Modified phase function model for kinoform lenses

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A more computationally tractable model of the kinoform lenses in hybrid refractive-diffractive systems is proposed by taking into consideration the actual phase function of the kinoform lenses for every wavelength. The principle and outline of this modified model are explained. We compare the results of this approach with the more conventional single order calculation and with the standard diffraction-order expansion by using a practical hybrid optical system example. © 2008 Optical Society of America

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1. Introduction

Recent advances in manufacturing techniques have brought about great developments in diffractive optical elements (DOEs) [1,2]. The diffractive optical elements usually have continuous surface relief, known as kinoform lenses. The hybrid refractive-diffractive optical components have been widely used in IR and visible wavebands [3,4] as they invariably bring a reduction in lens element count, mass, and sensitivity to manufacturing tolerances. The imaging performance of hybrid components is limited by radiation scattered into additional parasitic diffraction orders which serve to lower the contrast of the desired image. Therefore, the accurate evaluation of the imaging quality of the hybrid system is very important.

Most optical design software packages model the kinoform lens as an ideal pure diffraction element and use a single diffraction order to evaluate the performance of hybrid systems, which tends to give optimistic performance predictions. The classical exact ray tracing method [5] and the zone composition method [6] are indeed the exact ways of modeling the kinoform lens. Nevertheless, both of the two methods are not compatible with these commercial optical design software packages. The summation model of orders [7] is a fairly accurate solution as far as the scalar approximation of optical systems holds, and such summation should be performed coherently for all orders at each wavelength. However, the convergence of the diffraction-order series expansion is known to be often impractical, owing to its slow $1/M$ convergence, with M being the number of summed-up orders.

Here a more computationally tractable model of the kinoform lenses is given by taking into consideration the actual phase function of the kinoform lenses for every wavelength. It is shown that the evaluation of hybrid system performance using the modified model yields more realistic predictions than a single order computation. The principle and outline of this model is described in Section 2. An illustrative example is given in Section 3, and our remarks and observations are put forward in Section 4.

2. Principle of the Model

For the purpose of design and performance evaluation, the kinoform lens is often modeled as an ideal thin phase screen over the substrate surface. This model of a kinoform lens uses the grating equation for ray tracing. However, as is shown in Fig. 1, the actual wave surface of another wavelength passing through the kinoform lens deviates from the ideal phase and has discontinuities at the boundaries of the different zones.

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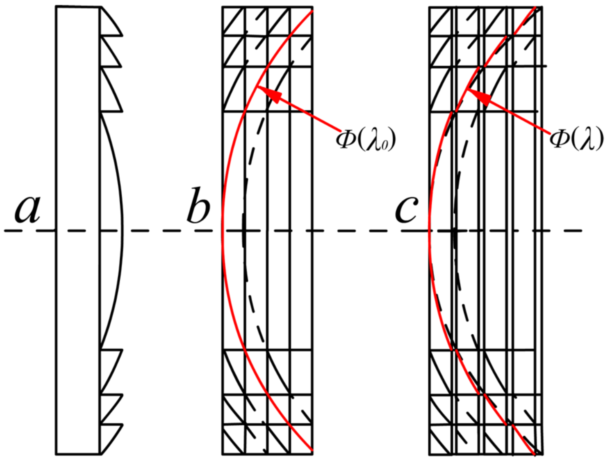

Fig. 1. (Color online) Wave surface of different wavelengths: (a) kinoform lens, (b) wave surface of nominal wavelength, (c) wave surface of different wavelength.

Such complex wave surface results in stray light and reduces the diffraction efficiency of the primary order, which is analyzed in many earlier articles [8,9]. To accurately predict the performance of the hybrid system, the actual complex wave surface must be taken into consideration. The computation procedure of the phase departure from the ideal constant one is as follows.

