin\epsilon, \tag{9}$$

and for each resonant mode, $\overline{N}_m$ was measured separately seven times in our experiment.

The refractive index of the liquid or gas was determined from a set of $\overline{N}_m$'s and corresponding mode equations, and its total error is[1]

$$\sigma = \sum_m [\overline{N}_m - N_m(n, d)]^2, \tag{10}$$

where $N_m(n,d)$ is the theoretical value of the mode index. For the given values of n_1 and d, theoretical values of the N_m's were obtained from Eq. (2). Changing n_1 and d, and taking the error sum σ to be the minimum, $\sigma_{\min}$, n_i, and d corresponding to $\sigma_{\min}$ were noted as $\overline{n}_1$ and $\overline{d}_0$, respectively. $\overline{n}_1$ is the measured value of the liquid or gas refractive index. In our calculation, the initial value of n_1 was chosen as[10]

$$n_1 = \frac{1}{3}(4\overline{N}_0 - \overline{N}_1) \quad 或 \quad n_1 = \frac{1}{10}(15\overline{N}_0 - 6\overline{N}_1 + \overline{N}_2), \tag{11}$$

and the initial value of d was obtained from Eq. (2) and Eq. (11), and the steps of n_1 and d were $\Delta_n = 10^{-6}$ and $k\Delta_d = 10^{-4}$, respectively. The measurement error can then be written as[1]

$$\Delta n_1 = \left[\frac{\sum_m |\Delta n_1(m)|^2}{(M-1)(M-2)}\right]^{1/2}, \tag{12}$$

$$\Delta n_1(m) = \frac{2\Delta_n[\overline{N}_m - N_m(\overline{n}_1, \overline{d})]}{N_m(\overline{n}_0 + \Delta_n, \overline{d}) - N_m(\overline{n}_1 - \Delta_n, \overline{d})}, \tag{13}$$

TABLE I. Measurement values of the refractive indices for some liquid and gas ($\lambda = 0.6326\,\mu m$, TE modes).

Measured method	dielectric	m	$\bar{N}_m$	$N_m(\bar{n}_1,\bar{d})$	$\bar{N}_m - N_m$	$\bar{n}_1 \pm \Delta n_1$	$\bar{d}$ (μm)
one side leaky mode method	triethanolamine	0	1.47097	1.47087	1.47247 1.2×10^{-4}	1.47247 $\pm1.5\times10^{-4}$	4.13
		1	1.46596	1.46613			
		2	1.45889	1.45882			
two sides leaky mode method		0	1.47207	1.47207	2.2×10^{-4}	1.47296 $\pm1.9\times10^{-4}$	6.16
		1	1.46930	1.46938			
		2	1.46524	1.46489			
		3	1.45832	1.45859			
	glycerin	0	1.46400	1.46407	2.2×10^{-4}	1.46561 $\pm1.7\times10^{-4}$	4.71
		1	1.45962	1.45944			
		2	1.45191	1.45169			
		3	1.44045	1.44077			
	air	0	0.99944	0.99909	7.4×10^{-4}	1.00026 $\pm3.4\times10^{-4}$	6.52
		1	0.99545	0.99559			
		2	0.98924	0.98974			
		3	0.98095	0.98148			
		4	0.96965	0.97076			
		5	0.95811	0.95750			
		6	0.94285	0.94158			

where M is the total number of measured modes.

The refractive index of triethanolamine was measured by using the one-sided leaky mode method, and the refractive indices of triethanolamine, glycerin, and air were measured by using the two-sided leaky mode method; the results are listed in Table I. The m-line photographs of the liquid measured are shown in Fig. 3, where (a) and (b) correspond to one and two-sided leaky mode, respectively.

It can be seen in Table I that the measurement error of the refractive indices is about 1–3×10^{-4} for the one and two-sided leaky mode methods. For triethanolamine, one and two-sided leaky mode methods give identical refractive indices within the error range. For air, the refractive index measured by using the two-sided leaky mode method is identical with the standard value (1.000294) at room temperature and normal pressure within the error range.

(a) (b)

FIG. 3. m-line photographs for measuring liquids (a) one-sided leaky mode; (b) two-sided leaky mode.

