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# Simplified High-Electric-Field Technique for Measuring the Liquid Crystal Anchoring Strength

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A simple passive approach, employing a small current-regulating capacitor connected in series with a liquid crystal cell, is proposed and its utility demonstrated as a new scheme of implementing the high-electric-field technique (HEFT) for determining the surface anchoring strength of liquid crystals. The series capacitor removes the cumbersome need for measuring the cell capacitance at high fields from the HEFT, thereby drastically simplifying the operation. The simplified HEFT allows a direct linear relation between the optical phase retardation and the reciprocal of the applied voltage, and as in the original HEFT, the linear extrapolation of this relationship to infinite voltage provides the anchoring strength.

KEYWORDS: anchoring strength, extrapolation length, liquid crystal, polar anchoring, anchoring energy, surface alignment, high-electric-field technique

When a liquid crystal is put into contact with a substrate, an easy axis emerges along which the orientation of the liquid crystal is restricted.[1] The anchoring strength coefficient $W$, usually referred to as the anchoring energy, is a surface elastic constant representing the stiffness of the orientational restriction by way of an orientational Hook's law as

$$\gamma(\delta\theta) = \gamma_0 + \frac{1}{2} W \delta\theta^2 \qquad (1)$$

where $\delta\theta$ denotes a small angular deviation of the liquid crystal director from the easy axis, and $\gamma(\delta\theta)$ is the orientation-dependent interfacial free energy. Equation (1) is simply an expression that the larger the anchoring strength, the more elastic energy should be stored at the interface for a given angular strain. Since the orientation of the director needs two independent angles for its complete specification, the above definition of the anchoring strength should accordingly be generalized, making $W$ a tensor quantity.[1] Coupled, then, with the Frank-Oseen elastic free energy describing the bulk director field, eq. (1) constitutes a closed set of equations that allows an analysis of director configuration for any given geometry. In the face of growing stringency on the control of surface alignment in the advanced liquid crystal technology, there is now a resurgence of interest in quantitative characterization of the anchoring strength.

During the past three decades, a wide variety of techniques for determining the anchoring strength have been proposed and applied with varying level of success.[1,2] Of these, the high-electric-field technique (HEFT) is now regarded as one of the standard methods for polar (out-of-plane) anchoring strength measurement.[2–4] The HEFT consists in simultaneously measuring the optical phase retardation $R$ for a light beam passing perpendicularly through the liquid crystal cell and the cell capacitance $C$ as a function of the applied voltage $V$. For the range of voltage, taken well above the Freedericksz threshold $V_{\text{th}}$, the director tends to align along the electric field except in the vicinity of the substrate, where the anchoring force is in effect. The residual retardation from the boundary layer eventually disappears in the limit of $V \to \infty$, and as intuitively clear, the smaller the anchoring strength, the faster the decrease of retardation. It actually follows from the scaling property of the Frank-Oseen elastic theory that the residual retardation per boundary layer should satisfy the linear relation,

$$R = \frac{I_0}{CV} - \frac{2\pi}{\lambda} \Delta n_{\text{eff}}(\theta_{\text{e}}) d_{\text{e}}, \qquad (2)$$

as long as the voltage is low enough not to induce significant deviations of the boundary director from the easy axis. Here $I_0$ is a bulk constant independent of the anchoring strength, $\lambda$ the wavelength of probe light, $\theta_{\text{e}}$ the pretilt angle measured from the surface normal, $\Delta n_{\text{eff}}(\theta_{\text{e}}) \approx \Delta n \sin^2 \theta_{\text{e}}$ the effective birefringence corresponding to $\theta_{\text{e}}$, and finally $d_{\text{e}}$ the extrapolation length that derives from the anchoring energy as

$$d_{\text{e}} = \frac{K_{11} \sin^2 \theta_{\text{e}} + K_{33} \cos^2 \theta_{\text{e}}}{W} \qquad (3)$$

with $K_{11}$ and $K_{33}$ being the splay and bend constants, respectively. According to eq. (2), when the anchoring is infinitely strong ($W = \infty$, $d_{\text{e}} = 0$), the $R$ vs $1/CV$ plots should yield a straight line that passes through the origin, when extrapolated. The only effect of finite anchoring strength, in this medium range of voltage, is to cause a uniform downward shift of the straight line, the degree of which directly provides the extrapolation length.

The appearance of $C$ in front of $V$ in eq. (2) is a manifestation of the fact that the electrical displacement $D \propto CV$, rather than the electric field, is the primary independent field variable in the liquid crystal elasticity due to its constancy across the cell. Since the cell capacitance significantly changes as the director reorients in response to the applied voltage, $C$ cannot be cast into $I_0$ without deteriorating the precision. In practical use of the HEFT, therefore, one needs to carry out an independent measurement of the capacitance for each applied voltage ranging from 1 V up to as high as 200 V. Since such measurement conditions are not usually supported by commercially available capacitance meters, this part of the HEFT has been regarded as a technical barrier that had hampered even wider use of the HEFT. The purpose of the present Letter is to describe a simple passive method to alleviate this difficulty.

