Calculation of the integral-order number of interference of a Fabry–Perot interferometer-fringe system

Zhu Shidong

Changchun Institute of Optics and Fine Mechanics, Academia Sinica, Changchun, China

Received April 28, 1986; accepted September 3, 1986

A modified method of excess fractions used for finding the correct integral orders of interference is given. In this modified method, besides an approximate value of the interferometer spacing $d' \pm \Delta d'$, either two known precise wavelengths satisfying the requirement of $|4(1/\lambda_2 - 1/\lambda_1)\Delta d'| \leq 1$ or three known precise wavelengths satisfying the requirement of $1 < |4(1/\lambda_2 - 1/\lambda_1)\Delta d'| \leq n$ are sufficient for finding the correct integral orders. I take the integral part of either $2(d' + \Delta d')/\lambda_1$ or $2(d' - \Delta d')/\lambda_1$ as m_{11}', continue using $m_{12}' + e_2' = (m_{11}' + e_1)\lambda_1/\lambda_2$, point out that the correct x in the expression $m_{11} = m_{11}' + x$ when calculating the correct integral order is the integer nearest $(e_2 - e_2' - z)/(\lambda_1/\lambda_2 - 1)$, give the expression $m_{12} = m_{12}' + x - z$, introduce integer z into the above-mentioned expressions, and give the selection rules of z. In comparison with the traditional methods, this modified method will avoid both repeatedly probing calculation and using even more wavelengths in measurements and checking computation, and it has rigorous, simple, and convenient features.

INTRODUCTION

The comparison of the wavelength of the primary standard with the wavelengths of other lines that are to serve as the secondary standard has been one of the important applications of a Fabry–Perot interferometer. For precision measurements of secondary-standard wavelengths, however, it is necessary to determine the precise value of the interferometer spacing. The customary methods in reducing interferometric data are to determine the integral orders of interference. In the traditional methods of exact fractions[1] and excess fractions[2] that can be used to determine the integral orders of interference, repeatedly probing calculation and using even more wavelengths in checking computation are necessary to determine the correct integral orders of interference. In the iterative method[3] used for the determination of the precise value of the interferometer spacing, many wavelengths must be used for the iterative measurements as well. The methods currently used are time consuming, strenuous, and inconvenient for the experimentalist. In this paper, a modified method of excess fractions is given. In this modified method, besides an approximate value of the interferometer spacing $d' \pm \Delta d'$, either two known precise wavelengths satisfying the requirement of $|4(1/\lambda_2 - 1/\lambda_1)\Delta d'| \leq 1$ or three known precise wavelengths satisfying the requirement of $1 < |4(1/\lambda_2 - 1/\lambda_1)\Delta d'| \leq n$ are sufficient for finding the correct integral orders; explicit expressions for calculation of the correct integral orders are given. In comparison with the traditional methods, this modified method will avoid both repeatedly probing calculation and using even more wavelengths in measurements and checking computation, and it has rigorous, simple, and convenient features.

MODIFIED METHOD OF EXCESS FRACTIONS

The path difference between successive rays emerging from a Fabry–Perot interferometer in vacuum is

$$m\lambda = 2d\cos\theta, \tag{1}$$

where m is the order number, which must be an integer for a bright fringe, λ is the wavelength, d is the interferometer spacing, and θ is the angle of the light rays to the normal of the interferometer plates. One does not take into account the phase change at reflection here. For $\theta = 0$, Eq. (1) becomes

$$(m_1 + e)\lambda = 2d, \tag{2}$$

where m_1 is the integral order of the innermost bright fringe and e, which is both less than unity and not less than zero, is the fractional order number at the center of the fringe system. Let λ_1, λ_2 be the known precise wavelengths. Then from Eq. (2) one has

$$(m_{11} + e_1)\lambda_1 = (m_{12} + e_2)\lambda_2 = 2d, \tag{3}$$

where m_{11}, m_{12} are the integral orders of the first bright rings and e_1, e_2 are the fractional orders at the center. For each line, the fractional orders may be obtained from measurements of the bright ring diameters. The integers m_{11}, m_{12} may be found by the modified method of excess fractions, which is just the subject of discussion in this paper. For this, as in the traditional method of excess fractions, an approximate value of the interferometer spacing $d' \pm \Delta d'$ is known from measurements performed with a good screw micrometer or a comparator, the uncertainty $\Delta d' > 0$, and a known precise wavelength λ_1 is chosen. Unlike the traditional method, one takes the integral part m_{11}' of either

$$m_{11}' + e_1' = \frac{2(d' + \Delta d')}{\lambda_1} \tag{4a}$$

or

$$m_{11}' + e_1' = \frac{2(d' - \Delta d')}{\lambda_1} \tag{4b}$$

as an approximate value of m_{11}, and one may write

$$m_{11} = m_{11}' + x, \tag{5}$$

where x is an unknown integer as well. It should be noted that, when m_{11}' is taken by Eq. (4a), $x \leq 0$ and one has

