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Supplemental optical specifications for imaging systems: parameters of phase gradient

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Abstract. Specifications of phase error, peak to valley (PV), and root mean square (rms) are not able to represent the properties of a wavefront reasonably because of their irresponsibility for spatial frequencies. Power spectral density is a parameter that is especially effective to indicate the frequency regime. However, it seems not convenient for opticians to implement. Parameters of phase gradient, PV gradient, and rms gradient are most correlated with a point-spread function of an imaging system, and they can provide clear instruction of manufacture. The algorithms of gradient parameters have been modified in order to represent the image quality better. In order to demonstrate the analyses, an experimental spherical mirror has been worked out. It is clear that imaging performances can be maintained while manufacture difficulties are decreased when a reasonable trade-off between specifications of phase error and phase gradient is made. [DOI: 10.1117/1.3275460]

Subject terms: optical specification; phase gradient; ripple.

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1 Introduction

With the development of optical testing and manufacturing, the tolerances of system components are getting tighter so that better performance can be achieved. Traditional optical specifications are being improved in order to meet those new requirements. Among the improvements, the most famous one can be the utilization of power spectral density (PSD).[1–6] PSD is a parameter especially designed and used for the National Ignition Facility (NIF), where ripple is such a critical type of phase error that does harm to system security. Recently, ZYGO Corporation put forth another parameter, called the peak-to-valley robust (PVr), in the latest MetroPro v. 8.3.1.[7] It is a new version of PV but is more stable and insensitive to testing resolution. PV, root mean square (rms), PSD, and PVr may be applied to identify a wavefront as a final result, but none of them could direct opticians to fight against ripples during the process. It is necessary to have a more practical specification. Parameters of phase gradient are introduced trying to play such a role.

2 Simplified Ripple

Ripple is a kind of phase error of spatial frequency regime between figure error and roughness and is found to be of great importance.[8] It produces small-angle scatter that dramatically reduces Strehl ratio and, therefore, modulation transfer function of an imaging system. In the case of a high-power laser system, such as NIF, small-angle scatter may cause nonlinear energy amplification. It is dangerous to get near to or exceed the damage threshold of optics.

In order to simplify the analyses below, a rotational symmetric phase is considered to be the representation of ripple, as shown in Fig. 1. The generator of the phase is of cosine style

$$W = A_0 \cos\left(2\pi \frac{y}{d_0}\right), \qquad (1)$$

where A_0 represents the amplitude and d_0, the spatial period of the phase.

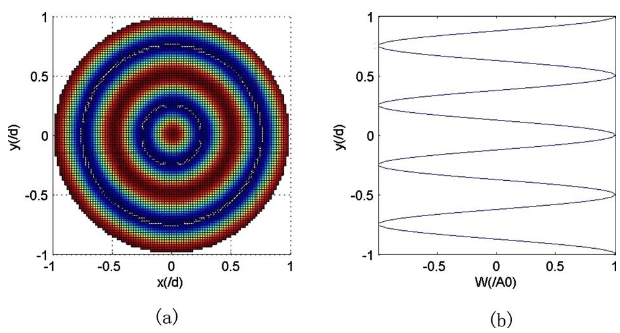


Fig. 1 (a) Map of a cosinelike phase and (b) profile of the generator.

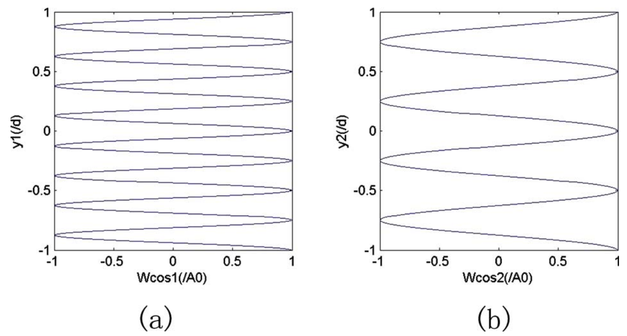


Fig. 2 Two cosinelike ripples of the same amplitude but different frequencies.