Let us consider a circuitry shown in Fig. 1, in which a fixed capacitor $C_{\text{a}}$ is connected in series with the liquid crystal cell as a passive current regulator. For a given total applied voltage $V_{\text{t}}$, the voltage drop across the liquid crystal cell is given by

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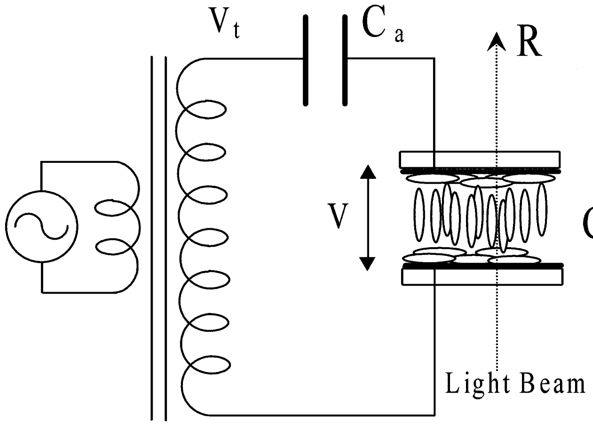


Fig. 1. Schematic diagram of the simplified high electric field technique (HEFT) using a small current-regulating capacitor $C_a$ connected in series with the liquid crystal cell.

$$V = \frac{C_a}{C + C_a} V_t. \tag{4}$$

Using this expression in eq. (2), we obtain

$$R = \left(\frac{1}{C} + \frac{1}{C_a}\right)\frac{I_0}{V_t} - \frac{2\pi}{\lambda}\Delta n_{\rm eff}(\theta_e)d_e. \tag{5}$$

Compared to eq. (2), the influence of the voltage dependence of $C$ on the linearity of $R$ vs $1/V_t$ plot is reduced by a factor of $C_a/(C + C_a)$. Hence, if $C_a$ is so chosen as to satisfy $C_a \ll C$, we may be allowed to ignore the change in the cell capacitance, thereby obtaining the direct linear relationship between the retardation and the reciprocal of the externally applied voltage. This is the basic idea of the simplified HEFT.

In order to quantitatively specify the condition for the additional capacitor and to evaluate the remaining errors, we note here the fact that the linearity in $1/CV$ applies not only to the retardation but also to any quantity that can be written as an integral of a function of the director over the cell thickness $d$. The reciprocal of the cell capacitance actually falls in this class of quantity:

$$\frac{1}{C} - \frac{1}{C_\infty} = \frac{1}{S}\int_0^d \frac{\Delta\varepsilon \sin^2\theta}{\varepsilon_1(\varepsilon_1\cos^2\theta + \varepsilon_2\sin^2\theta)}\,dz, \tag{6}$$

where $\varepsilon_1$ and $\varepsilon_2$ are the dielectric constants parallel and perpendicular to the director, $\Delta\varepsilon = \varepsilon_1 - \varepsilon_2$ the dielectric anisotropy (assumed positive here), $S$ the area of the electrode, and $C_\infty = S\varepsilon_1/d$ the limiting capacitance of the cell when the director is aligned completely homeotropically. In the high voltage range relevant to the HEFT, the two boundaries in a cell are orientationally decoupled, and the contribution from each boundary to the right-hand-side of eq. (6) becomes additive. For a cell consisting of identically treated substrates, for example, application of the Frank-Oseen elasticity theory to eq. (6) yields the capacitance version of eq. (2) as follows:

$$\frac{1}{C} - \frac{1}{C_\infty} = \frac{2J_0}{CV} - \frac{2\Delta\varepsilon\sin^2\theta_e}{S\varepsilon_1(\varepsilon_1\cos^2\theta_e + \varepsilon_2\sin^2\theta_e)}d_e. \tag{7}$$

Here, $J_0$ is a constant independent of the voltage and the anchoring condition, which is roughly given, under the one-constant approximation ($K_{11} = K_{33}$), by

$$J_0 \approx \frac{\Delta\varepsilon}{\sqrt{\varepsilon_1\varepsilon_2}}\frac{V_{\rm th}}{\pi}(1 - \cos\theta_e), \tag{8}$$

with $V_{\rm th} = \pi\sqrt{K_{11}/\Delta\varepsilon}$ being the threshold voltage of the Freedericksz transition. Given the form of eq. (7), one might suspect that the capacitance should by itself be a useful alternative to retardation in anchoring measurement. However, the presence of nonvanishing bulk contribution $1/C_\infty$ even in the high-voltage limit seriously restricts its utility. Actually, in order for this to be practical, $1/C_\infty$ should be independently known with a precision better than the anchoring contribution on the right-hand-side of eq. (7), which is equivalent to controlling the cell thickness with the accuracy better than $d_e$.

