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Physics Letters A 164 (1992) 465–469
North-Holland

Cyclotron resonance of a magnetopolaron in a harmonic quantum dot in resonant magnetic fields

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Received 4 September 1991; revised manuscript received 8 January 1992; accepted for publication 25 February 1992
Communicated by J. Flouquet

Cyclotron resonance of a magnetopolaron in a harmonic quantum dot with the resonant magnetic field normal to the plane of the quantum dot is investigated theoretically. It is shown that for a resonant magnetic field, the two cyclotron masses (m^*_+ and m^*_-) in a harmonic quantum dot are each split into two cyclotron masses, respectively. The cyclotron mass (m^*_+) decreases with decreasing size of the quantum dot, the stronger the magnetic field strength, the larger the cyclotron mass. The lower branch of the cyclotron mass (m^*_-) increases linearly with increasing magnetic field strength. The upper branch of the cyclotron mass (m^*_-) is larger than the bare band mass and enhances as the magnetic field strength increases.

In recent years, there has been much interest in the investigation of quasi-zero-dimensional electronic systems (quantum dots) derived from originally two-dimensional electronic systems in $Al_xGa_{1-x}As/GaAs$ heterostructures or similar systems [1–3]. The electron energy spectrum of such quantum dots is fully quantized. These quantum dots are often referred to as artificial atoms with the atomic potential in place of the artificially constructed dot potential. Because of the potential device applications and a lot of new physical effects in such structures, understanding the electronic properties of these systems is of particular importance.

The interaction of the electrons with longitudinal-optical (LO) phonons in quantum dots has been investigated by various authors [4–9]. Recently, Roussignol et al. [4] have shown experimentally and explained theoretically that phonon broadening is quite important in very small semiconductor quantum dots. Klein et al. [6] studied the size dependence of electron–phonon coupling in semiconductor nanospheres, they derived an expression for the vibrational LO and SO eigenfunctions for a sphere in the continuum approximation. Zhu and Gu [9] have shown that the polaron effects on quantum dots are larger than those on lateral quantum wires. However, all studies were performed in the absence of an external magnetic field. In this paper we consider the effect of an external magnetic field, B (applied normal to the plane of the quantum dot), on zero-dimensional polarons in the weak-coupling limit. For the sake of analytic simplicity, we will model the relevant vibrational modes by the corresponding bulk modes, i.e., we will neglect any size quantization of the phonons. This assumption has been used by Schmitt-Rink, Miller and Chemla [7] and Bockelmann and Bastard [8] to treat the phonon broadening of optical spectra and the phonon scattering in quantum dots. Taking into account the effect of phonon confinement would certainly vary the results in comparison with those of the bulk-phonon model. We will treat this effect in a forthcoming paper.

Since the electrons are much more strongly confined in one direction (taken as the z direction) than in the other two directions, we will consider only the motion of the electrons in the x–y plane. Kumar et

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al. [10] have shown that even if the defining cap layer is square shaped, the confining potential seen by electrons in a quantum dot has a nearly circular symmetry. The energy levels are found to be insensitive to the charge in the dot at a fixed gate voltage, and the evolution of energy levels with increasing magnetic field is similar to that for a harmonic potential. These results make the harmonic confining potential model very appealing. In the presence of a magnetic field, this model potential offers exact analytic information on the single-particle energy states. We will use this harmonic confining potential model in the present work. In addition, the effects of a strong modification of the Coulomb forces through the electronic and ionic surface polarization can be significant if there is a large dielectric discontinuity between the quantum dot and the surrounding medium [11]. This is not the case for microfabricated dots made with GaAs wells and AlGaAs barriers, therefore, we ignore such effects which will be taken into account in the forthcoming paper.

We assume that the confining potential in a single quantum dot is harmonic:

$$V(\boldsymbol{\rho})=\tfrac{1}{2}m^*\omega_0^2\rho^2\,, \tag{1}$$

where m^* is the bare band mass and $\boldsymbol{\rho}$ is the coordinate vector of a two-dimensional quantity. In the presence of a magnetic field in the z direction, the Hamiltonian of electron–phonon systems is given by

$$H=H_0+H_1\,, \tag{2}$$

$$H_0=\frac{1}{2m^*}(\boldsymbol{p}+e\boldsymbol{A})^2+\tfrac{1}{2}m^*\omega_0^2\rho^2+\sum_q \hbar\omega_{\rm LO}b_q^+b_q\,, \tag{3}$$

$$H_1=\sum_q[V_q\exp(\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r})b_q+V_q^+\exp(-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r})b_q^+]\,, \tag{4}$$

where b_q^+ creates a bulk LO photon of wave vector $\boldsymbol{q}$, $\boldsymbol{q}=(\boldsymbol{q}_\parallel, q_z)$ and $\boldsymbol{r}=(\boldsymbol{\rho}, z)$ is the coordinate of the electron,

$$V_q=\mathrm{i}(\hbar\omega_{\rm LO}/q)(\hbar/2m^*\omega_{\rm LO})^{1/4}(4\pi\alpha/V)^{1/2}\,. \tag{5}$$

