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Model for Interaction Between ... d Cold Atom in QE ...*

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Abstract *A model has been establishe ... ction between a single-mode opti ... and a 2-energy-level cold atom with exact analytic solutions gi ... esses of momentum and ener ... ex ... between the optical field and the cold atom due to the interacti ... m are discussed in detail, an ... a fo ... s been given for the variation of momentum and energy exchar ... h time t in dress state while ... ot ... cts of photon recoil and Doppler effect are taken into conside ...*

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The researches on the Bose–Einstein c ... cold neutral atoms developed fast in recent ... the momentum and energy exchanges bet ... tons and mass center motion of atoms in C ... photon recoil and Doppler effect will inevit ... effect on the dynamic behavior of atoms. ... dealt with the numeral resolutions far a ... nance condition for this problem, and ref ... [7] dealt with the effect of quantum dissi ... on this problem.

This paper establishes a model for inter ... a single-mode optical field and a 2-energy- ... with exact analytic solutions given, and t ... momentum and energy exchanges between ... and the cold atom due to the interaction be ... discussed in detail.

Let $a^\dagger$ and a be creation and annihilatio ... a photon, respectively, $|e\rangle$ and $|g\rangle$ exited an ... of atom, ω_a the energy difference between ... excited states, and $|p\rangle$ momentum eigens ... mass center motion, the Hamiltonian of at ... cal field can then be written as

$$H = \sum_p \Big[\Big(\varepsilon_p + \frac{1}{2}\hbar\omega_a\Big)|e_p\rangle\langle e_p| + \Big(\varepsilon_p - \frac{1}{2}\hbar \ldots$$

$$+ \hbar\omega a^\dagger a + \hbar g \sum_p (a^\dagger \sqrt{a^\dagger a}\,|g_p\rangle\langle e_{p+\lambda k}| \ldots$$

$$+ \sqrt{a^\dagger a a}\,|e_{p+\hbar k}\rangle\langle g_p|\,,$$

where $\varepsilon_p = P^2/2m$ is the mass center mo ... an atom in a free space, $|A_P\rangle = |A\rangle \otimes |P\rangle$ ... resents the atom momentum states of the ... internal state of an atom; k is the value ... of single-mode optic field $E \sim a\,e^{ikx} + a$ ... is the coefficient for coupling between the ... the atom. It can be seen from $e^{ikx}|p\rangle =$ ... under the rotational wave approximation, ... between the optic fiel ... an ... tom will result in the momentum exchange p ... oce ... own in Fig. 1.

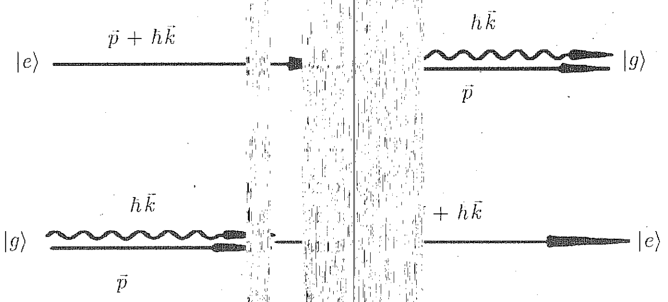

Fig. 1 Generatio ... of ... $\vec{k}$ during transition of atom from exited sta ... wi ... ntum $\vec{p}+\hbar\vec{k}$ to ground state with momentu ... $\vec{p}$, ... conjunctive process.

It is obvious that ... th ... ber of photons n and the value of momentu ... ished, $|g_{p-\hbar k/2}, n+1\rangle$ and $|e_{p+\hbar k/2}, n\rangle$ $(|A_p, \ldots\rangle \equiv \ldots (1/\sqrt{n!})a^{\dagger n}|n\rangle)$ form a 2-D invariable subsp ... ltonian of the system, and the eigenvalue of ... in the subspace can be expressed as follows:

$$E_n(p,k,\pm) = E \ldots \Big)\hbar\omega$$

$$\pm \sqrt{\Delta \ldots 4 + g^2(n+1)^2}\,,$$

$$\Delta(p,k) = \omega_a + \ldots /n \ldots$$

$$E(p) = \varepsilon_p + \varepsilon_k / \ldots, \quad \ldots k^2\hbar^2/2m\,, \tag{2}$$

the corresponding eig ... fu ... i.e. the dress state of mass center motion, is

$$|x_{+n}(p,k)\rangle = \ldots s\Big(\ldots \hbar k/2, n+1\rangle$$

$$\ldots \sin \ldots +\hbar k 2, n\rangle\,,$$

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$$|x_{+n}(p,k)\rangle = -\sin\Big(\frac{\theta_n}{2}\Big)|g_{p-\hbar k/2}, n+1\rangle + \cos\Big(\frac{\theta_n}{2}\Big)|e_{p+\hbar k2}, n\rangle\,, \tag{3}$$