Because of the rotational symmetry of the phase, the simplified ripple shares most of the properties with its generator. Thus, the description of ripple is replaced by the generator in this paper. According to geometric optics, the diameter of the image spot, after propagation of distance L, will be

$$D=\left[\left(\frac{dW}{dy}\right)_{\max}-\left(\frac{dW}{dy}\right)_{\min}\right]L\propto\frac{A_0}{d_0}. \tag{2}$$

That is, the spot size is proportional to the difference between the maximum and minimum gradient of the phase, therefore, PV of the phase, A_0, and spatial frequency, $1/d_0$.

3 Properties of Standardized Specifications

3.1 *Parameters of Wavefront Errors*

PV and rms are the most popular optical specifications. Both of them are parameters describing how flat a wavefront phase is. When the phase of an exit pupil has rms of σ, the Strehl ratio of the point spread function (PSF) is

$$S=\exp\left[-\left(\frac{2\pi}{\lambda}\sigma\right)^2\right]. \tag{3}$$

However, the formula could be imprecise. Suppose two phase profiles of length $2d$ are both of cosine ripple styles but of different frequencies,

$$W_{\cos 1}=A_0\cos\left(2\pi\frac{y_1}{d_1}\right),\quad -d\leqslant y_1\leqslant d,$$

$$W_{\cos 2}=A_0\cos\left(2\pi\frac{y_2}{d_2}\right),\quad -d\leqslant y_2\leqslant d, \tag{4}$$

where $d_1=1/2d_2=1/4d$, as shown in Fig. 2. According to Eq. (2), $W_{\cos 2}$ performs better than $W_{\cos 1}$. Although specified by PV and rms, the two phases share the same results, $\mathrm{PV}=2A_0$ and $\mathrm{rms}=(\sqrt{2}/2)A_0$. This leads to the conclusion that PV and rms are not responsible for spatial frequency and nor capable of specifying an image system, especially when ripple dominates.

A more accessible example is in optical design field. When optimizing an imaging system, different results will be given with merit function of wavefront and spot radius (OPDX and TRAC in ZEMAX, respectively), as shown in Fig. 3. In order to get a better wavefront, more high-order aberrations are introduced into the system to balance residual phase errors. The combined wavefront has tighter PV and rms but the image spot increases because the wavefront is not as smooth as the one aiming at spot radius.

3.2 *Power Spectral Density (PSD)*

PSD is also an optical specification listed in ISO 10110. It is superior to PV and rms for the advantage of sensitivity to frequency. Although it is easy to calculate, two-dimentional PSD is hard to interpret. Therefore, a slice sampling has to be done through a phase map and then, one-dimentional PSD can be worked out of the profile. The standardized expression of PSD is

$$\mathrm{PSD}=\mathrm{A}/\upsilon^{\mathrm{B}}, \tag{5}$$

where A and B are constants and υ stands for spatial frequency. When abscissa and ordinate are in log-log form, the expression transforms to a straight line:

$$\log\mathrm{PSD}=\log\mathrm{A}-\mathrm{B}\log\upsilon. \tag{6}$$

Essentially, PSD is a parameter that decomposes a wavefront phase into a series of cosine and sine segments and the amplitude of each segment defines the value of corresponding frequency of PSD. If PSD of a phase does not meet the requirement, then it is not easy to give instructions to improve it. Furthermore, when a zone of a phase is changed after polishing, the whole shape of PSD could vary, as shown in Fig. 4. Figures 4(a) and 4(c) are two profiles of the same mirror before and after a certain polishing process at the edge. They are identical except for the edge zone. However, the respective outlines of PSD in Figs. 4(b) and 4(d) are dramatically different from each other. That is why PSD sometimes confuses opticians during manufacture.

4 Merits of Specifications of Phase Gradient

The importance of phase gradient was published years ago.[9,10] Unfortunately, this kind of parameter has never played a deserving role in the optical specification field. Similar to PV and rms of phase error, phase gradient can be specified by PV gradient and rms gradient. For a high-energy system with a phase map of area S, the two-dimentional gradient was treated as a scalar,[11]

$$\mathrm{Grad}=\sqrt{\left(\frac{\partial W}{\partial x}\right)^2+\left(\frac{\partial W}{\partial y}\right)^2}. \tag{7}$$

Therefore,

$$\mathrm{RMSG_{REF}}=\sqrt{\frac{\iint_S[\mathrm{Grad}-\overline{\mathrm{Grad}}]^2dxdy}{S}}, \tag{8}$$

where

$$\overline{\mathrm{Grad}}=\frac{\iint_S(\mathrm{Grad})dxdy}{S}. \tag{9}$$

This definition is not reasonable for imaging systems. A phase with the same gradient amplitude all over the aperture could make the image a blur, as shown in Fig. 5. The