In the symmetric gauge $\boldsymbol{A}=(-\tfrac{1}{2}By, \tfrac{1}{2}Bx)$, the energy levels of the unperturbed Hamiltonian H_0 are given by [12,13]

$$E_{nm}^{(0)}=(2n+|m|+1)\hbar\omega_c(\tfrac{1}{4}+\omega_0^2/\omega_c^2)^{1/2}+\tfrac{1}{2}m\hbar\omega_c\,, \tag{6}$$

where ω_c is the cyclotron frequency, m is the angular quantum number, $m=0, \pm1, \pm2, \ldots$, and n is the radial quantum number, $n=0, 1, 2, \ldots$. The corresponding wave functions are given by

$$|n, m, 0_q\rangle=\frac{1}{\sqrt{2\pi}}\,\mathrm{e}^{\mathrm{i}m\theta}\left(\frac{2m^*\omega_c(\frac{1}{4}+\omega_0^2/\omega_c^2)^{1/2}}{\hbar(n+|m|)!}\right)^{1/2}$$

$$\times x^{|m|}L_n^{|m|}(x^2)\exp(-\tfrac{1}{2}x^2)|0_q\rangle\,, \tag{7}$$

where

$$x=\rho[m^*\omega_c(\tfrac{1}{4}+\omega_0^2/\omega_c^2)^{1/2}/\hbar]^{1/2}\,, \tag{8}$$

$L_n^{|m|}$ are associated Laguerre polynomials, $|0_q\rangle$ is the vacuum state of the phonon, which satisfies $b_q|0_q\rangle=0$.

Since the electron–phonon interaction is weak in these systems, in the sense that the Fröhlich coupling constant (α) is of the order of 0.1, we will use second-order Rayleigh–Schrödinger perturbation theory to obtain the electronic self-energy shift, δE_{nm}, given by

$$\delta E_{nm}=\sum_{n'}\sum_{m'}\frac{|\langle n, m, 0_q|H_1|n', m', 1_q\rangle|^2}{E_{nm}^{(0)}-E_{n'm'}^{(0)}-\hbar\omega_{\rm LO}} \tag{9}$$

$$=-\alpha(\hbar\omega_{\rm LO})^2\left(\frac{\hbar}{2m^*\omega_{\rm LO}}\right)^{1/2}$$

$$\times\sum_{n'}\sum_{m'}Q_{nmn'm'}\{[2(n'-n)+(|m'|-|m|)]$$

$$\times(\tfrac{1}{4}+\omega_0^2/\omega_c^2)^{1/2}\hbar\omega_c$$

$$+\tfrac{1}{2}(m'-m)\hbar\omega_c+\hbar\omega_{\rm LO}\}^{-1}\,, \tag{10}$$

where

$$Q_{nmn'm'}=\int_0^\infty \mathrm{d}q_\parallel\,[V_{nmn'm'}(q_\parallel)]^2\,, \tag{11}$$

$$V_{nmn'm'}(q_\parallel)=2\left(\frac{n'!n!}{(n'+|m'|)!(n+|m|)!}\right)^{1/2}$$

$$\times\int_0^\infty \mathrm{d}x\,x^{|m'|+|m|+1}L_{n'}^{|m'|}(x^2)$$

$$\times L_n^{|m|}(x^2)J_{m-m'}(-q_\parallel lx)\,, \tag{12}$$

where $l=[\hbar/m^*(\omega_0^2+\tfrac{1}{4}\omega_c^2)^{1/2}]^{1/2}$ and J_n are Bessel functions of the first kind.

