

Effects of the space–bandwidth product on the liquid-crystal kinoform

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Abstract: The effects of the space–bandwidth product on the phase modulation magnitude are our main topic of investigation here. The large phase modulation produced with the liquid-crystal display (LCD) is realized with the kinoform method. One model is established in order to analyze the effects of the space–bandwidth product. The maximum phase change is drastically increased while the pixel size is less than 50 μm or so. But the effect of the area is almost linear with the maximum phase change. Then the experiments are completed for the purpose of verifying the theoretical analysis. We achieve a phase change of 2.05λ ($\lambda=633\text{ nm}$), which is half of the calculated value in a $1\text{cm}\times 1\text{cm}$ area, as it is affected by the cross talk and the pixel shape.

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References and links

1. A. Tanone, Z. Zhang, C.-M. Uang, F. T. S. Yu, D. A. Gregory, "Phase modulation depth for a real-time kinoform using a liquid crystal television," *Opt. Eng.* **32**, 517-521 (1993).
 2. J. Amako and T. Sonehara, "Kinoform using an electrically controlled birefringent liquid-crystal spatial light modulator," *Appl. Opt.* **30**, 4622-4628 (1991).
 3. J. A. Davis, K. O. Valadéz, and D. M. Cottrell, "Encoding amplitude and phase information onto a binary phase-only spatial light modulator," *Appl. Opt.* **42**, 2003-2008 (2003).
 4. G. Paul-Hus and Y. Sheng, "Optical real-time kinoform for on-axis phase-only correlation using liquid crystal television," *SPIE* **2043**, 287-295 (1993).
 5. Z. Cao, L. Xuan, L. Hu, Y. Liu, Q. Mu, and D. Li, "Investigation of optical testing with a phase-only liquid crystal spatial light modulator," *Opt. Express* **13**, 1059-1065 (2005), <http://www.opticsexpress.org/abstract.cfm?URI=OPEX-13-4-1059>.
 6. N. Lindlein, "Analysis of the disturbing diffraction orders of computer-generated holograms used for testing optical aspherics," *Appl. Opt.* **40**, 2698-2708 (2001).
 7. Q. Gu, J. Cao and Y. Sun, "Lagrange invariant, interference invariant and space-bandwidth product," *SPIE* **2866**, 104-107.
 8. L. Hu, L. Xuan, Y. Liu, Z. Cao, D. Li, and Q. Mu, "Phase-only liquid crystal spatial light modulator for wavefront correction with high precision," *Opt. Express* **12**, 6403-6409 (2004), <http://www.opticsexpress.org/abstract.cfm?URI=OPEX-12-26-6403>.
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1. Introduction

The liquid-crystal (LC) spatial light modulator (SLM) used to produce the computer-generated hologram (CGH) has been reported in many papers [1-3]. It is often used to produce the reconstruction image of the object. Other times it is used as a space filter [4] and a phase

shifter. We have illustrated that the LC SLM is feasible for optical testing [5]. Although it has some advantages compared with the traditional CGH, there are still some limits to using it for testing optical surfaces. The pixel size or the space–bandwidth product mainly limits its application in optical testing.

In this paper, we mainly discuss how to produce large phase modulation with the kinoform method and the effects of the space–bandwidth product.

2. Producing large phase modulation with the LC kinoform

2.1 Measurement of the phase retardation

To realize the phase-only modulation, we reconstruct the LCD (VGA). Two polarizers are removed, the color film is cleared, the rubbing orientation is aligned in parallel, and the LC is changed with large Δn . Finally, the LCD is changed to a phase-only LC SLM, which is actively addressed, consisting of $640 \times 480 \times 3$ pixels. The thickness of the LC is $5\mu\text{m}$, and the pixel size is $100\mu\text{m} \times 300\mu\text{m}$.

Figure 1 shows the phase retardation as a function of the video signal (grey level of the LCD) measured in a spectroscopic ellipsometry fabricated by JOBIN YVON. The phase modulation depth is more than 1λ ($\lambda=633\text{nm}$). Thus it can be used to produce the kinoform.

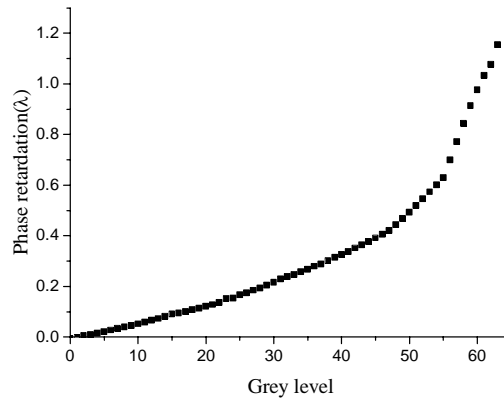


Fig. 1. Measured phase retardation as a function of grey level.