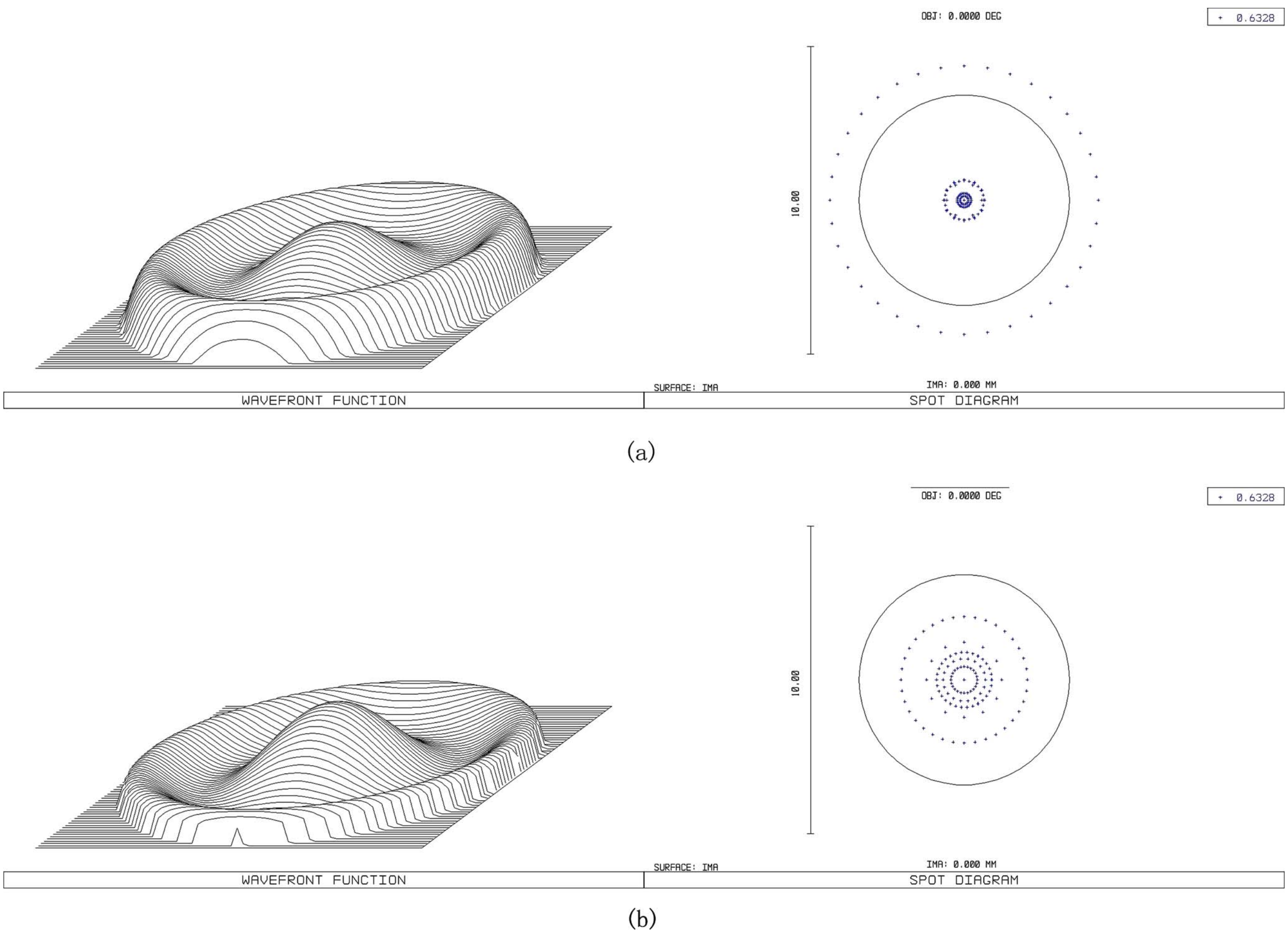


Fig. 3 Different optimized results given by ZEMAX with merit function of (a) OPDX and (b) TRAC, respectively: (a) PV=0.0615λ, rms=0.0114λ, rms radius=2.358 μm, geometrical radius=4.369 μm; and (b) PV=0.0845λ, rms=0.0181λ, rms radius=1.239 μm, geometrical radius=2.058 μm.

phase is of conical style, and the gradient amplitude of the phase is the same if the origin is ignored. The image formed by the phase is a ring because the rays emerged from the phase propagate in different directions. However, when specified by Eq. (8), the phase gives a rms gradient of zero, which is obviously unreasonable.