The electron–phonon coupling lifts the singularity of the electronic self-energy structure while the cyclotron frequencies ($\omega_{c\pm}=[(\frac{1}{2}\omega_c)^2+\omega_0^2]^{1/2}\pm\frac{1}{2}\omega_c$) approach the phonon frequency (ω_{LO}). The relevant magnetic field is called the resonant terms. In this paper we only investigate the cyclotron resonance in this resonant region since the features of a magnetopolaron beyond the resonant region have been investigated in our previous papers [14,15]. When seeking for the value of E_{nm} in the resonant region, one is allowed to neglect all the minor off-resonant terms, and consider only the divergent terms in eq. (9) due to the singularity [16–18]. It is obvious that this is not a feature of the actual polaron resonance phenomenon. $E^{(0)}_{nm}-E^{(0)}_{n'm'}$ in eq. (9) can be replaced by $E_{nm}-E_{n'm'}$ so as to obtain E_{nm} [19]. For the ground state, the energy, in the resonant region, can be written as

$$E_{00}=\hbar\omega_c(\tfrac{1}{4}+\omega_0^2/\omega_c^2)^{1/2}+\alpha(\hbar\omega_{LO})^2 \times\left(\frac{\hbar}{2m^*\omega_{LO}}\right)^{1/2}\frac{Q_{0000}}{E_{00}-E_{01}+\hbar\omega_{LO}} \tag{13}$$

and

$$E_{00}=\hbar\omega_c(\tfrac{1}{4}+\omega_0^2/\omega_c^2)^{1/2}+\alpha(\hbar\omega_{LO})^2 \times\left(\frac{\hbar}{2m^*\omega_{LO}}\right)^{1/2}\frac{Q_{0000}}{E_{00}-E_{0\,-1}+\hbar\omega_{LO}}\,. \tag{14}$$

When only the lowest-energy level is occupied, the selection rules allow only two excitations, from the state (0, 0) to (0, −1), and from (0, 0) to (0, 1) [20]. Consequently, the excited state (0, 1) energy, in the resonant region, can be written as

$$E_{01}=2\hbar\omega_c(\tfrac{1}{4}+\omega_0^2/\omega_c^2)^{1/2}+\tfrac{1}{2}\hbar\omega_c +\alpha(\hbar\omega_{LO})^2\left(\frac{\hbar}{2m^*\omega_{LO}}\right)^{1/2}\frac{Q_{0101}}{E_{01}-E_{00}-\hbar\omega_{LO}}, \tag{15}$$

and the excited state (0, −1) energy, in the resonant region, can be written as

$$E_{0-1}=2\hbar\omega_c(\tfrac{1}{4}+\omega_0^2/\omega_c^2)^{1/2}-\tfrac{1}{2}\hbar\omega_c +\alpha(\hbar\omega_{LO})^2\left(\frac{\hbar}{2m^*\omega_{LO}}\right)^{1/2}\frac{Q_{0\,-1\,0\,-1}}{E_{0\,-1}-E_{00}-\hbar\omega_{LO}}\,. \tag{16}$$

From eqs. (12)–(16), we can obtain the four solutions of the cyclotron resonance frequency, which imply four renormalized cyclotron masses (two m^*_+ and two m^*_-). That is, the two cyclotron masses (m^*_+ and m^*_-) are each split into two cyclotron masses, respectively. Since their expressions are too complicated, we only give the numerical results.

The numerical results of the cyclotron mass in GaAs harmonic quantum dots are presented in figs. 1–3. We assume that the Landau radius is always smaller than the sizes of the quantum dots, which means that the electrons are always confined in the quantum dots. Therefore, we choose two sizes of the quantum dots (r_0 is the polaron radius) in order to ensure that the curves m^* versus B have a physical meaning, which implies that in the range of B variations, the effective confinement lengths are always larger than the fundamental cyclotron radius and the cyclotron radius of the first excited levels. Fig. 1 shows the two-splitting cyclotron resonance mass (m^*_+) of GaAs harmonic quantum dots in the resonant magnetic field region as a function of the magnetic field strength (B) for two effective confinement lengths. From the figure we can see that in the resonant magnetic field, the cyclotron mass increases with increasing magnetic field strength. The smaller the quantum dot, the smaller the cyclotron mass. The figure also shows that the upper branch of the cyclotron mass (m^*_+) illustrates a significant enhancement of the polaron effect, which is very similar to that in the two-dimensional case. And the

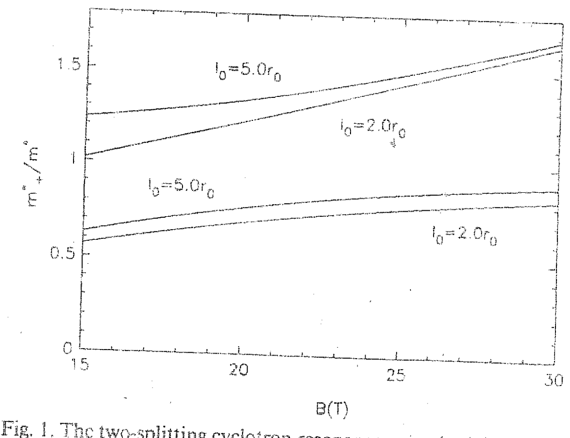

Fig. 1. The two-splitting cyclotron resonance mass (m^*_+) of GaAs harmonic quantum dots in the resonant magnetic field region as a function of the magnetic fields strength (B) for two effective confinement lengths. (r_0 is the polaron radius.)