0740-3232/87/020352-04$02.00

Table 1. Selection Rules of z Based on Condition (7)

$\left(\frac{\lambda_1}{\lambda_2}-1\right)x_m = a$	$e_2 > e_2'$	$e_2 = e_2'$	$e_2 < e_2'$
$-1 \le a < 0$	1	0	0
$0 < a \le 1$	0	0	-1

$$-\frac{4\Delta d'}{\lambda_1} = x_m \le x \le 0; \tag{6a}$$

whereas when m_{11}' is taken by Eq. (4b), $x \ge 0$ and one has

$$0 \le x \le x_m = \frac{4\Delta d'}{\lambda_1}. \tag{6b}$$

If another known precise wavelength λ_2 is chosen that satisfies the following requirement:

$$\left|\left(\frac{\lambda_1}{\lambda_2} - 1\right)x_m\right| \le 1, \tag{7}$$

i.e.,

$$\lambda_1\Big/\left(1 + \frac{1}{|x_m|}\right) \le \lambda_2 \le \lambda_1\Big/\left(1 - \frac{1}{|x_m|}\right),$$

it is necessary only to calculate an approximate order of interference for the known line λ_2 as follows:

$$m_{12}' + e_2' = (m_{11}' + e_1)\frac{\lambda_1}{\lambda_2}. \tag{8}$$

Similarly to the reduction of the traditional method of excess fractions,[3] one can obtain the following relation:

$$m_{12} + e_2 = m_{12}' + e_2' + \left(\frac{\lambda_1}{\lambda_2} - 1\right)x. \tag{9}$$

It must be noted that I do not carry out the comparison between the calculated and measured fractions for the possible values of x permitted traditionally by either Eq. (6a) or Eq. (6b). Instead I introduce an integer z into Eq. (9) as follows:

$$m_{12} + e_2 = [m_{12}' + x - z] + \left[e_2' + z + \left(\frac{\lambda_1}{\lambda_2} - 1\right)x\right], \tag{10}$$

where the value of z is selected by equating the value of the second bracketed expression on the right-hand side of Eq. (10) to the measured fraction on the left-hand side of Eq. (10) and $|x| \le |x_m|$. If the requirement of expression (7) is satisfied, the value of z may be selected according to Table 1.

Now, by equating the second bracketed expression on the right-hand side of Eq. (10) to the measured fraction on the left-hand side of Eq. (10), one has

$$e_2 = e_2' + z + \left(\frac{\lambda_1}{\lambda_2} - 1\right)x; \tag{11}$$

thus the correct value of x is directly obtained:

$$x = \mathrm{Int}\left[\frac{e_2 - e_2' - z}{\dfrac{\lambda_1}{\lambda_2} - 1}\right], \tag{12}$$

where Int indicates that the integer nearest the value of the bracketed expression is taken. Then, by equating the integral order on the left-hand side of Eq. (10) to the first bracketed expression on the right-hand side of Eq. (10), one has

$$m_{12} = m_{12}' + x - z; \tag{13}$$

thus the correct integral order number m_{12} is found.

If another known precise wavelength λ_2 is chosen that does not satisfy the requirement of expression (7) but that satisfies the following requirement:

$$1 < \left|\left(\frac{\lambda_1}{\lambda_2} - 1\right)x_m\right| \le n, \tag{14}$$

then

$$\lambda_1\Big/\left(1 + \frac{n}{|x_m|}\right) \le \lambda_2 < \lambda_1\Big/\left(1 + \frac{1}{|x_m|}\right)$$

or

$$\lambda_1\Big/\left(1 - \frac{1}{|x_m|}\right) < \lambda_2 \le \lambda_1\Big/\left(1 - \frac{n}{|x_m|}\right),$$

where n is an integer both greater than 1 and less than $|x_m|$. Under this condition, the values of z may be selected according to Table 2.