2.2 Kinoform

To realize the large phase change for optical testing, the kinoform method is used. For the CGH, its phase function can be written as [6]

$$\varphi_{CGH} = \varphi_{out} - \varphi_{in}, \quad (1)$$

where φ_{CGH} is the phase function of CGH and φ_{out} and φ_{in} are the wavefront coming out from the CGH and the wavefront going into it, respectively. If we use the reference wave to test an optical surface, the phase distribution of the kinoform can be written as

$$\varphi_{kin} = \varphi_{sur} - \varphi_{ref}, \quad (2)$$

where φ_{ref} and φ_{sur} are the phase functions of reference wave and optical surface, respectively. After φ_{kin} is modulated by 2π , the remainder is quantified to N levels and the φ_{kin} will be achieved while the driving voltage is applied to the LC SLM. For the LC SLM, the space–bandwidth product mainly affects the magnitude of the phase change.

2.3 Large phase change produced by using the kinoform method

Only a 1 cm² area can be used to produce the large phase change, as the area is limited by the size of the crystal polarizer [5]. The phase change magnitude is measured with the ZYGO interferometer (Fig. 2), and the phase change magnitude is 2.81λ ($\lambda=632.8\text{nm}$). This indicates that the LC SLM does produce the large phase change with the kinoform method.

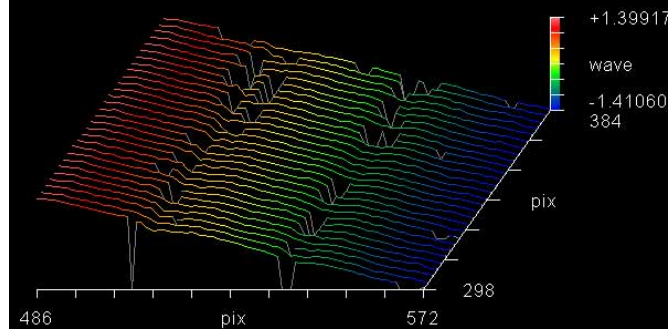


Fig. 2. Phase map measured by the ZYGO interferometer.

3. Effects of the space–bandwidth product

3.1 Space–bandwidth product

For a two-dimensional signal, the space–bandwidth product (denoted by SW) can be defined as [7]

$$SW = \iint dx dy \iint dv_x dv_y = S \times W. \quad (3)$$

Here S is the area of the spatial signal in the spatial domain, and the two-dimensional frequency band area of the signal in the spatial frequency domain (v_x, v_y) is W . If the area of the spatial signal and the spatial frequency domain are the rectangle, the space–bandwidth product can be rewritten as:

$$SW = \Delta x \times \Delta y \times \Delta v_x \times \Delta v_y, \quad (4)$$

where Δx and Δy are the signal extensions along the axis x and y in space, and Δv_x and Δv_y are its essential bandwidth along the axis v_x and v_y . For the LC SLM, its space–bandwidth product is mainly determined by the pixel size and the area.

3.2 Effects of the pixel size and the area of the LC SLM

The LC SLM used for optical testing is required to produce the large phase change with high accuracy. The high accuracy produced by the LC SLM has been investigated in another paper [8]. We mainly discuss the effects of the pixel size and the area on the magnitude of the phase change.

The sphere is used because the phase distribution used for optical testing is more like a sphere wavefront. As the diffractive efficiency is not important in the optical testing, we assume only two pixels are used to realize one wavelength phase change at the edge of the LC SLM. And this can ensure there are more than two pixels to realize one wavelength phase change in the other area of the LC SLM. Under this assumption, the capability of producing the phase change is maximum.

The geometric graph of the model is shown in Fig. 3. The radius of the sphere is R , D is the width of the LC SLM, and W is the width of the two pixels at the edge. If we assume that the pixel size and the area of the LC SLM are known, then R can be calculated as

$$R = \left[\left(\frac{2LW - W^2 - \lambda^2}{2\lambda} \right)^2 + L^2 \right]^{1/2}. \quad (5)$$

Here $L=D/2$ and λ is the relevant wavelength. Thus the maximum phase change under the assumption can be obtained by

$$Phase_{\max} = R - (R^2 - L^2)^{1/2}. \quad (6)$$

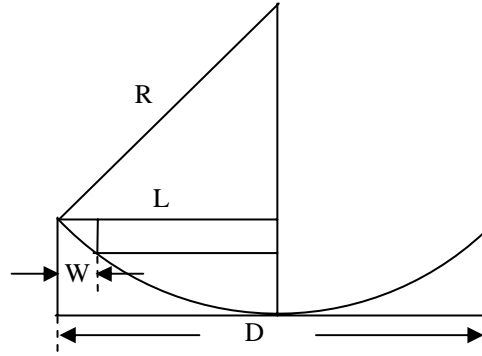


Fig. 3. The geometric graph of the model.

