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Using computer-generated holograms to test cubic surface

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ABSTRACT

Computer-generated holograms (CGH) have been widely used to evaluate symmetrical aspherical surfaces in combination with Fizeau interferometers. Because the CGH can create any wavefront shape, it can also be used in unsymmetrical aspherical surfaces testing. Taking the cubic surface as an example, this paper gives a thinking of testing unsymmetrical surfaces. First we deduce the expression of the aberration for the cubic phase when propagating and the CGH null lens design has been carried out while taking into consideration the higher order aberrations. The separation of the diffraction orders of the CGH is discussed. We fabricated the CGH using e-beam photography and tested a 13-mm diameter cubic surface.

1. Introduction

Wavefront coding technique [1] presented by Edward et al. in the end of last century is a milestone in pupil imaging technology field. And from then on, optical/digital imaging technology has been recognized and used widely. Although wavefront coding method can greatly extend the scene depth of a incoherent optical imaging system, correct the aberration caused by defocus, improve the environmental adaptability, reduce the volume and weight of the system by using fewer elements, it is a trouble to manufacture the asymmetric phase plate which is the key element of the wavefront coding system. The unique shapes of the phase plate, such as "cubic" or "petal" surface, increase the difficulty of optical processing. So a reasonable and feasible technique to test the phase plate accurately is desirable.

Currently, the method reported in the references of testing the phase plate is mainly contact measurement using the profiler [2]. Generally speaking, it is also accurate, but there are obvious shortcomings: (a) diamond probe may scratch the surface workpiece under test; it is particularly serious when the workpiece is made of PMMA for example. (b) The measuring speed is slow, typically 1 mm/s. Under such circumstances, temperature drift will bring error to the testing result.

Computer-generated holograms (CGH) have been used for years in optical testing of aspheric surfaces [3–5], and are on their way to becoming a state-of-the-art technology for aspheric tests in interferometry. With the development of diffraction optics, computer technology, and micro-processing technology, the accuracy of the CGH testing can achieve a hundredth of a wave, and the cost is reduced greatly. CGH is capable of producing an optical wavefront with any desired shape. So it is fascinating and will have good prospects to test free-form [6] surfaces using CGH.

The feasibility using CGH as null lens to test an asymmetric cubic phase plate is discussed in this paper. First we deduce the theoretical wavefront formula from the rays being reflected by the cubic phase plate. This is because higher aberrations will occur when the cubic phase travel through a distance. And the higher aberrations must be considered when the CGH function is computed. Then we review the principle of testing the unsymmetrical cubic surface using CGH and list the result of the optical design for the cubic surface testing system. The separation of the diffraction orders of the CGH phase function is discussed. The samples of the amplitude null CGH and alignment CGH are given in the paper, and the minimum line spacing of the CGH is 12.5 μm. If we use the e-beam writer to fabricate the CGH, the precision of the pattern distortion can achieve ±0.1 μm, so its effect on the wavefront error is just about 0.008 waves.

We tested a cubic surface successfully. It is believed that this approach can be used to measure other unsymmetrical surfaces.

2. Wavefront aberration of cubic phase in propagation [7]

The cubic phase introduces higher order aberrations as it propagates. There are two types of aberrations introduced: mapping distortion and optical path length (OPL) difference at each point. Mapping distortion means the coordinate changes as the wavefront propagates. OPL difference means that the rays travel

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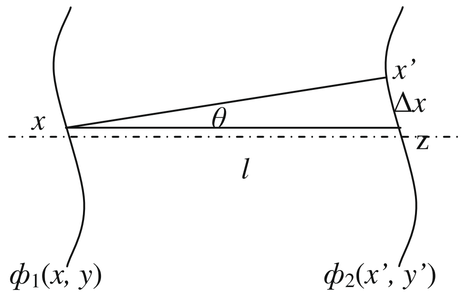

Fig. 1. Cubic wavefront propagation in space.

different optical path lengths as the wavefront propagates. Both of these two aberrations occur when the wavefront is not spherical or planar as it propagates.

As is shown in Fig. 1, the cubic phase has the function of $\phi_1(x,y)$ at (x,y) plane.

$$\phi_1(x,y) = \alpha(x^3 + y^3) \tag{1}$$

Here α is the coefficient for the cubic surface, and its unit is mm^{-2}. In this function, α is very small (In fact for the under test cubic phase plate, the magnitude of α is at 10 to the power of -5). The radius of the cubic phase plate "R" is small.