where

$$\sin\theta_n = \frac{g(n+1)}{\sqrt{\Delta^2(p,k)/4 + g^2(n+1)^2}}\,. \tag{4}$$

It can be seen from Eq. (3) that the eigenfunction of Hamiltonian of the system is the linear combination of two different eigenstates of mass center momentum $|g_{p-\hbar k/2,n+1}\rangle$ and $|e_{p+\hbar k/2,n}\rangle$. The Rabi vibration resulting from the interaction will inevitably cause the transition of momentum $\hbar k$. This is why this model can be used to explain the momentum and energy exchanges between the photon and atomic mass center motion.

It can also be seen from Eq. (3) that the spontaneous radiation probability from $|e_p, 0\rangle$ to $|g_{p-\hbar k}, 1\rangle$ is

$$P = P(k) = \frac{4g^2}{(\omega_a + pk/M - \omega)^2 + 4g^2} \times \cos\Big[\Big(\frac{\Delta^2(p,k)}{4} + 4g^2\Big)^{1/2} t\Big]\,. \tag{5}$$

This shows that the maximum value of spontaneous radiation probability $P_{\max} = \cos(2gt)$ for momentum translation $\hbar k$ where pk/M represents the atomic Doppler effect. Therefore, with Doppler effect taken into consideration, the condition for resonance is $\omega = \omega_a + pk/M$.

Let us deal with energy translation now. When $t = 0$, the atom in the ground state with momentum p enters the single-mode optical field in interference state $|a\rangle$. And the initial energy of the atom is $E(0) = \varepsilon_p - \hbar\omega_a/2$. The wave function for the optical-atomic system under the resonance condition is

$$|\psi(t)\rangle = \mathrm{e}^{-\mathrm{i}[E(p+\hbar k/2) - \hbar\omega/2]t/\hbar - |a|^2/2} \times \Big\{\sum_{n=0}^{\infty} \frac{\alpha^{n+1}}{\sqrt{(n+1)!}}\{\cos[(n+1)gt]|g_p, n+1\rangle - \mathrm{i}\sin[(n+1)gt]|e_{p+\hbar k}, n\rangle\} + |g_p, 0\rangle\Big\}\,. \tag{6}$$

The exact value of atomic energy at moment t is

$$E(t) = \varepsilon_p + \frac{1}{2}\varepsilon_k + \frac{\hbar kp}{2M} - \frac{\hbar}{2}\Big(\omega_a + \frac{kp}{M} + \frac{\varepsilon_k}{\eta}\Big) \times \mathrm{e}^{-2|a|^2\sin^2(gt)}\cos[(\sin 2gt)]\,. \tag{7}$$

The physical meaning of formula (7) is very obvious, that is, when the initial energy of the atom is $E(0) = \varepsilon_p - \hbar\omega_a/2$, the atom starts absorbing energy first after entering the cavity due to the photon recoil and Doppler effect, and the energy absorbed during Δt is

$$\Delta E = |a|^2(g\Delta t)^2\frac{\hbar}{2}\Big(\omega_a + \frac{kp}{M} + \frac{\varepsilon_k}{\hbar}\Big)\,. \tag{8}$$

The release of the absorbed energy back to the system starts at $t = \pi/2g$, and the transfer of energy is an oscillation process with a period of π/g, as shown in Fig. 2 below.

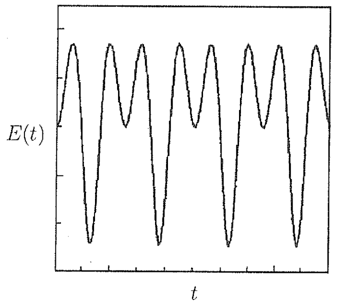

Fig. 2 Energy $E(t)$ versus time.

The oscillation process with collapse and restore is right the characteristics of Jaynes–Cummings (JC) model, and this shows that the exchanges of energy and momentum under the resonance condition does not change this characteristics. However, the energy of atomic mass center motion must be taken into consideration in this case. On this occasion, the resonance condition can be expressed as follows: $\omega = \omega_a + pk/2M + \hbar k^2/2M$. The results indicate that the resonance condition must include the effect of Doppler effect and photon recoil.

It should be noted that a complete state with momentum p cannot be established through experiments. The momentum diffuse will damage the condition of resonance. However, we have presented in this paper a comparable result with the experiment at a certain approximation.

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