The phase function of a rotationally symmetric diffractive phase profile representing a kinoform lens is given by the following power series:

$$\phi_0(r) = \frac{2\pi}{\lambda_0} \sum_{i=1}^{n} a_i r^{2i}, \tag{1}$$

where $\Phi_0(r)$ is the phase at radius r, λ_0 is the nominal wavelength, and the maximum value of n used in the software packages is usually less than five. The surface relief profile $d(r)$, corresponding to the desired $\Phi_0(r)$, can be determined approximately by the relation [10,11]

$$d(r) = \frac{\lambda_0}{2\pi(n_0 - 1)} \{\phi_0(r) \mathrm{mod} 2\pi\}. \tag{2}$$

However, the surface relief profile $d(r)$ has the correct phase depth only at the design wavelength λ_0, thus the phase function $\Phi(r)$ will change due to both wavelength shift and material dispersion. Here $\{\Phi_0(r) \mathrm{mod} 2\pi\}$ is the phase function $\Phi_0(r)$ modulo 2π and n_0 are the refractive index of the optical material used for the kinoform lens. Usually, the thickness $d(r)$ is the order of the wavelength.

When a different wavelength, λ, is chosen, phase function $\Phi(r)$ will change because of the material dispersion [12,13]:

$$\phi(r) = \frac{2\pi}{\lambda}(n-1)d(r). \tag{3}$$

From Eqs. (2) and (3), we have

$$\phi(r) = \frac{\lambda_0}{\lambda} \frac{1}{\alpha} \{\phi_0(r) \mathrm{mod} 2\pi\}, \tag{4}$$

where α is the dispersion coefficient:

$$\alpha = \frac{n_0 - 1}{n - 1}. \tag{5}$$

Accordingly, because of the material dispersion, the phase difference in each zone is

$$\Delta\phi(r) = \phi(r) - \{\phi_0(r) \mathrm{mod} 2\pi\}, r_i \le r < r_{i+1}, \tag{6}$$

where r_i is the zone radius of the mth full period zone of the kinoform lens corresponding to $\Phi_0(r)$, which is determined by the relation

$$\phi_0(r_i) = \pm m 2\pi, \qquad m = 0, 1, 2, 3\ldots . \tag{7}$$

Thus the modified analysis method of kinoform lenses with the effect of the material dispersion is straightforward: For each nonnominal wavelength, the phase departure is computed according to Eq. (6) and added into the ideal phase function of the kinoform lens. In many optical system design software packages, a special surface can be defined to express this modified phase function. Then monochromatic performance analyses can be predicted by the optical system design software packages.

As for ray tracing, the kinoform surfaces are very similar to standard surfaces, except that the rays are further deviated by the derivative of the phase function Φ as a function of coordinates X and Y:

$$l' = l + \frac{\lambda}{2\pi} \frac{\partial\phi}{\partial x} = l + \frac{\lambda}{2\pi}\left(\frac{\partial\phi_0}{\partial x} + \frac{\partial\Delta\phi}{\partial x}\right), \tag{8}$$

$$m' = m + \frac{\lambda}{2\pi} \frac{\partial\phi}{\partial y} = m + \frac{\lambda}{2\pi}\left(\frac{\partial\phi_0}{\partial y} + \frac{\partial\Delta\phi}{\partial y}\right). \tag{9}$$

where l and m are the direction cosines and Φ is the actual phase of the kinoform in radians.

Thus the optical path difference (OPD) and the wavefront deformation $W(x,y)$ can be found by tracing rays. According to Eqs. (8) and (9), it is clear that there will be additional OPD and additional wavefront deformation because of the phase difference $\Delta\phi$. If W_0 represents the wavefront deformation corresponding to Φ_0, and ΔW represents the additional wavefront deformation corresponding to $\Delta\phi$, the pupil function of the system $P(x,y)$ can be written as

$$P(x,y) = E(x,y)e^{-ikW(x,y)} = E(x,y)e^{-ik(W_0+\Delta W)}, \tag{10}$$

where $E(x,y)$ represents the amplitude distribution over the exit pupil.

As described in many optical design texts [14] an autocorrelation of the pupil function can be used to obtain the MTF for a given wavelength λ_i:

$$\mathrm{MTF}(\lambda_i,v,u)=\left|\frac{\int\int_{-\infty}^{+\infty}P(x,y)P^{*}(x+\lambda_i vR,y+\lambda_i uR)\mathrm{d}x\mathrm{d}y}{\int\int_{-\infty}^{+\infty}|P(x,y)|^2\mathrm{d}x\mathrm{d}y}\right|, \tag{11}$$

where λ_i is the wavelength under consideration, R is the radius of the wavefront and v and u are the spatial frequencies in the x and y directions, respectively.