The surface slope of the cubic phase at this plane can be expressed as:

$$\phi'(x,y) = \phi'_x\hat{i} + \phi'_y\hat{j} = 3\alpha x^2\hat{i} + 3\alpha y^2\hat{j} \tag{2}$$

The second derivative of the cubic phase is :

$$\phi''(x,y) = \phi''_x\hat{i} + \phi''_y\hat{j} = 6\alpha x\hat{i} + 6\alpha y\hat{j} \tag{3}$$

As the wavefront propagates to a new plane (x',y'), which is a distance l away along *z-axis*, $\phi_2(x',y')$ is the new wavefront phase in the new coordinate.

$$x' = x + \phi'_x(x)\cdot l \quad y' = y + \phi'_y(y)\cdot l \tag{4}$$

$$\begin{aligned}\phi_2(x',y') &= \phi_{map}(x',y') + \phi_{OPD}(x',y')\\ &= \phi_1(x,y) + l/\cos\theta - l\end{aligned} \tag{5}$$

where θ is the angle between the ray and *z-axis*.

$$\begin{aligned}\tan\theta &= \sqrt{(\phi'_x(x)l)^2 + \left(\phi'_y(y)l\right)^2}\Big/ l\\ &= \sqrt{\phi'^2_x(x) + \phi'^2_y(y)},\ \text{and}\\ 1/\cos\theta &= \sqrt{\phi'^2_x(x) + \phi'^2_y(y) + 1}\end{aligned} \tag{6}$$

By using Taylor expansion, we have

$$\begin{aligned}\phi'_x(x) &= \phi'_x(x' - \Delta x) = \phi'_x(x') - \phi''_x(x')\cdot\Delta x\\ &= \phi'_x(x') - \phi''_x(x')\cdot\phi'_x(x')\cdot l\end{aligned} \tag{7}$$

Assume l and θ are small, if α and R are also very small, then $\phi'_x(x) \approx \phi'_x(x'), \phi'_x, \phi'_y$ is small. So there is:

$$\begin{aligned}\phi_{OPD}(x',y') &= l\cdot\sqrt{\phi'^2_x(x') + \phi'^2_y(y') + 1} - l\\ &\approx l + \frac{1}{2}l\left[\phi'^2_x(x') + \phi'^2_y(y')\right] - l = \frac{9}{2}l\alpha^2\left(x'^4 + y'^4\right)\end{aligned} \tag{8}$$

From Eqs. (1)–(8), we can get:

$$\begin{aligned}\phi_2(x',y') &= \phi_{map}(x',y') + \phi_{OPD}(x',y')\\ &= \alpha(x'^3 + y'^3) - \frac{9}{2}\alpha^2 l(x'^4 + y'^4) + 27\alpha^3 l^2(x'^5 + y'^5)\\ &\quad - 27\alpha^4 l^3(x'^6 + y'^6) + \cdots\end{aligned} \tag{9}$$

That is to say besides the cubic term "$x'^3 + y'^3$", the higher terms "$x'^4 + y'^4$", "$x'^5 + y'^5$"...are introduced to form the aberration.

3. Cubic surface testing

The under test cubic surface's equation is

$$Z = \alpha(x^3 + y^3), \quad \alpha = 7\times 10^{-5}\ \mathrm{mm}^{-2}, \quad x,y \in\ -6.5 \sim 6.5\ \mathrm{mm}$$

Its substrate is a plane. So we consider that just one CGH null lens can constitute the testing system and the collimated beam from a Zygo interferometer can be used. The optical layout is displayed in Fig. 2.

As CGH is a diffractive element, the diffraction order separation is concerned. We can use the aperture inside the interferometer to solve the equation of the diffraction order separation. Fig. 3 is the equivalent optical path for testing system. We can easily design the system using Zemax-EE 8.0 or Code V 8.30.

In our presentation in Section 2, we have considered that the cubic phase introduces higher order aberrations as it propagates. So the higher order aberrations must be considered when the test system is designed. The CGH function can be calculated using Eq. (9). The residual wavefront error of the test system is only 0.0057λ *P–V*, and the RMS value is 0.001λ. At the plane of the aperture, the simulated separation of CGH diffraction orders is shown in Fig. 4.