Substitution of eq. (7) into eq. (5) leads to a series expansion of $R$ in powers of $1/V_t$. Retaining the terms up to second order with the negligence of the anchoring contribution in eq. (7) as small, we obtain

$$R = \left(\frac{1}{C_\infty} + \frac{1}{C_a} + \frac{2J_0}{C_\infty V}\right)\frac{I_0}{V_t} - \frac{2\pi}{\lambda}\Delta n_{\rm eff}(\theta_e)d_e. \tag{9}$$

Consequently, the relative error in the slope of $R$ vs $1/V_t$ plot due to the voltage dependence of cell capacitance is found to be

$$\delta k = \frac{C_a}{C_\infty + C_a}\frac{2\Delta\varepsilon}{\sqrt{\varepsilon_1\varepsilon_2}}\frac{V_{\rm th}}{\pi V}(1 - \cos\theta_e), \tag{10}$$

while the absolute error in the retardation is given by $\delta k R$. Using an approximate formula for $R$,

$$R \approx d\frac{2\pi\Delta n}{\lambda}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\frac{V_{\rm th}}{\pi V}(1 - \cos\theta_e), \tag{11}$$

which is obtainable under the one-constant approximation with the neglect of anchoring effects, we reach an expression for the equivalent error in the extrapolation length as follows:

$$\delta d_e \approx -d\frac{C_a}{C_\infty + C_a}\frac{2\Delta\varepsilon}{\varepsilon_1}\left(\frac{V_{\rm th}}{\pi V}\right)^2\tan^2\frac{\theta_e}{2}. \tag{12}$$

This equation indicates that the error is always negative and decreases rapidly with an increase in the applied voltage relative to the Freedericksz threshold. As well known, the range of voltage for the HEFT to be valid is determined by the condition that the boundary layer be well approximated by a semi-infinite nematic system, and is specifically given by $V \geq 6V_{\rm th}$. By adopting the lower bound voltage and a typical value of $2\Delta\varepsilon/\varepsilon_1 \approx 1$ in eq. (12), we obtain an inequality giving the upper bound for the error in the extrapolation length:

$$0 \leq -\delta d_e \leq \frac{d}{400}\frac{C_a}{C_\infty + C_a}. \tag{13}$$

For a cell with $d = 40\,\mu$m, for example, we find $0 \leq -\delta d_e \leq 5$ nm even with a moderate choice of the series capacitor as $C_a \approx C_\infty/20$. A least-squares linear fitting performed over a decade of voltage above $6V_{\rm th}$ further reduces the error in $d_e$ by a factor of 6, thereby making the final error less than 1 nm. Since the extrapolation lengths for the majority of alignment layers of technical interest fall in the range from 10 nm to 1 $\mu$m, this level of error should be practically permissible.

The validity of the simplified HEFT has been experimentally tested for a homogeneously aligned cell with a pair of rubbed polyimide alignment layers ($\theta_e \approx \pi/2$). A 38.0 $\mu$m-thick cell with the effective electrode area of 1.0 $cm^2$ was fabricated using a polyester film spacer and was filled

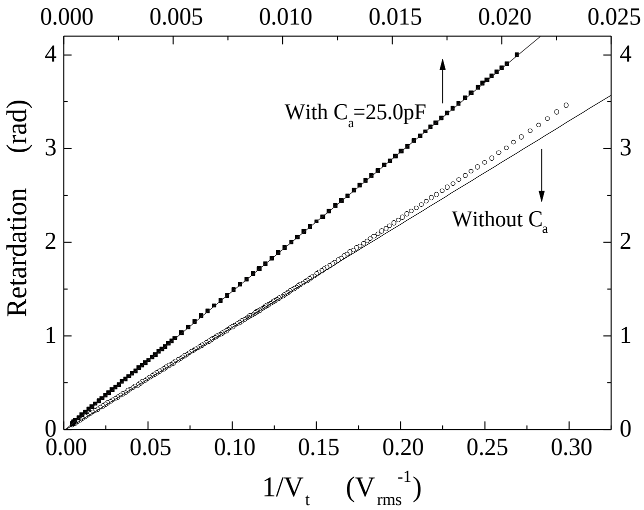


Fig. 2. $R$ vs $1/V_t$ plots with and without the current-regulating series capacitor. The retardation is given per boundary basis. The solid lines indicate the results of least-squares linear fitting: $R = 193.34/V_t - 0.012$ (with $C_a = 25.0$ pF) and $R = 11.01/V_t - 0.013$ (without $C_a$).