If the entrance pupil is illuminated with a constant amplitude light beam we have $E(x,y)=1$. Then we obtain

$$\begin{aligned}\mathrm{MTF}(\lambda_i,v,u)=\bigg|\int\int&\exp\{ik[W_0(x+\lambda_i vR,y+\lambda_i uR)\\&-W_0(x,y)]\}\\&\times\exp\{ik[\Delta W(x+\lambda_i vR,y+\lambda_i uR)\\&-\Delta W(x,y)]\}\mathrm{d}x\mathrm{d}y\bigg|,\end{aligned} \tag{12}$$

From the analysis above, we can see that the nonnominal wavelength results in phase difference $\Delta\phi$, the additional wavefront deformation ΔW is generated over the exit pupil, and the final impact on the system is represented by the MTF according to Eq. (12).

From the monochromatic MTF analysis of the optical system, the polychromatic MTF can be performed by use of incoherent weighted summation over wavelength:

$$\mathrm{MTF}_{\mathrm{POLY}}=\frac{\sum_{i=1}^{N}W_i\cdot\mathrm{MTF}_i}{\sum_{i=1}^{N}W_i}, \tag{13}$$

where W_i represents the spectral weight of wavelength λ_i. It is apparent that the accurate pupil function is derived from the actual phase function of the kinoform for each wavelength, and then the accurate MTF can be found by the autocorrelation of the pupil function. So the performance predictions of the hybrid system are exact.

3. Example: Hybrid Miniature CCD Camera

Because of the distinct dispersive behavior and other advantages, such as reductions in system weight and cost, kinoform lenses have been widely used in optical systems in recent years. The following example is intended to illustrate the application of the modified model. It is a miniature and lightweight remote sensing CCD camera with the following specifications: effective focal length 320 mm, $F/5$ and field of view $\pm 9°$. This camera is a visible to near-infrared system with a wide bandwidth of 0.500–0.900 um. The primary wavelength is 0.700 um and the four nonnominal wavelengths are as follows: 0.525 μm, 0.615 μm, 0.785 μm, and 0.870 μm. A 37-zone kinoform on the back face of a plane–parallel plate is employed to correct combined aberrations. The system layout is sketched in Fig. 2. Here the optical system design program ZEMAX [15] is used to model and analyze this system.

As represented in Section 2, the actual phase of the kinoform lens will depart from the ideal phase for each nonnominal wavelength. According to Eq. (6), the phase departures are computed and the actual phase plots of the central part of the kinoform are shown in Fig. 3. To represent the accurate phase for each nonnominal wavelength in the software program ZEMAX, we compiled and used a user-defined surface, “Binary new”, to replace the surface “Binary 2”, which is a surface type used for a rotationally symmetric diffractive phase profile and which has a constant phase for each wavelength. There are independent phase polynomial expansions for different zones to express the discontinuous actual wave surface of the kinoform lens in the user defined surface “Binary new”.

With the help of the program ZEMAX, optical performances of the system can be predicted and plotted. By ray tracing, as described in Section 2 [Eqs. (8) and (9)], the curves of the optical path differences in the radial direction are shown as a function of the pupil coordinate in Fig. 4. Figure 4(a) corresponds to the original calculation with the ideal phase function. Figure 4(b) corresponds to the modified model and clearly shows that there is much OPD jaggedness in the plots. However, this can be easily accepted because of the phase jumps $\Delta\phi$ from both the wavelength shift and the material dispersion, which have been illustrated in Fig. 3. According to Eq. (11), the monochromatic MTF is calculated by the autocorrelation of the pupil function. The MTF curves onaxis are given as a function of frequency in Fig. 5. It is clear that there are sharp drops at very low frequencies of the MTF curves. Obviously, this phenomenon is due to the additional wavefront deformation ΔW.