The area of the LC SLM used in the paper is $22\text{cm} \times 16\text{cm}$. So the area that can be used to produce the sphere wave is $16\text{cm} \times 16\text{cm}$. Assuming $\lambda=632.8\text{nm}$ and the area of 256cm^2 , the maximum phase change as a function of the pixel size is shown in Fig. 4. The maximum phase change is drastically increased while the pixel size is less than $50\mu\text{m}$ or so. Figure 5 shows the maximum phase change as a function of the area (the pixel size is $300\mu\text{m}$). The phase modulation changes linearly with the area. Accordingly, we need to reduce the pixel size of the LC SLM to much less than about $50\mu\text{m}$ in order to use it for optical testing. Of course, we also need the large area to increase the phase change magnitude.

Figure 6 shows the maximum phase change as functions of the pixel size and the area. If

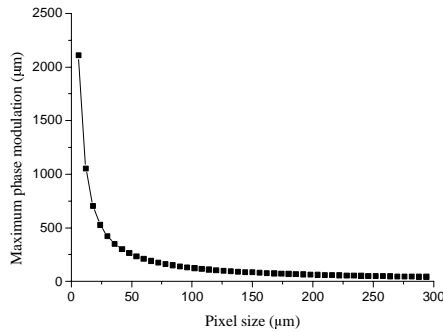


Fig. 4. The maximum phase change as a function of pixel size.

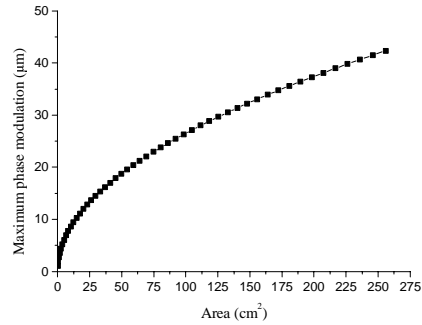


Fig. 5. The maximum phase change as a function of the area.

we assume that the effective area is 256cm^2 and the pixel size is $9\mu\text{m}$ (the minimum size as we know), the maximum phase change is $1405\mu\text{m}$, which is adequate for optical testing. But we just consider the effects of the pixel size theoretically and do not include the pixel shape and other actual factors. In practice, the phase change magnitude may be much smaller than expected. Nevertheless, the LC SLM is still feasible for optical testing, as its area can be much larger than what we used in the paper.

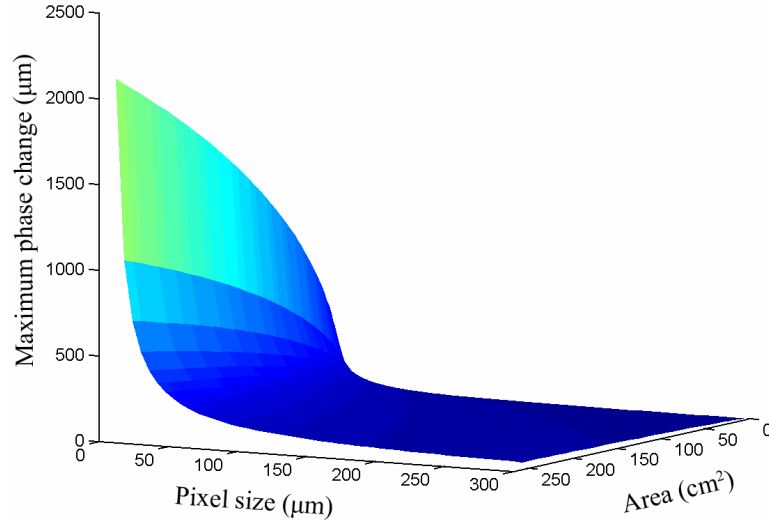


Fig. 6. The maximum phase change as functions of the pixel size and the area.

4. Experiments

In order to validate the theoretical analysis, a sphere wavefront is produced with the LC SLM. The area used to produce it is $1\text{cm}\times 1\text{cm}$ (32×32 pixels). The calculated maximum phase change is $2.8\mu\text{m}$ [4.4λ ($\lambda=632.8\text{nm}$)]. The distribution of the pixel and grey level is shown in Fig. 7(a). Figure 7(b) is the measured phase distribution with the ZYGO interferometer. The poor phase distribution and the little modulation magnitude are achieved.