To align the null CGH with the interferometer, a reflection type alignment CGH is used as shown in Fig. 5. As the CGH is unsymmetrical, a "+" sign is used to mark the *x*-axis.

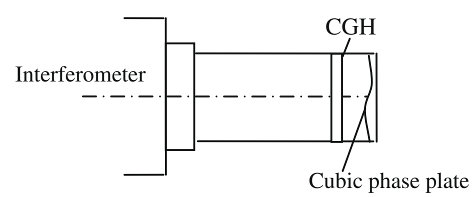

Fig. 2. Sketch of the testing system for cubic surface.

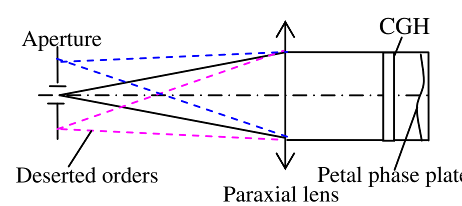

Fig. 3. The equivalent optical path for testing system.

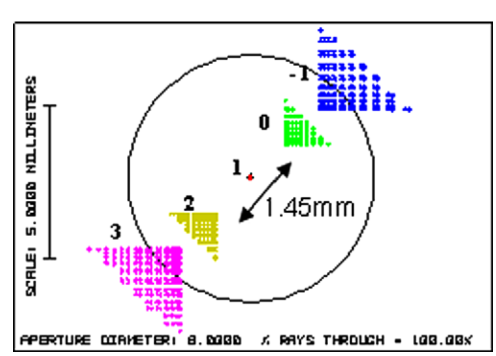

Fig. 4. The separation of CGH diffraction orders.

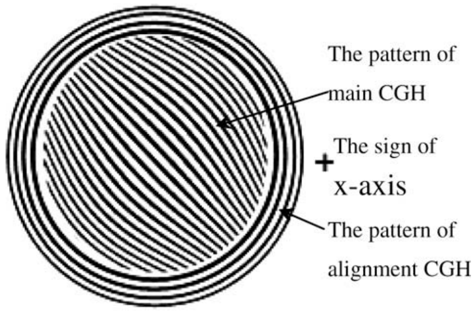

Fig. 5. The CGH pattern (plotted at a scale showing every 10 lines).

Fig. 6. The fabricated CGH null lens.

Fig. 6 shows the fabricated CGH by an e-beam writing system. The figure shows that all the CGH patterns were written onto the same BK7 substrate. The null CGH is designed to be used as an amplitude type in the 1st-order transmission mode. It has a circular aperture with 13.1 mm (main CGH) and a 50% duty cycle and the minimum line spacing of the CGH is 12.5 μm. The alignment CGH is designed as an amplitude type in the 3rd-order reflection mode. The CGH was installed in a lens frame made of aluminum alloy and the CGH was adhered to a drive ring. The drive ring can rotate among the frame. So the CGH owns a rotational degree of freedom. The frame was fixed to a six degree of freedoms adjusting rack.

The substrate unevenness has been measured in reflection and the result is $\delta s = 0.105\lambda$. It is easy to calculate that if measured in transmission the result will be 0.054λ ($(n-1)\delta s$). And it can be backed out from the final test result of the cubic surface.

When standard plane wave of Zygo interferometer traveled though the fabricated null CGH, we obtained the experimental result of the orders' separation. The photograph is shown in Fig. 7.

The cubic surface with the CGH null lens was measured by using the interferometric test configuration shown in Fig. 2. Fig. 8 shows the interferogram of the test result. From the test result captured by MetroPro program with Zygo GPI-XP interferometer, we get the *P–V* figure error is 1.05λ and the RMS error is 0.09λ. After removing the hologram substrate errors, we can get a result 0.996λ (*P–V*).

4. Discussion

The design result of the test system is perfect also there are some approximations in the derivation of the phase function that CGH need to produce during propagation. That is the residual wavefront error of the test system is only 0.0057λ *P–V*, and the RMS value is 0.001λ.

