A simple method for evaluating the wavefront compensation error of diffractive liquid-crystal wavefront correctors

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Abstract: A simple method for evaluating the wavefront compensation error of diffractive liquid-crystal wavefront correctors (DLCWFCs) for atmospheric turbulence correction is reported. A simple formula which describes the relationship between pixel number, DLCWFC aperture, quantization level, and atmospheric coherence length was derived based on the calculated atmospheric turbulence wavefronts using Kolmogorov atmospheric turbulence theory. It was found that the pixel number across the DLCWFC aperture is a linear function of the telescope aperture and the quantization level, and it is an exponential function of the atmosphere coherence length. These results are useful for people using DLCWFCs in atmospheric turbulence correction for large-aperture telescopes.

OCIS codes: (010.1080) Adaptive optics; (230.3720) Liquid-crystal devices; (010. 1285) Atmospheric correction.

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1. Introduction

Liquid-crystal adaptive-optics systems (LCAOSs) have been widely investigated for the purpose of atmospheric turbulence correction [1–4]. Liquid-crystal wavefront corrector (LCWFC), as a key element of a LCAOS, can be used to correct atmospheric turbulence for large-aperture telescopes because of the millions of pixels it contains. The finite number of wavefront correction elements used for the correction of atmospheric turbulence, however, causes the permanent existence of a wavefront compensation error. Hudgin gave the relationship between compensation error and actuator size as follows [5]:

$$f = \alpha(\frac{r_s}{r_0})^{5/3}, \tag{1}$$

where r_s is the actuator spacing, r_0 is the atmosphere coherence length, and α is a constant depending on the response function of the actuator. For continuous surface deformable mirrors (DMs), the response function of the actuator is a Gaussian function and α ranges from 0.3 to 0.4 [6]. For a piston-only response function, α is 1.26 [7]. Researchers have always used a piston-only response function to evaluate an LCWFC and have proved that actuators need to be four to five times as large as those of the DM [7,8]. However, the case is totally different when diffractive LCWFC (DLCWFC) is used where the kinoform or phase-wrapping technique is employed to expand the correction capability [9,10]. Therefore, Eq. (1) is no longer suitable. In this paper, a simple method suitable for evaluating the wavefront compensation error of DLCWFC was developed for atmospheric turbulence correction.

2. Effect of quantization on wavefront error

First, the wavefront error generated in the phase wrapping due to quantization was considered. Generally, the phase stroke of LCWFCs is approximately 2π radian [11]. Therefore, it is necessary to use the kinoform method to achieve correction of the wavefront distortion with a scale of more than one wavelength [12]. That is to say, the modulo 2π operation is used to wrap the distortion within one wavelength and then the modulated wavefront is quantized. Since LCWFC is a two-dimensional device, quantification is performed along the x and y axes respectively by taking the pixel as the unit. According to diffraction theory, correction precision as a function of quantization level can be deduced [13]. If the pixel size is not considered, the root mean square (RMS) error of the diffracted wavefront as a function of quantization level can be simplified as [13]

$$\Delta W = \frac{\lambda}{2\sqrt{3}N}, \tag{2}$$

where N is the quantization level and λ is the wavelength. If $N = 30$, then the RMS error can be as small as $\lambda/100$. For $N = 8$, RMS = 0.036λ and the corresponding Strehl ratio is 0.95. Figure 1 shows the diffracted wavefront RMS error as a function of quantization level N. As seen, that the wavefront RMS error reduces drastically at first, and then gradually approaches a constant when the quantization level becomes larger than 10. Wavefront RMS error can be calculated for a known quantization level. For DLCWFC, wavefront compensation error is directly determined by the quantization level without the need to consider the pixel number. Therefore, the distribution of the quantization level on atmospheric turbulence should be calculated first for a given pixel number, telescope aperture, and atmospheric coherence length. The wavefront compensation error can then be calculated using Eq. (2).