The n different values of x satisfying the requirement of $|x| \le |x_m|$ can be obtained from Eq. (12) for the possible values of z, as given in Table 2; however, there is one and only one correct value of x among these. Under the present condition, besides the known line λ_2, the third known precise wavelength λ_3 should be chosen in order to find the correct value of x. If one understands another value of λ_2 by λ_3, the calculation for λ_3 can use all formulas with regard to λ_2. Under the conditions of both $x \le |x_m|$ and expression (14), the n different values of x can be obtained from Eq. (12) for the combination of $\lambda_1\lambda_3$ as well. Of course, the value n of λ_3 may be different from that of λ_2. Certainly, the n values of x given by Eq. (12) for the combination of $\lambda_1\lambda_3$ will be totally different from those of $\lambda_1\lambda_2$, except for the sole value of x that is in common among them, provided that λ_1 is the same one. This common value of x is just the correct value of x. Therefore, under condition (14), three known precise wavelengths are sufficient for finding the correct integral orders of interference.

Evidently, it is simpler and more convenient to choose two known wavelengths satisfying the requirement of expression (7) than three known wavelengths satisfying the require-

Table 2. Selection Rules of z Based on Condition (14)

$\left(\frac{\lambda_1}{\lambda_2}-1\right)x_m = a$	$e_2 > e_2'$	$e_2 = e_2'$	$e_2 < e_2'$
$-n \le a < 0$	$1, 2, \ldots, n$	$0, 1, \ldots, (n-1)$	$0, 1, \ldots, (n-1)$
$0 < a \le n$	$0, -1, \ldots, -(n-1)$	$0, -1, \ldots, -(n-1)$	$-1, -2, \ldots, -n$

Table 3. Correct Orders of Interference m_{1i} and e_i for the Known Wavelengths when $d = 12.05361436$ mm

i	λ_i (Å)	m_{1i}	e_i
1	5570.2890	43,278	0.237
2	5506.2627	43,781	0.472
3	5443.6916	44,284	0.707
4	5322.7206	45,291	0.178
5	5207.0093	46,297	0.649
6	5635.8218	42,775	0.002
7	5702.9149	42,271	0.767
8	5709.5452	42,222	0.678

ment of expression (14) for finding the correct integral orders.

It should be noted that the correct m_{11} is always found by Eq. (5), and, when the correct m_{12} is found by Eq. (13), one must use the value of z used for finding the correct x. After the correct m_{11} and m_{12} have been found as shown above, one may find the precise value of the spacing d with the relation

$$2d = (m_{11} + e_1)\lambda_1 = (m_{12} + e_2)\lambda_2.$$

Evidently, since the integral orders are not in doubt, the precision of d found by this method is confined only by the precisions of both the known wavelengths and the measured fractions.

NUMERICAL EXAMPLES

In order to show the manner of calculation of the modified method of excess fractions, one takes a checking example numerically. As is known, when the precise value of the interferometer spacing is $d = 12.05361436$ mm, the correct m_{1i} and e_i may be found by Eq. (2) for the known wavelengths, as given in Table 3.

Conversely, one assumes that an approximate value of the spacing $d' \pm \Delta d' = 12.059 \pm 0.006$ mm and the fractional orders of the known wavelengths listed in Table 3 have been measured. Now our task is to find the correct integral orders with the modified method of excess fraction. Let $\lambda_1 = 5570.2890$ Å; then using Eq. (4a) may give $m_{11}' = 43{,}319$ and using Eq. (6a) may give $-43 = x_m \leq x \leq 0$. When $\lambda_2 = 5506.2627$ Å is taken as another line, using Eq. (8) may give $m_{12}' = 43{,}822$, $e_2' = 0.949$. Now, when $(\lambda_1/\lambda_2 - 1)x_m = -0.5$ satisfies the requirement of expression (7) and $e_2 = 0.472 < e_2'$, one may select $z = 0$, according to Table 1; hence one may obtain $x = -41$ from Eq. (12), $m_{11} = 43{,}319 - 41 = 43{,}278$ from Eq. (5), and $m_{12} = 43{,}822 - 41 - 0 = 43{,}281$ from Eq. (13).