Considering phase gradient as a vector, the specifications are redefined in this paper. Assuming an image formed by a phase, the compactness of the image denotes the image quality. The image center will be at $(\overline{\partial W/\partial x}, \overline{\partial W/\partial y})$, where

$$\overline{\frac{\partial W}{\partial x}} = \frac{\iint_S (\partial W/\partial x) dx dy}{S},$$

$$\overline{\frac{\partial W}{\partial y}} = \frac{\iint_S (\partial W/\partial y) dx dy}{S}. \tag{10}$$

The gradient deviation of each point from the energy center is

$$\text{GradDev}_{\text{2-D}} = \sqrt{\left(\frac{\partial W}{\partial x} - \overline{\frac{\partial W}{\partial x}}\right)^2 + \left(\frac{\partial W}{\partial y} - \overline{\frac{\partial W}{\partial y}}\right)^2}. \tag{11}$$

PV gradient is defined as the maximum gradient deviation between any two points,

$$\text{PVG}_{\text{2-D}} = \left(\left\{\left[\left(\frac{\partial W}{\partial x}\right)_{\text{point 1}} - \left(\frac{\partial W}{\partial x}\right)_{\text{point 2}}\right]^2 + \left[\left(\frac{\partial W}{\partial y}\right)_{\text{point 1}} - \left(\frac{\partial W}{\partial y}\right)_{\text{point 2}}\right]^2\right\}^{1/2}\right)_{\max}. \tag{12}$$

And the image compactness, which is defined as rms gradient in this paper, can be expressed as the standard deviation of GradDev in Eq. (11),

$$\text{RMSG}_{\text{2-D}} = \sqrt{\frac{\iint_S (\text{GradDev}_{\text{2-D}} - \overline{\text{GradDev}_{\text{2-D}}})^2 dx dy}{S}}, \tag{13}$$