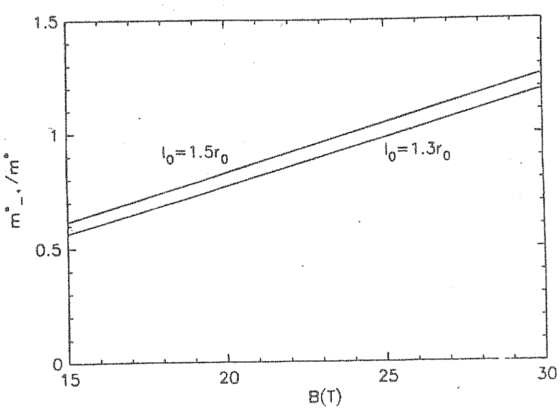

Fig. 2. The lower branch (m^*_{-1}) of the two-splitting cyclotron resonance mass (m^*_-) of GaAs harmonic quantum dots in the resonant magnetic field region as a function of the magnetic field strength (B) for two effective confinement lengths. (r_0 is the polaron radius.)

lower branch of the cyclotron mass (m^*_+) has a negative polaron mass renormalization. Figs. 2 and 3 demonstrate the two-splitting cyclotron resonance mass (m^*_-) of GaAs harmonic quantum dots in the resonant magnetic field region as a function of the magnetic field strength (B) for two effective confinement lengths. It is shown that the lower branch (m^*_{-1}) of the cyclotron mass (m^*_-) increases linearly with enhancing magnetic field strength. The upper branch (m^*_{-2}) also increases monotonically with increasing magnetic field strength and is much

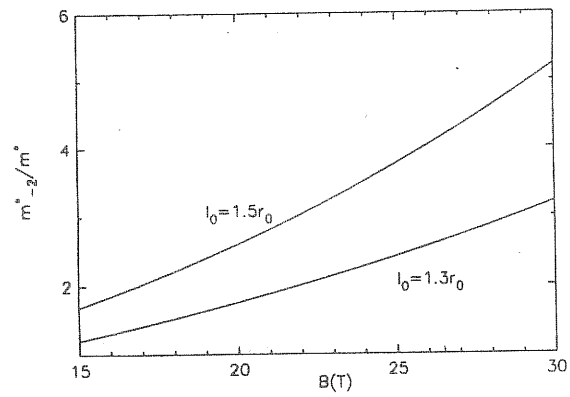

Fig. 3. The upper branch (m^*_{-2}) of the two-splitting cyclotron resonance mass (m^*_-) of GaAs harmonic quantum dots in the resonant magnetic field region as a function of the magnetic field strength (B) for two effective confinement lengths. (r_0 is the polaron radius.)

larger than the bare band mass in small quantum dots.

In conclusion, we have investigated the cyclotron resonance of a magnetopolaron in a harmonic quantum dot in resonant magnetic fields. We find that for resonant magnetic fields, the two cyclotron masses (m^*_+ and m^*_-) in a harmonic quantum dot are each split into two cyclotron masses, respectively. The cyclotron mass (m^*_+) decreases with decreasing size of the quantum dot, the stronger the magnetic field strength, the larger the cyclotron mass. The lower branch of the cyclotron mass (m^*_-) increases linearly with increasing magnetic field strength. The upper branch of the cyclotron mass (m^*_-) is larger than the bare band mass and enhances as the magnetic field strength increases. The results indicate that the polaronic effects can appear in experiments in quantum dots rather than in a two-dimensional quantum well. It should be emphasized that the use of the 2D disk approximation for the electronic wave function is not an essential restriction of this paper, it has been done only for sake of analytic convenience and clarity of the final-results. Introduction of the finite width of the electronic wave function in the z direction into the above formalism is straightforward and it will reduce the effective electron–phonon coupling. But the qualitative features in this paper are independent of this approximation. Finally, it is hoped that this paper will stimulate more experimental work which will be helpful in an understanding of the role of electron–LO-phonon interaction in quasi-zero-dimensional systems.

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