From the onaxis monochromatic MTF of the system, obtained as explained above, the polychromatic MTF can be found by using incoherent weighted

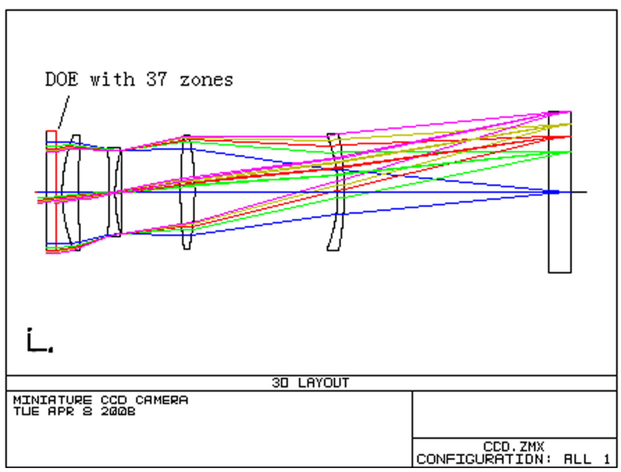

Fig. 2. (Color online) Layout of the hybrid miniature CCD camera system.

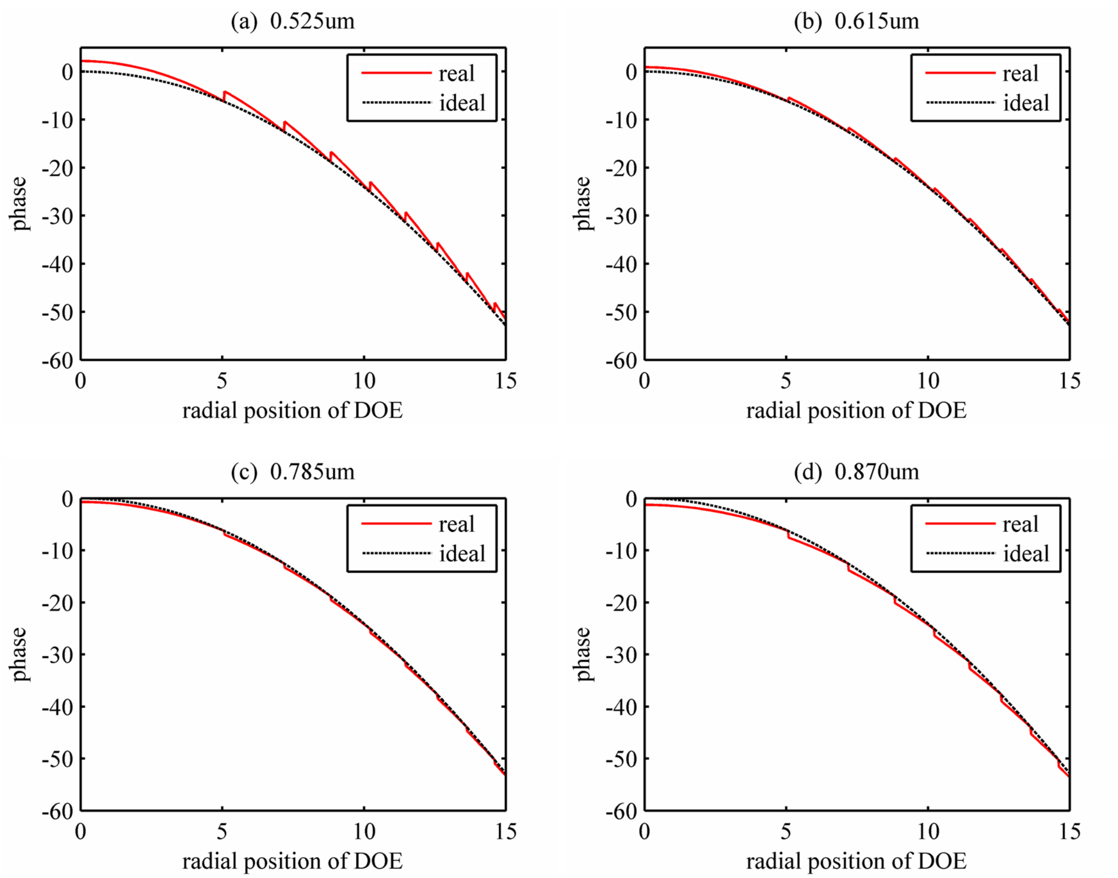

Fig. 3. (Color online) Phase plots versus radius of kinoform. Here only the first eight zones of the 37-zone kinoform are plotted. The dotted black curve is the ideal phase under nominal wavelength and the solid red curve is the actual phase under nonnominal wavelength.