where

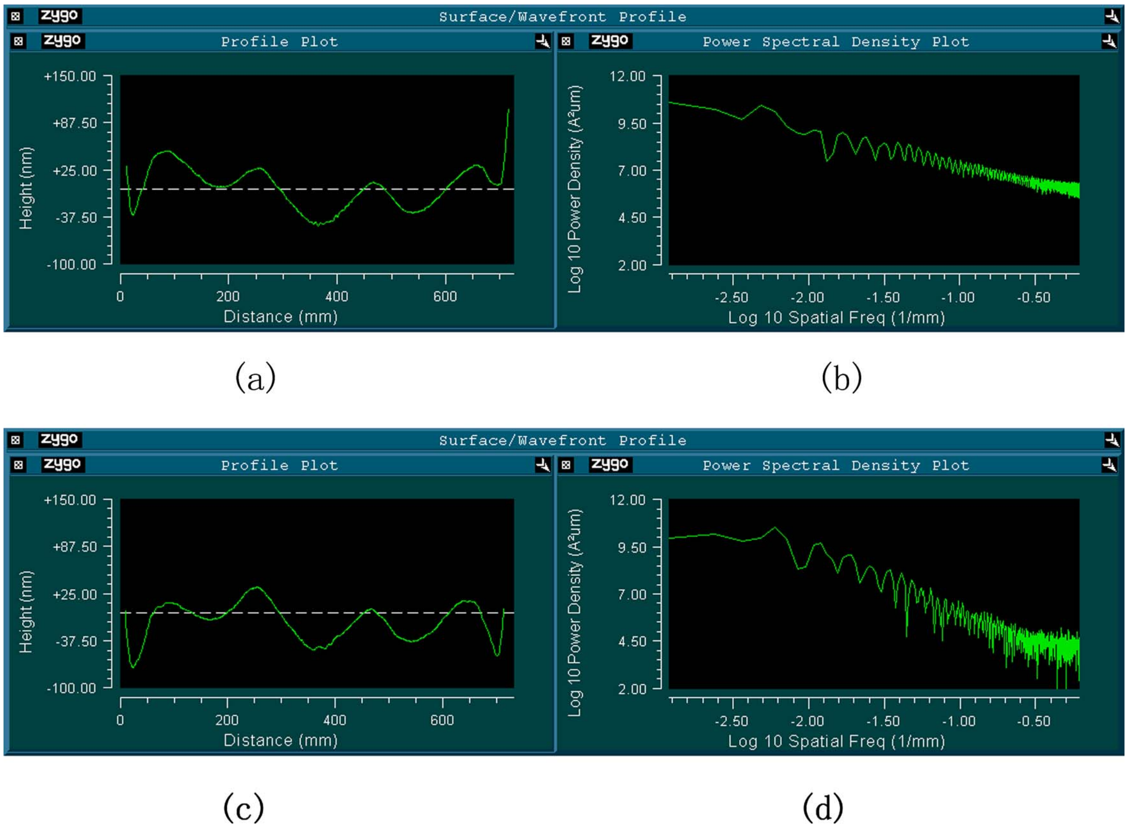


Fig. 4 Profiles of a mirror before and after certain polishing at the edge. Note the similarities of (a) and (c) and dramatic slope differences between (b) and (d).

$$\overline{\text{GradDev}_{2\text{-D}}}=\frac{\iint_S \text{GradDev}_{2\text{-D}}\,dxdy}{S}. \tag{14}$$

The modified rms gradient of the phase in Fig. 5 is the radius of the image, which is an acceptable result. Similarly, for a phase profile of length d, the one-dimentional PV gradient and rms gradient are defined as

$$\text{PVG}_{1\text{-D}}=\left[\left(\frac{dW}{dy}\right)_{\text{point 1}}-\left(\frac{dW}{dy}\right)_{\text{point 2}}\right]_{\max}, \tag{15}$$

and

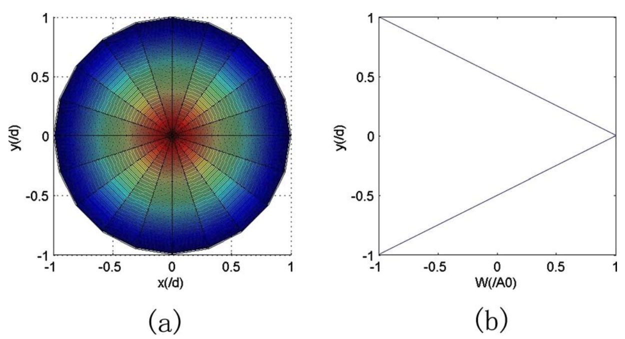


Fig. 5 (a) Map of a cone-shaped phase and (b) profile of the generator.

$$\text{RMSG}_{1\text{-D}}=\sqrt{\frac{\int_d(\text{GradDev}_{1\text{-D}}-\overline{\text{GradDev}_{1\text{-D}}})^2dy}{d}}, \tag{16}$$

where

$$\text{GradDev}_{1\text{-D}}=\left|\frac{dW}{dy}-\overline{\frac{dW}{dy}}\right|,$$

$$\overline{\text{GradDev}_{1\text{-D}}}=\frac{\int_d\left|dW/dy-\overline{dW/dy}\right|dy}{d},$$

$$\overline{\frac{dW}{dy}}=\frac{\int_d(dW/dy)dy}{d}. \tag{17}$$

Precisely speaking, the modified rms gradient should have the name of the standard deviation of phase gradient deviation. Because rms and standard deviation stand for the same parameter of optical specifications, the convention is kept in this paper.