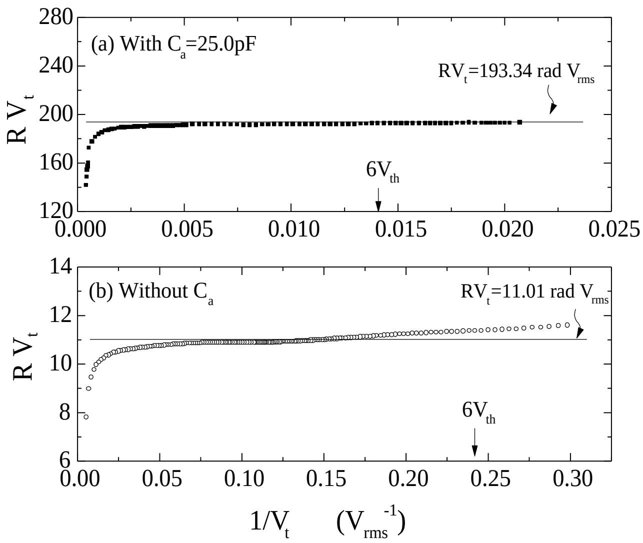


Fig. 3. $RV_t$ as a function of $1/V_t$: (a) with $C_a = 25.0$ pF and (b) without $C_a$. The steep drop of $RV_t$ near $1/V_t \approx 0$ reflects field-induced deviations of the boundary director from the easy axis as a result of a finite anchoring strength.

with 4-pentyl-4′-cyanobiphenyl (5CB). A 11.1 kHz sinusoidal AC voltage up to 2.5 kV was applied with the use of a 1 : 100 step-up transformer, and the retardation was measured with an automatic transmission ellipsometer using a 3 mW He–Ne laser and a photoelastic modulator. We employed a 25.0 pF series capacitor that resulted in $C_\infty/C_a = 16.5$. Figure 2 shows the $R$ vs $1/V_t$ plots measured at 30.0°C with and without the series capacitor. Apparently, the plot in the absence of a series capacitor shows significant deviations from linearity, particularly at lower voltages even for $V_t > 6V_{th}$; here, $V_{th} = 0.70V_{rms}$ and $\Delta n = 0.15$. On the other hand, we see that addition of the series capacitor nicely recovered the linearity in agreement with eq. (10). A least-squares linear fitting applied to this latter plot over the range of $0.0004V_{rms}^{-1} < V_t^{-1} < 0.02V_{rms}^{-1}$ yielded an extrapolation length of $d_e = 8.3\pm0.6$ nm. This value was confirmed to be identical to that simultaneously obtained by the conventional HEFT within the experimental error. The degree of improvement brought about by the series capacitor can be made even more visible by plotting $RV_t$ instead of $R$ as shown in Fig. 3. As clear from eq. (5), the constancy of $RV_t$ provides a direct measure for the goodness of the approximation to neglect the change in the cell capacitance. This approximation is obviously poor for the case without the series capacitor shown in Fig. 3(b), whereas, in Fig. 3(a) with the series capacitor, $RV_t$ remains virtually constant over a wide range of voltage except in the highest voltage region where the finite anchoring effect sets in.

In conclusion, we developed a new scheme of the high-electric-field technique (HEFT) that avoids simultaneous measurements of the cell capacitance by introducing a small capacitor in series with the liquid crystal cell.[5)] The essential role of the series capacitor is to passively regulate the AC current flowing through the cell against voltage-dependent changes in the cell capacitance, thereby making the total applied voltage directly proportional to the electrical displacement. Although an active current regulation is also feasible in the same sense as described in this Letter, the passive scheme using a series capacitor allows an easier yet sufficiently accurate implementation of the simplified HEFT for its wider use in liquid crystal industry, now facing various surface-anchoring issues on the daily basis. Finally, we want to emphasize that the significance of the present scheme is not limited to such a practical utility, but may be of more fundamental import as to open up new possibilities for further extension of the HEFT based on application of tailored external voltages realizing a variety of surface-sensitive modulation techniques.


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2) H. Yokoyama: Mol. Cryst. & Liq. Cryst. **165** (1988) 265.
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5) Recently, Nastishin *et al*. [J. Appl. Phys. **86** (1999) 4199] proposed a modified HEFT in which the capacitance measurement is avoided *a posteriori* by use of eq. (7) in eq. (2) while regarding $C_\infty J_0$ as an adjustable parameter. Although this technique introduces an additional fitting parameter, it has an advantage to be applicable even to samples with inhomogeneous tilt distribution and/or unpatterned electrodes.