summation over wavelength. For comparison purposes, Fig. 6 shows the polychromatic MTF of a CCD camera system from a different model: the black solid curve, the blue dotted curve, and the red dashed curve correspond to the diffraction limit, the conventional calculation with only +1 order (with 100%

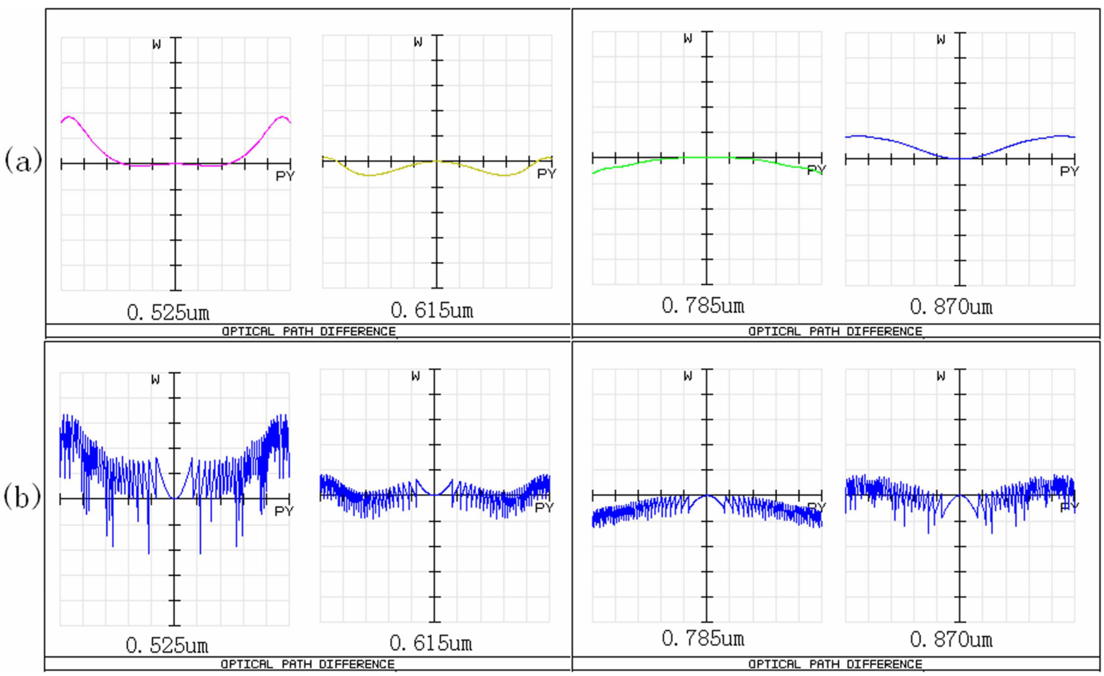

Fig. 4. (Color online) Plots of optical path differences as a function of pupil coordinate: (a) original model and (b) modified model.