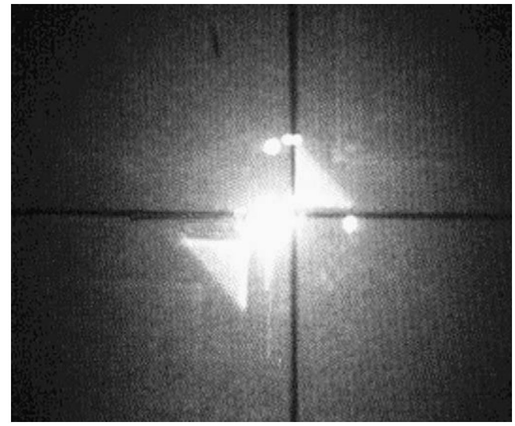

Fig. 7. The experimental result of the orders' separation.

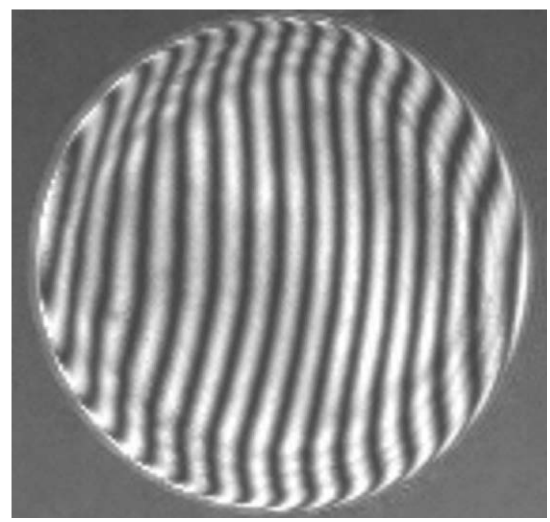

Fig. 8. The measurement result of the cubic surface.

To clarify that the derived phase function that CGH need to produce during propagation is correct and the assumptions affect little to the final wavefront errors, we can use Zemax-EE 8.0 to have a simulation.

In Fig. 3, the wavelength is 632.8 nm; the caliber of the cubic phase plate is 13 mm; $\alpha = 0.00007\ \mathrm{mm}^{-2}$; the distance between CGH and the cubic surface is 10 mm. we can know that if there is no higher terms (compared to the cubic term) during propagation, then the CGH only need to produce the cubic term "$x'^3 + y'^3$" to compensate the under test surface. But the fact is not so easy.

From Eq. (9) we can calculate the cubic term of the wavefront aberrations is:

$$w_3 = \alpha(x^3 + y^3) = 7 \times 10^{-5}\left(6.5^3 + 6.5^3\right) = 38.4475\ \mu\mathrm{m} = 60.758\lambda \tag{10}$$

The fourth power term of the wavefront aberrations is:

$$w_4 = -\frac{9}{2}\alpha^2 l(x^4 + y^4) = -4.5 \times 0.00007^2 \times 6\left(6.5^4 + 6.5^4\right) = -0.7464\lambda \tag{11}$$

Table 1
Relationship between the aberration of CGH and the theoretical aberration.

	The cubic term	The fourth term	The fifth term
The theoretical wavefront aberration terms from Eq. (9)	60.758λ	−0.7464λ	0.0122λ
The wavefront aberration terms in Zemax-EE 8.0	−60.758λ	0.7471λ	−0.0123λ

The fifth power term of the wavefront aberrations is:

$$w_5 = 27\alpha^3 l^2 (x^5 + y^5) = 27 \times 0.00007^3 \times 6^2 \left(6.5^5 + 6.5^5\right) = 0.0122\lambda \quad (12)$$

In Zemax-EE 8.0, each term of the CGH corresponding with one term calculated from Eq. (9). Table 1 shows a comparison between the terms in Zemax-EE 8.0 and the terms calculated theoretically from Eq. (9). We can see they are very close to each other. The deviation is less than 0.001λ.

When the cubic term, the fourth power term and the fifth power term of the wavefront aberrations are considered, the final residual wavefront error of the test system is only 0.0057λ *P–V*. If better result is needed, we must add higher power terms.