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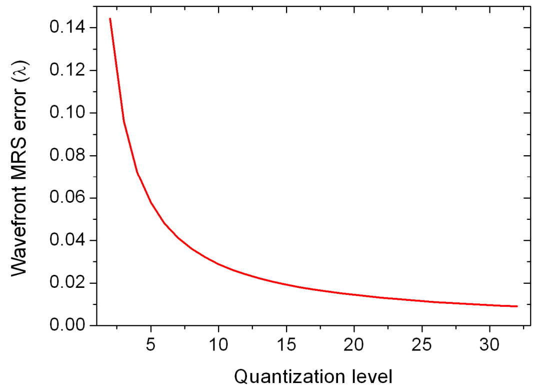

Fig. 1. Wavefront RMS error as a function of quantization level

3. Determination of the total pixel number of DLCWFCs

3.1 Zernike polynomials for atmospheric turbulence

Kolmogorov turbulence theory was employed to analyze the distribution of quantization level across an atmospheric turbulence wavefront. Noll described Kolmogorov turbulence by using Zernike polynomials [14]. According to him, Zernike polynomials is redefined as

$$\begin{aligned} Z_{even\ j} &= \sqrt{2(n+1)}R_n^m(\rho)\cos(m\theta),\ m \neq 0 \\ Z_{odd\ j} &= \sqrt{2(n+1)}R_n^m(\rho)\sin(m\theta),\ m \neq 0, \\ Z_j &= \sqrt{(n+1)}R_n^0(\rho), \qquad m=0, \end{aligned} \tag{3}$$

where

$$R_n^m(\rho) = \sum_{s=0}^{(n-m)/2} \frac{(-1)^s (n-s)!}{s!\left[\frac{(n+m)}{2}-s\right]!\left[\frac{(n-m)}{2}-s\right]!} \cdot r^{n-2s}. \tag{4}$$

The parameters n and m are integral and have the relationship of $m \leq n$ and $n-|m| = even$. An atmospheric turbulence wavefront can be described by a Kolmogorov phase structure function as below [14]:

$$D(r) = 6.88\left(\frac{r}{r_0}\right)^{5/3}. \tag{5}$$

By combining the phase structure function and the Zernike polynomials, covariance between the Zernike polynomials Z_j and $Z_{j'}$ with amplitudes a_j and $a_{j'}$, respectively, can be deduced as [15]

$$\langle a_j a_{j'} \rangle = \begin{cases} \dfrac{K_{zz'}\delta_{mm'}\Gamma[(n+n'-5/3)/2](D/r_0)^{5/3}}{\Gamma[(n-n'+17/3)/2]\Gamma[(n'-n+17/3)/2]\Gamma[(n+n'+23/3)/2]} & j-j' = even \\ 0, & j-j' = odd \end{cases}, \tag{6}$$

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where $K_{zz'} = 2.698(-1)^{(n+n'-2m)/2}\sqrt{(n+1)(n'+1)}$; D is the telescope diameter; and $\delta_{mm'}$ is the Kronecker delta function. The coefficients of the Zernike polynomials can be easily computed using Eq. (6). If the first J modes of the Zernike polynomials are selected, the atmospheric turbulence wavefront is represented as

$$\phi_t = \sum_{j=1}^{J} a_j Z_j . \tag{7}$$

Therefore, the atmospheric turbulence wavefront ϕ_t can be calculated using Eqs. (6) and (7). As the phase-wrapping technique is employed, the atmospheric turbulence wavefront can be wrapped into 2π and quantized. Thus, the distribution of quantization level across a telescope aperture D can be determined.

3.2 Calculation of the required pixel number of DLCWFCs

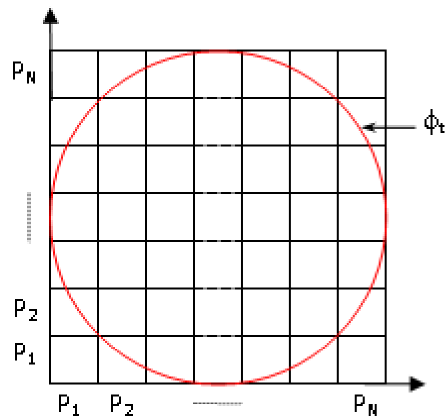

Fig. 2. The field of DLCWFC. The circle represents wavefront of atmospheric turbulence and $P_1 \ldots P_N$ are the pixel numbers of the DLCWFC.