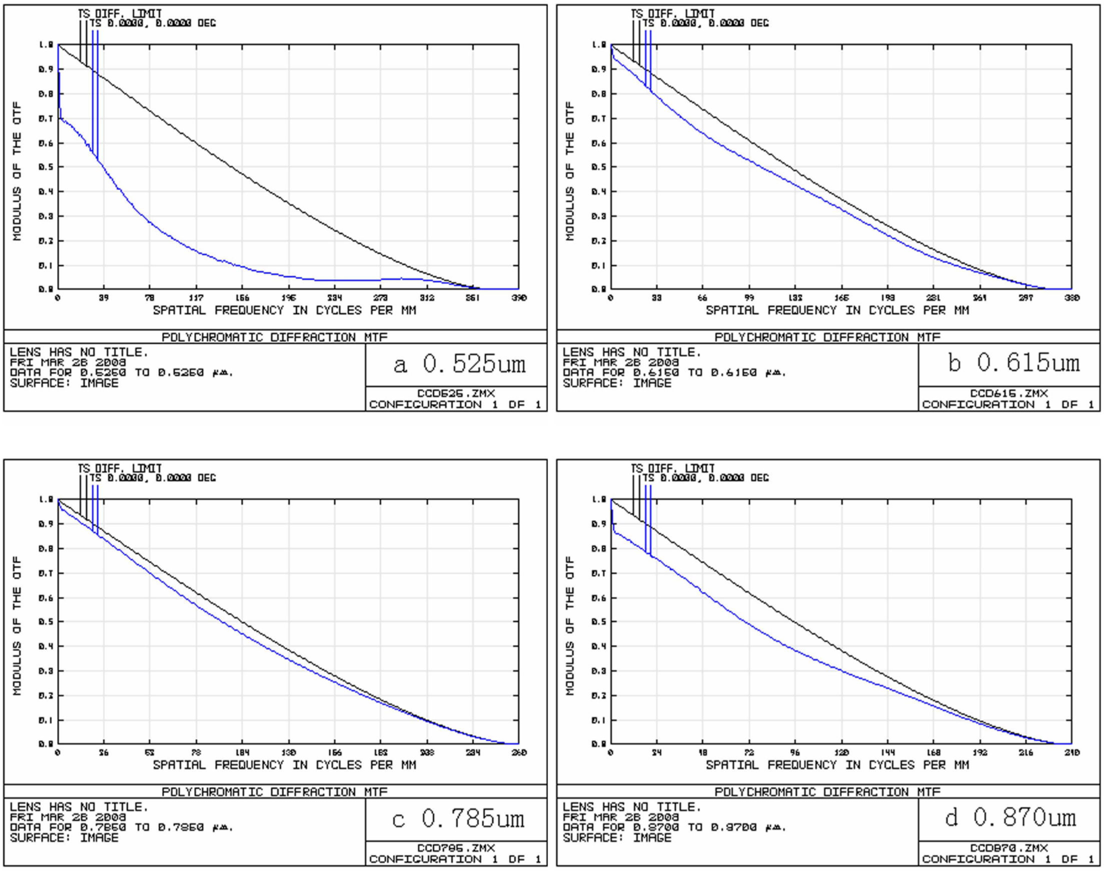

Fig. 5. (Color online) Onaxis monochromatic MTF of the hybrid system under different nonnominal wavelength.

efficiency at all wavelengths, as assumed by most commercial optical design software), and our modified model. The curves of the low spatial frequency region are given in Fig. 6(b). It is clear that there are great differences between the two computation methods. Finally, the comparison between the modified phase function model and the standard diffraction-order expansion is represented in Fig. 7. In this plot, the blue dotted curve corresponds to the standard diffraction-order expansion method and the results of the incoherent summation of five orders (taken symmetrically around the +1 order: −1). It is obvious that the two curves are very close to each other. However, the slow 1/5 convergence of the diffraction-order expansion method results in less accuracy than the modified phase function model.

4. Conclusions

In the framework of scalar optics, a more computationally tractable model of kinoform lenses is proposed to