Figure 6(a) illustrates a phase composed of complicated aberrations, including tilt, defocus, coma, astigmatism, and spherical aberration. After differential operations in the x- and y-axes of each point, Fig. 6(b) is drawn as the gradient distribution $(\partial W/\partial x,\partial W/\partial y)$ of the phase. The cross indicates the distribution center $(\overline{\partial W/\partial x},\overline{\partial W/\partial y})$, which is also the energy center. On the other hand, the phase error is

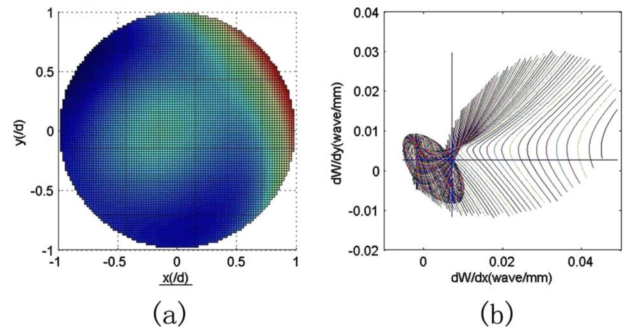


Fig. 6 (a) Map of a complicate phase and (b) its gradient distribution.

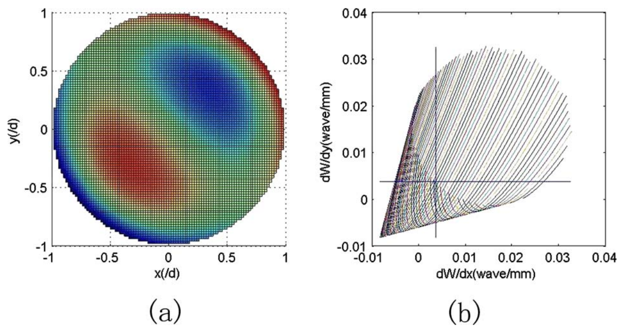


Fig. 8 (a) Map of a coma phase and (b) its gradient distribution.

transformed to a surface error and imported into ZEMAX in order to assure the exit pupil of an imaging system shares the same phase error. Figure 7 illustrates the PSF of the system. Note that Fig. 7 is extremely similar to Fig. 6(b). Therefore, the specifications of phase gradient make great sense because gradient distribution is a reasonable and reliable representation of the performance of an imaging system: the modified PV gradient and rms gradient denote geometrical diameter and rms diameter of the PSF, respectively. This can also be proved by specifying the ripple profiles in Eq. (4): $\mathrm{PVG}_1=16\pi A_0/d$, $\mathrm{RMSG}_1=4\sqrt{2}\pi A_0/d$ of $W_{\cos 1}$ and $\mathrm{PVG}_2=8\pi A_0/d$, $\mathrm{RMSG}_2=2\sqrt{2}\pi A_0/d$ of $W_{\cos 2}$. For this point, PV gradient and rms gradient are superior to PV and rms.

When the phase map of an optical component under testing is obtained, opticians can easily distinguish the over- and underpolished zones by the lowlands and highlands of the map. Following these instructions, PV and rms can be improved step by step.[12,13] Similarly, a new map working as the guider of phase gradient can be refined and drawn. To make it clearer, a simpler phase is shown in Fig. 8. This phase only contains the eighth and ninth Zernike terms (first-order coma and tilt). When the phase gradient deviation in Eq. (11) of each point replaces the original phase value, a map as the phase gradient guider is drawn, as shown in Fig. 9. This is the advantage of phase gradient on PSD. The map of the phase represents how flat the phase is, whereas the map of phase gradient deviation denotes how compact the rays will be. A flatter phase does not always result in better image quality. That is why the highlands and lowlands in Figs. 9(a) and 9(b) are not coincident. If phase and phase gradient specifications work at the same time, then there must be a trade-off between them. And one must check that the map of phase gradient deviation is a guider that only points out where to polish to improve the gradient specifications. Improper polishing at the highlands in Fig. 9(b) can make things worse. It is necessary for opticians to have in mind that phase gradient is a vector; thus, the correct polishing methods must be chosen wisely and executed carefully at the right places.