In practice, people hope to calculate the desired pixel number of DLCWFC expediently for a given telescope aperture, D, quantization level, N, and atmospheric coherence length, r_0. The relation between the pixel number of DLCWFC and D, N, and r_0 should therefore be deduced. As shown in Fig. 2, the DLCWFC aperture can be represented by the pixel number across the aperture, which is called P_N in this paper. The circle represents the atmospheric turbulence wavefront ϕ_t. Since the atmospheric turbulence wavefront is random, the ensemble average $<\phi_t>$ should be used in the calculation. The modulated and quantized atmospheric turbulence wavefront can be expressed as

$$\mathrm{mod}(\langle \phi_t \rangle) = f(N, D, r_0, \langle P_N \rangle), \tag{8}$$

where mod() denotes modulo 2π. If $<\phi_t>$ is known, $<P_N>$ can be expressed as a function of the telescope aperture, D, quantization level, N, and atmospheric coherence length, r_0. By using Eqs. (6) and (7), $<\phi_t>$ can be calculated. The first 136 modes of the Zernike polynomials were also used in the calculation. For the randomness of the atmospheric turbulence wavefront, different quantization levels were used during quantization depending on the wavefront's fluctuation degree. We use N to represent the quantization level of the atmospheric turbulence wavefront. In this study, N is defined as the minimum quantization level so that the sum of those quantization levels larger than N should occupy 95% of the quantization levels included in the atmospheric turbulence wavefront. Fifty atmospheric turbulence wavefronts were used to achieve statistical results. First, the relation between pixel number P_N and telescope

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aperture D was calculated for r_0 = 10cm and N = 16 as shown in Fig. 3, where the line is the fitted curve and the dot is the calculated data. As seen, $<P_N>$ is a linear function of D when N and r_0 are fixed. That is to say, the larger the aperture of the telescope, the more pixel numbers will be needed if DLCWFC is used to correct atmospheric turbulence. Specifically, for a telescope with a diameter of two meters, the total pixel number will be 96 × 96; while for a telescope with a diameter of four meters, the total pixel number will be 168 × 168. P_N as a function of N was also computed for r_0 = 10cm and D = 2m as shown in Fig. 4. The figure illustrates that when D and r_0 are fixed, $<P_N>$ is a linear function of N. This means that the more quantization level is used, the more pixel numbers will be needed.

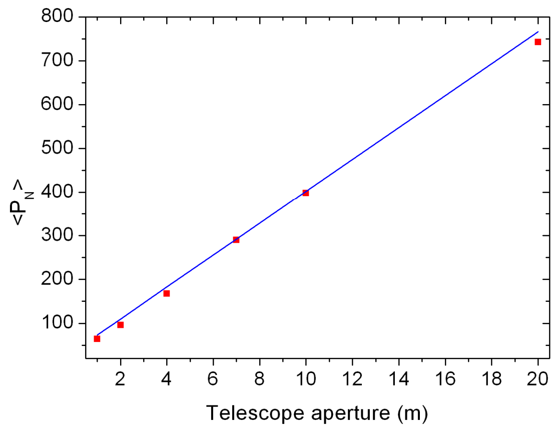

Fig. 3. $<P_N>$ as a function of telescope aperture D, ■ represents calculated data for r_0 = 10 cm and N = 16. The solid curve represents fitted data.