Let us examine the case satisfying the requirement of expression (14). If $\lambda_4 = 5322.7206$ Å is taken as another line, one may have $m_{14}' = 45{,}334$ and $e_4' = 0.085$. Now, $(\lambda_1/\lambda_4 - 1)x_m = -2$ and $e_4 = 0.178 > e_4'$; hence one may select $z = 1, 2$, according to Table 2 and may obtain $x_1 = -20$ and $x_2 = -41$ from Eq. (12). At present, the third known line must be employed to find the only correct value of x. Choosing $\lambda_5 = 5207.0093$ Å as another line, one may have $m_{15}' = 46{,}341$ and $e_5' = 0.509$. Now, $(\lambda_1/\lambda_5 - 1)x_m = -3$ and $e_5 = 0.649 > e_5'$, hence one selects $z = 1, 2, 3$ and may obtain $x_1 = -12$, $x_2 = -27$, $x_3 = -41$. When the values of x of the combination $\lambda_1\lambda_5$ are compared with those of the combination $\lambda_1\lambda_4$, one will obviously see that $x = -41$ is the correct one. Thus one may obtain $m_{11} = 43{,}278$, $m_{14} = 45{,}334 - 41 - 2 = 45{,}291$, and $m_{15} = 46{,}341 - 41 - 3 = 46{,}297$.

The same calculation may be carried out for the other wavelengths. The calculated results together with the conditions of selection for all the known wavelengths listed in Table 3 are summarized in Table 4.

By inspection of Table 4, one may see that the only correct value of x is found for all wavelengths satisfying expression (7); this shows that such two known lines are sufficient for finding the correct value of x. For all wavelengths satisfying expression (14), n values of x satisfying $|x| \leq |x_m|$ are found, and only one correct value of x is common to all these wavelengths. This shows such three known lines are sufficient for finding the correct value of x.

DISCUSSION

It should be pointed out that the key point of the modified method of excess fractions may be understood by introducing an integer z into Eq. (10). The reason for introducing z is that, because of the difference of $(\lambda_1/\lambda_2 - 1)x$ and e_2', the values of the bracketed expression in Eq. (9) can be lower than zero or not lower than 1, so the comparison between calculated and measured fractions will be difficult. Hence I introduce the proper integers z not only by making the value of the second bracketed expression on the right-hand side of Eq. (10) lower than 1 and not lower than zero but also by equating it to the measured fraction on the left-hand side of Eq. (10); this can make finding the correct x convenient. As we have already seen, the selection rules of z are just given by this principle with the addition of $|x| \leq |x_m|$.

Second, in order to find the correct value of x, the values of z must be correctly selected. In the traditional method of

Table 4. Calculated Results of the Modified Method Together with the Conditions of Selection Used

i	λ_i (Å)	m_{1i}'	e_i'	$\left(\frac{\lambda_1}{\lambda_i} - 1\right)x_m$	e_i/e_i'	z	x	m_{1i}
1	5570.2890	43,319					(−41)	43,278
2	5506.2627	43,822	0.949	−0.5	<1	0	−41	43,781
3	5443.6916	44,326	0.660	−1	>1	1	−41	44,284
4	5322.7206	45,334	0.085	−2	>1	1, 2	−20, −41	45,291
5	5207.0093	46,341	0.509	−3	>1	1, 2, 3	−12, −27, −41	46,297
6	5635.8218	42,815	0.525	0.5	<1	−1	−41	42,775
7	5702.9149	42,311	0.813	1	<1	−1	−41	42,271
8	5709.5452	42,262	0.678	1.05	=1	0, −1	0, −41	42,222

excess fractions, the sign of the possible values of x, i.e., $|x| \leq 2\Delta d'/\lambda_1$, can be either positive or negative; thus this makes selection of z difficult. In order to eliminate the ambiguity of the sign of the possible values of x, I give the new assumption of the calculation of m_{11}'. It will be convenient to take m_{11}' according to either Eq. (4a) or Eq. (4b). Now, certainly $x \leq 0$ for Eq. (4a) or $x \geq 0$ for Eq. (4b); hence the convenient criterion of selecting z can be provided.