IV. ANALYSIS AND DISCUSSION

Regarding the liquid or gas layer as a thin-film interferometer, from multibeam interference principles we derived the light-energy density in the liquid or gas layer:[5,11]

$$w_1 = w_2 G(N)\left[1 + F(N)\sin^2\left(\frac{\psi}{2}\right)\right]^{-1}, \tag{14}$$

$$G(N) = \frac{1-\gamma_{12}^2}{(1-|r_{10}r_{12}|)^2}, \tag{15}$$

$$F(N) = \frac{4|r_{10}r_{12}|}{(1-|r_{10}r_{12}|)^2}, \tag{16}$$

$$r_{ij} = \frac{h_i n_j^{2\alpha} - h_j n_i^{2\alpha}}{h_i n_j^{2\alpha} + h_j n_i^{2\alpha}}, \tag{17}$$

$$h_i = n_i \cos\theta_i = (n_i^2 - N^2)^{1/2}, \tag{18}$$

where w_2 is the light-energy density of the incident light wave in the prism, and r_{10} and r_{12} are the reflection coefficients at the 1–0 and 1–2 interfaces,[11] respectively. In Eq. (14), $G(N)$ and $F(N)$ are both slowly-varying functions of N, but $\sin^2[\psi(N)/2]$ is a rapidly varying function of N. Obviously, the change of w_1 with N exhibits a resonant effect. That is, the curve of the w_1 vs N relation is a resonance curve. When $\psi(N_m) = 2m\pi$, w_1 is at its maximum, and the m-lines on the screen are brightest. The peak value N_m of the mode index of the m-order resonant leaky mode. In practical measurements, an observer is required to carefully observe the change of m-line light intensity near the peak value, and accurately determine the peak value position where the m-line light intensity is strongest. Thereby in addition to the systematic errors, the sensitivity for observing a change in m-line light intensity also influences the measurement precision of the mode index. For this, we shall give a derivation and theoretical analysis in the following.

The light intensity of a certain m-line is assumed as I. Since the light intensity is proportional to the light energy density, i.e. $I \propto w_1$, from Eq. (14), we then obtain the light intensity I:

$$I = AG\left[1 + F\sin^2\left(\frac{\psi}{2}\right)\right]^{-1}, \tag{19}$$

where A is the coefficient of proportionality. Near the peak value, assuming

$$\psi = 2m\pi + \Delta\psi, \quad |\Delta\psi| \ll 1, \tag{20}$$

and neglecting the change of G and F with N, Eq. (19) becomes

$$I \approx AG_m\left[1 - F_m\left(\frac{\Delta\psi}{2}\right)^2\right], \tag{21}$$

where $G_m = G(N_m)$, $F_m = F(N_m)$. From Eq. (21), we derive that

$$\left.\begin{aligned} &\frac{\Delta I}{I_m} = F_m\left(\frac{\Delta\psi}{2}\right)^2, \\ &\Delta I = I - I_m, \quad I_m = AG_m, \end{aligned}\right\} \tag{22}$$

where I_m is the peak value of the m-line light intensity. Assuming the detectable relative change of the m-line light intensity to be $(\Delta I/I_m)$, the corresponding sensitivity of observation can then be defined as

$$\eta = \left(\frac{\Delta I}{I_m}\right)^{-1}. \tag{23}$$

From Eqs. (6), (16), (22), and (23), we derive the measurement deviation of the mode index caused by the observing sensitivity:

$$\Delta N_\eta = \frac{\lambda}{4\pi d_f \tan\theta_1\sqrt{\eta}} \cdot \frac{1-|r_{10}r_{12}|}{\sqrt{|r_{10}r_{12}|}}. \tag{24}$$

TABLE II. Calculation values of N's.

[1]R. Ulrich and R. Torge, Appl. Opt. **12**, 2901 (1973).
[2]Jin Feng and Li Yushan, Chinese Physics **2**, 804 (1982).
[3]T. -N. Ding and E. Garmire, Appl. Opt. **22**, 3177 (1983).
[4]T. -N. Ding and E. Garmire, Opt. Commun. **48**, 113 (1983).
[5]Jin Feng, Acta Opt. Sin. **8**, 337 (1983).
[6]Jin Feng and Li Yushan, Acta Opt. Sin. **7**, 637 (1987).
[7]Jin Feng and Li Lina, Acta Opt. Sin. **8**, 1072 (1988).
[8]Li Lina, Jin Feng, and Li Yushan, Quant. Electron. **6**, 57 (1989).
[9]Jin Feng and Fan Junqing, *Integrated Optics* (Vol.1) (Press of National Defense Industry, Beijing, China, 1981), 1.2.
[10]Jin Feng, Acta Opt. Sin. **3**, 656 (1983).
[11]M. Born and E. Wolf, *Principles of Optics* (Pergamon Press, 1975), 1.5 and 7.6.

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