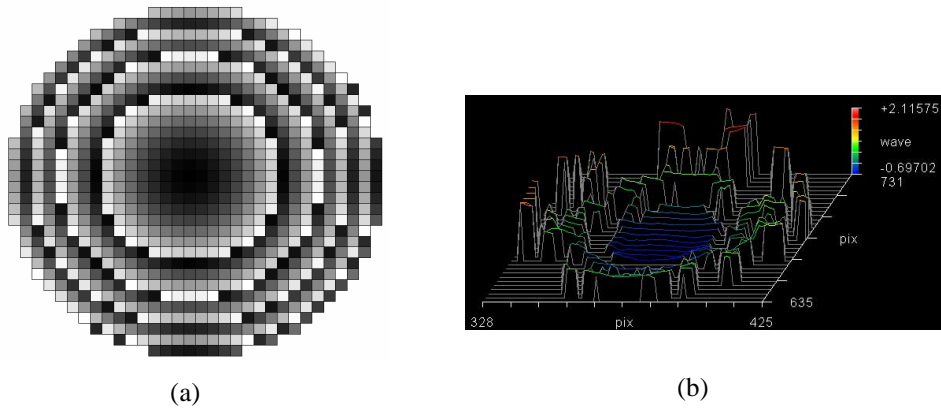


Fig. 7. (a) The distribution of the pixel and the grey level; (b) the measured phase distribution.

For the purpose of analyzing the effects of the actual factors, the phase changes of 3.56λ [Fig. 8(a)] and 2.05λ [Fig. 8(b)] are also produced in the same area. It is shown that the phase distribution is better while the modulation magnitude is decreased. The effects of other factors cannot be found when the phase change is 2.05λ . Although this modulation magnitude is much less than the calculated (4.4λ), almost half of the calculated magnitude is still obtained. Then, we may say that there is still $700\mu\text{m}$ (half of $1405\mu\text{m}$) to be obtained in the experiment with an area of 256cm^2 , and this is enough for optical testing with the large optical surface.

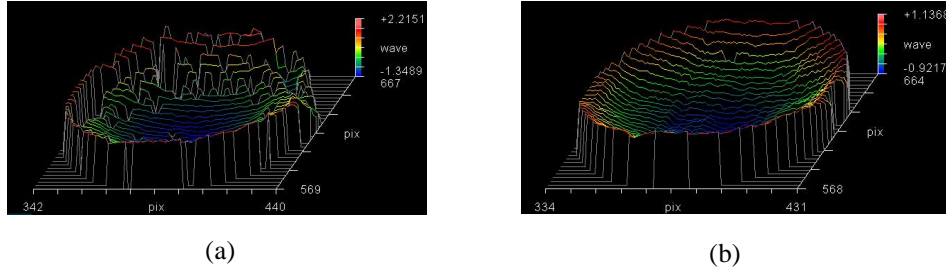


Fig. 8. The measured phase map of the sphere wave: (a) phase change is 3.56λ ; (b) phase change is 2.05λ .

5. Discussions

There is a significant difference between the prediction and the measurement. We consider that it may be caused by one of the following:

1. Cross talk. The cross talk between the neighboring pixels can disturb the wavefront, especially in its outer part. Such cross talk is very likely to happen in an LC SLM whose pixels are not addressed individually and separately, and hence a large gap in phase cannot be resolved. There are too few pixels to realize one wavelength phase change at the outer part of Fig. 7(a), and the phase change of the neighboring pixels is too large. So the distinct cross talk is produced and the bad result [Fig. 7(b)] is found.
2. Pixel shape. As the phase distribution of the kinoform in this paper is circularly symmetric but the pixel is square, the circle is formed by a lot of little squares while the LC SLM is used. Apparently with the larger pixel size, fitting errors are larger. This may also affect the phase distribution produced by the kinoform.

6. Conclusions

In order to use the LC SLM for optical testing, the kinoform method used to produce the large phase changes is analyzed and the effects of the space–bandwidth product are investigated. This indicates that the maximum phase change is drastically increased while the pixel size is less than $50\mu\text{m}$ or so. But the effect of the area is almost linear with the maximum phase change. Then, the experiments are completed for the purpose of verifying the theoretical analysis. Only half of the calculated maximum phase value is obtained, as there are some effects from cross talk and pixel shape. But it is still feasible for optical testing because half of the theoretical value is sufficient. We will use the LC SLM to test optical surfaces in the future.

Acknowledgments

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