Incidentally, I take the integral part rather than the nearest integer as m_{11}'. This is not only more convenient but also appears more appropriate. In fact, when an approximate value of the spacing is at a limit of the tolerance, if one takes the nearest integer, the correct x might go beyond the tolerance. For example, if one has the exact spacing $d = 1.100$ mm, an approximate value of the spacing $d' \pm \Delta d' = 1.106 \pm 0.006$ mm, and $\lambda_1 = 5123.4567$ Å, one has $m_{11} + e_1 = 2d/\lambda_1 = 4293.976$, $m_{11}' + e_1' = 2(d' + \Delta d')/\lambda_1 = 4340.819$, and $x_m = -4\Delta d'/\lambda_1 = -46.8 = -47$. Taking the integral part m_{11}' as an approximate value of m_{11}, one has $m_{11}' = 4340$, so $m_{11} - m_{11}' = -47$ does not go beyond the tolerance x_m. But, taking the nearest integer as an approximate value of m_{11}, one has $m_{11}' = 4341$, so $m_{11} - m_{11}' = -48$ obviously goes beyond the tolerance x_m. Although such cases are rare, they are, after all, possible; hence one prefers the integral part to the nearest integer.

In addition, a few remarks should be made about the proper selection of λ_2 satisfying expression (7). From expression (7) it was easily seen that the smaller $|x_m|$, i.e., the smaller $\Delta d'$ of an approximate spacing, can permit λ_2 to be farther away from λ_1. Provided that expression (7) is satisfied, to select λ_2 as far as possible away from λ_1 will be better for ensuring the precision of the calculation, because the large coefficient $(\lambda_1/\lambda_2 - 1)$ of x will give more contributions of x to the calculated fractions.

Finally, when the modified method is used for finding the integral orders by using a Fabry–Perot interferometer for wavelength comparison, apart from an approximate value of the spacing $d' \pm \Delta d'$ and a primary standard wavelength λ_s, another approximate wavelength λ_1' satisfying the requirement of $|4(1/\lambda_1' - 1/\lambda_s)\Delta d'| \leq 1$ or another two approximate wavelengths λ_1', λ_2' satisfying the requirement of $1 < |4(1/\lambda_i' - 1/\lambda_s)\Delta d'| \leq n$ $(i = 1, 2)$ are sufficient for finding the correct integral orders, provided that the uncertainties $(m_{11} + e_1)\Delta\lambda_i'/\lambda_i'$ are sufficiently small compared with unity; for example, $(m_{11} + e_1)\Delta\lambda_i'/\lambda_i' \sim 0.1$. The discussion about this problem may be similar to that in Ref. 2, but I will not discuss this in detail here.

SUMMARY

A modified method of excess fractions used for finding the correct integral orders of interference has been given. In this modified method, apart from an approximate value of the interferometer spacing $d' \pm \Delta d'$, either two known precise wavelengths satisfying the requirement $|(\lambda_1/\lambda_2 - 1)x_m| \leq 1$ or three known precise wavelengths satisfying the requirement $1 < |(\lambda_1/\lambda_2 - 1)x_m| \leq n$ are sufficient for finding the correct integral orders. I take the integral part m_{11}' of either $2(d' + \Delta d')/\lambda_1$ or $2(d' - \Delta d')/\lambda_1$ as an approximate value of m_{11}, continue using $m_{12}' + e_2' = (m_{11}' + e_1)\lambda_1/\lambda_2$, point out that the correct x in the expression $m_{11} = m_{11}' + x$ calculating the correct integral order is the integer nearest $(e_2 - e_2' - z)/(\lambda_1/\lambda_2 - 1)$, give the expression $m_{12} = m_{12}' + x - z$, introduce integer z, and give the selection rules of z. In particular, introducing z and giving the requirements of the wavelengths employed will make the calculation of the correct integral orders exceedingly simple and convenient. By comparison with the traditional methods, this modified method will avoid both repeatedly probing calculation and using even more wavelengths in measurements and checking computation, and it has rigorous, simple, and convenient features. Obviously, this modified method can be applied to each case in which the traditional method of excess fractions is applicable, for example, in the comparison of the unknown wavelengths with the standard wavelength.

ACKNOWLEDGMENT

The author is grateful to Wang Naihong for his interest in this work.

REFERENCES

1. K. W. Meissner, "Interference spectroscopy. Part 1," J. Opt. Soc. Am. 31, 405–427 (1941).
2. M. Born and E. Wolf, *Principles of Optics*, 6th ed. (Pergamon, Oxford, 1980), Sec. 7.6.4.
3. A. Fischer, R. Kullmer, and W. Demtröder, "Computed controlled Fabry–Perot wavemeter," Opt. Commun. **39**, 277–282 (1981).