As listed in Section 3, the real test result is that the *P–V* figure error is 1.05λ and the RMS error is 0.09λ. It is still not good enough to be used in practice. Through our analysis there may be caused by the following reasons:

(1) The fixture makes the PMMA cubic phase plate occur elastic deformation more or less. We tested a PMMA flat plate with the same thickness and diameter in transmission and found the deformation value is about 0.03λ.

$$\Delta W_{def}(x, y) = 0.03\lambda \quad (13)$$

(2) We must consider the hologram pattern distortion, which limits test accuracy. The phase error due to a line shift is:

$$\Delta W_{pde}(x, y) = \frac{\varepsilon(x, y)}{s(x, y)} m\lambda \quad (14)$$

where $\varepsilon(x, y)$ = CGH position error in direction perpendicular to ruled fringes
$s(x, y)$, local center-to-center ruled fringe spacing
$\Delta W_{pde}(x, y)$, wavefront phase error due to pattern distortion at position (x, y) on CGH
In our case, $m = 1$; $\varepsilon(x, y) = \pm 0.2$ μm; $s(x, y)_{min} = 29$ μm. So $\Delta W_{pde}(x, y) = 0.0069\lambda$ (15)

(3) The phase error due to CGH substrate unevennessFrom Fig. 9, the substrate error is δs; n is substrate index of refraction. The phase error can be written as

$$\Delta W_{sfe} = (n - 1) \times \delta s \quad (16)$$

Here $n = 1.515$ (for BK7 glass), $\delta s = 0.105\lambda$.
So $\Delta W_{sfe} = (n - 1) \times \delta s = 0.054\lambda$ (17)

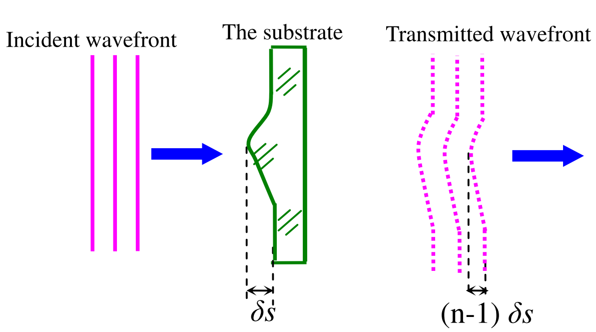

Fig. 9. Effect of substrate irregularity.

Table 2
Alignment tolerance analysis for CGH and the surface under test.

Δw (nm)		CGH	Cubic phase plate
dz	50 μm	0.696 nm	0.190 nm
Tilt *x*	5 min	0.190 nm	0.570 nm
Tilt *y*	5 min	0.190 nm	0.570 nm
Tilt *z*	5 min	21.058 nm	21.058 nm
Decenter *x*	10 μm	41.892 nm	41.892 nm
Decenter *y*	10 μm	41.892 nm	41.892 nm
RSS_{CGH} and RSS_{cubic}		62.880 nm	62.881 nm
RSS_{total}		88.926 nm	

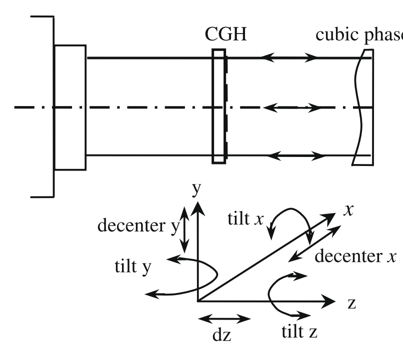

Fig. 10. Illustration of definitions of alignment parameters for CGH and cubic phase plate.

(4) The alignment error
When align the elements of the test system, errors will arise due to tilt, decenter and rotation. We have done tolerance analysis carefully. The results are listed in Table 2. The definition of alignment parameters is shown in Fig. 10.

The alignment error can be calculated as ΔW_{ale}

$$= 88.926/632.8 = 0.1405\lambda \quad (18)$$

As CGH substrate unevenness error can be backed out from the test result, so we calculate the accuracy of the test system just from Eqs. (12), (14), and (17), and it is:

$$\Delta = \sqrt{\Delta W_{def} + \Delta W_{pde} + \Delta W_{ale}} = \sqrt{(0.03\lambda)^2 + (0.0069\lambda)^2 + (0.1405\lambda)^2} = 0.1410\lambda \quad (19)$$

5. Conclusions

This design and tests have demonstrated that the custom-designed CGH null lens is fully feasible by using the commercially available Zygo interferometer for the unsymmetrical cubic surface. This method also applies to other unsymmetrical surfaces, such as the "petal" surface (Generalized cubic surface: $Z = \alpha(x^3 + y^3) + \beta(x^2y + xy^2)$). As the cubic phase plate is unsymmetrical, it is hard to align the elements in the test system. So the alignment error is large. This method is still not good enough to be used in practice to test unsymmetrical surfaces. However it will be of great prospect to apply the CGH test to the industrial production.

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