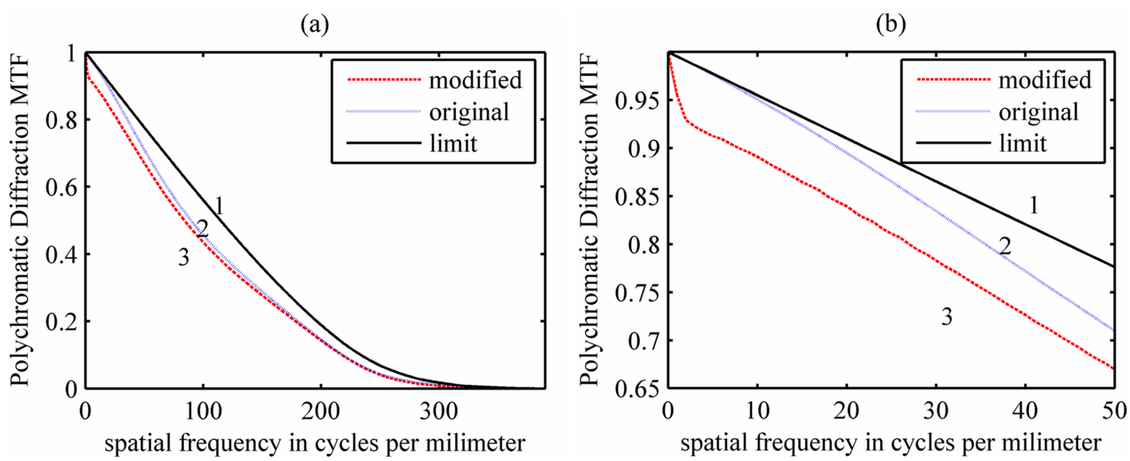

Fig. 6. (Color online) (a) Onaxis polychromatic MTF of the hybrid system: solid line (1), diffraction limit; dotted line (2), conventional calculation; dashed line (3), modified model. (b) Magnified view of the low spatial frequency region.

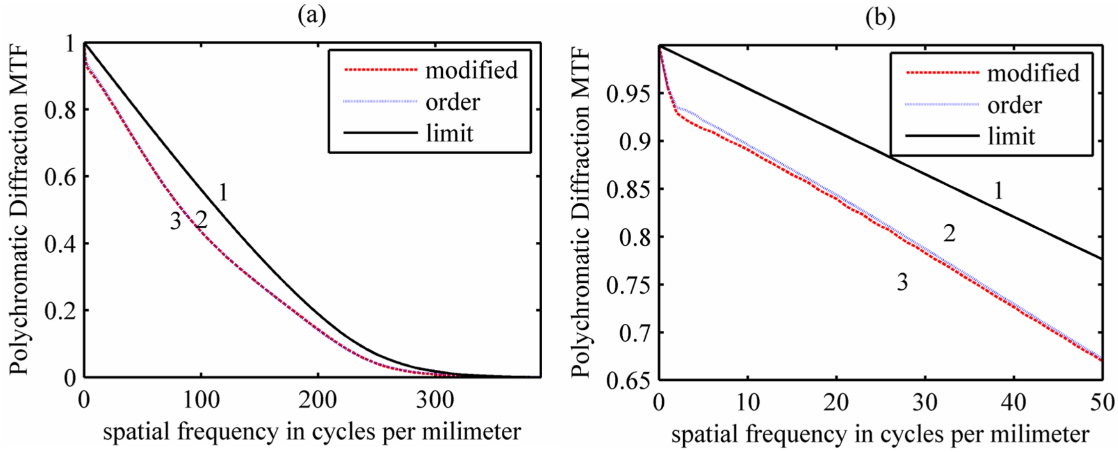

Fig. 7. (Color online) (a) Onaxis polychromatic MTF of the hybrid system: solid line (1), diffraction limit; dotted line (2), standard diffraction-order expansion calculation; dashed line (3), modified model. (b) Magnified view of the low spatial frequency region.

analyze hybrid systems by modifying the corresponding phase expressions for each wavelength. Through the analysis of an actual design example, the modified model is found to be useful and indeed necessary for hybrid systems. Obviously, it will give a more exact performance evaluation of the hybrid system.

Here the implementation steps of the model are presented. We used the optical system design software program ZEMAX for illustration but other programs in which the diffractive surface is modeled as an ideal phase screen could be used. However, this model, based on the exact phase modification for each wavelength, suffers from two limitations: The kinoform lenses are expected to have a fairly small number of zones and restricted for a rotationally symmetrical hybrid system. For a kinoform lens with three phase polynomial terms, the extra data columns in ZEMAX will support a maximum of 80 zones. If there are more zones, another diffractive surface type must be employed; however, the accuracy of such an implementation depends on the sampling grid that is used. Second, while the model is feasible as a design, its optimization function will be the scope of our future investigation.

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