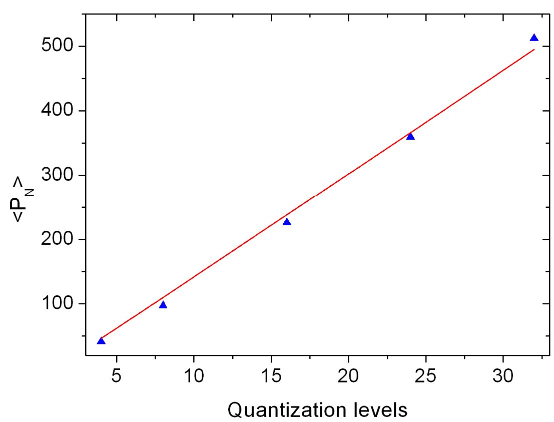

Fig. 4. $<P_N>$ as a function of quantization level N, ▲ represents calculated data for D = 2 m and r_0 = 10 cm. The solid curve represents fitted data.

The relationship between $<P_N>$ and r_0 was also calculated with variables N and D. Fig. 5 shows only three curves with three pairs of fixed N and D. This time the function is no longer linear but exponential. With more pairs of N and D fixed, more curves can be obtained (not shown in the figure). It was found that the relationship between $<P_N>$ and r_0 can be expressed by the following formula:

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$$\langle P_N \rangle = A + B r_0^{-6/5}, \tag{9}$$

where A and B are the coefficients. Furthermore, we found that A is only related to N and can be expressed as $A = 6.25N$. As shown above, $<P_N>$ is a linear function of D and N, thus coefficient B can be expressed as,

$$B = a + bN + cD + dND, \tag{10}$$

where a, b, c, and d are coefficients. By substituting the known value of N, D, and the calculated coefficient B, the values of a, b, c, and d are determined to be 15, −23, −1.5, and 0.91, respectively, using the least-square method. Thus, $<P_N>$ can be expressed as

$$\langle P_N \rangle = 6.25N + \left(15 - 1.5D - 23N + 0.91ND\right) r_0^{-6/5}, \tag{11}$$

where the unit of D and r_0 is in centimeters. The total pixel number of DLCWFC can be calculated by $P_N \times P_N$. By combining Eqs. (2) and (11), the compensation error of DLCWFC can be evaluated for atmospheric turbulence correction. The necessary parameters of DLCWFC for a specific telescope can easily be found to correct atmospheric turbulence using the new derived formula. However, these two formulas are not suitable for modal types of LCWFCs [16], or other types that do not use the diffraction method to correct atmospheric turbulence.

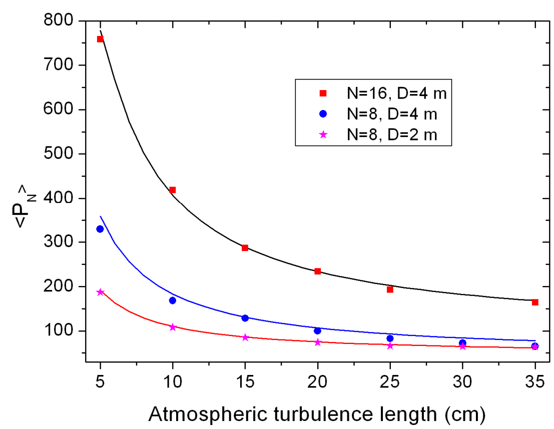

Fig. 5. $<P_N>$ as a function of atmosphere coherence length r_0, line is fitted curve. ■, ●, and ★ represent computed data with $N = 16$ and $D = 4$ m, N = 8 and $D = 4$ m, and $N = 8$ and $D = 2$ m, respectively.

Normally, a quantization level of 8 is considered suitable for atmospheric turbulence correction for three reasons. First, a higher correction accuracy can be obtained. When $N = 8$, the RMS error can be reduced to 0.035λ and the Strehl ratio can be increased to 95%. Second, a higher diffraction efficiency can be obtained. According to diffractive optics theory [13], diffraction efficiency as a function of quantization level can be calculated as shown in Fig. 6. It indicates that the diffraction efficiency increases sharply when the quantization level is less than 8 and then changes slowly. Diffraction efficiency is as large as 95% for $N = 8$. Finally, total pixel number can be controlled in a reasonable range if low quantization level is used. It can be seen from Figs. 1 and 6 that a smaller wavefront RMS error and higher diffraction efficiency can be achieved with a larger quantization level. Therefore, a higher quantization level such as 16 or 32 can be used to acquire higher correction accuracy and diffraction efficiency. In that case, however, the required pixel number of DLCWFC will be increased drastically, leading to a significantly slow computation and data transformation rate of

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LCAOS. Figure 7 shows the relation between P_N, D, and r_0 for N = 8. As seen, desired pixel number increases apparently when atmospheric coherence length becomes smaller and telescope aperture becomes larger. For instance, the total pixel number of DLCWFC is 1700 × 1700 when r_0 = 5 cm and D = 20 m. However, if the atmospheric coherence length is increased to 10cm (i.e., r_0 = 10 cm), total pixel number can be reduced down to 768 × 768. Therefore, the strength of atmosphere turbulence is a key factor which must be considered when designing LCAOS for a ground-based telescope.

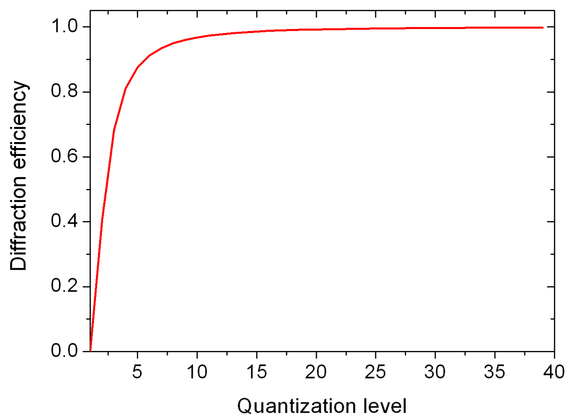

Fig. 6. Diffraction efficiency as a function of quantization level.

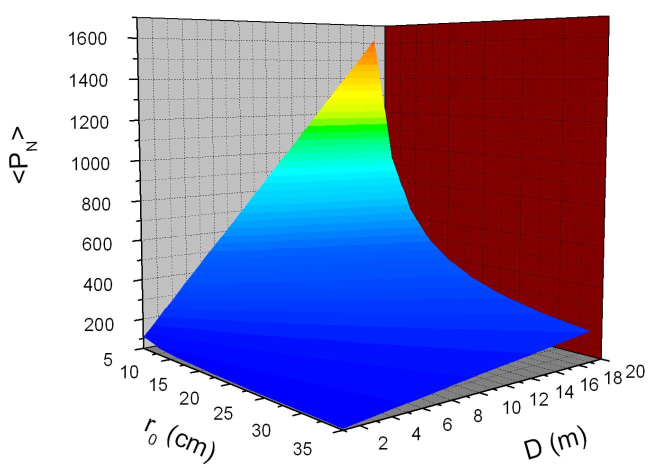

Fig. 7. $<P_N>$ as functions of atmosphere coherence length r_0 and telescope aperture D for N = 8.

4. Conclusions

In summary, compensation error of a diffractive type of LCWFC was analyzed for correcting atmospheric turbulence using Kolmogorov turbulence theory and Zernike polynomials method. By employing the theory of diffractive optics, diffracted wavefront error as a function of the quantization level was obtained first. In addition, we have found that pixel number across the DLCWFC aperture is not only a linear function of the telescope's aperture and

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quantization level, but an exponential function of atmosphere coherence length. Finally, a simple formula which represents relation between pixel number of DLCWFC and telescope aperture, quantization level and atmosphere coherence length has been derived. Using this formula, one can evaluate the compensation error of DLCWFC for atmospheric turbulence correction, or the desired pixel number of DLCWFC for a specific telescope. Therefore, these results should be useful for those who use DLCWFC to correct atmospheric turbulence for large aperture telescope.

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