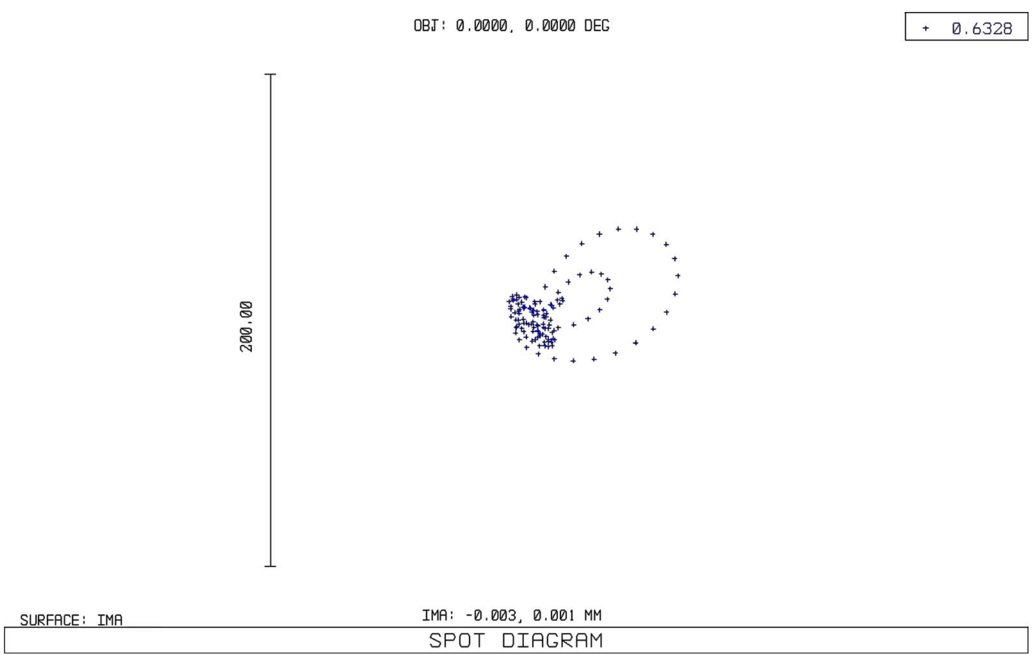


Fig. 7 PSF of an imaging system with phase error of exit pupil in Fig. 6(a).

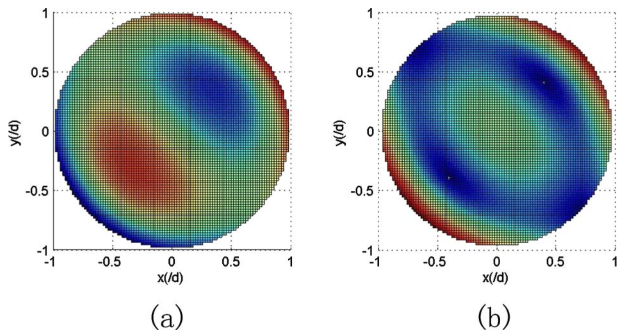


Fig. 9 (a) Map of the coma phase and (b) map of its phase gradient deviation.

Anyway, specifications of phase gradient give clear instructions and are easy to implement.

It is interesting when a phase contains tilt, such as the one in Fig. 6. The energy center is not at the origin. Unlike PV and rms, it is not necessary for PV gradient and rms gradient to remove tilt for reasonable results. Tilt is automatically removed because it causes equal gradient (the same amplitude and direction) all over the phase. This makes the algorithms of the two parameters simpler. When dealing with a phase with tilt removed, gradient deviation is still executed instead of gradient because tilt is calculated based on the flatness of a phase error and the energy center of the phase without tilt may not locate at the origin, as shown in Fig. 8(b).

5 Experiment and Results

A 240-mm-diam F/1.5 spherical mirror has been manufactured with the specifications of phase, phase gradient and PSD. The mirror was tested with a digital interferometer to acquire accurate phase error, which gave PV and rms. A program based on Eqs. (12) and (13) dealt with the phase map and provided PV gradient and rms gradient. An off-axis aberration-free Foucault apparatus[14] was set up for the star test. A pinhole of a few microns worked as the object and was regarded as isoplanatic so that the spot diameter was equal to the diameter of star image minus the diameter of star. The apparatus exceeded PV of 60 nm, rms of 12 nm, PV gradient of 5 nm/mm, and rms gradient of 0.5 nm/mm, and the error was removed from testing results. The size of star image was measured with a microscope, which gave precision of 2.5 μm. The experimental data are refined and listed in Table 1 and illustrated in Fig. 10.

On the basis of the data in Table 1 and Fig. 10, conclusions can be made as follows:

1. Comparing data 2 with data 3, PV and rms are almost the same while spot size decreases a half, which is not a proper judgment.
2. Watching the trends of data in each column, rms gradient is the parameter that denotes image quality the best despite that PV gradient should indicate the spot diameter.
3. PSD is not stable during manufacture.

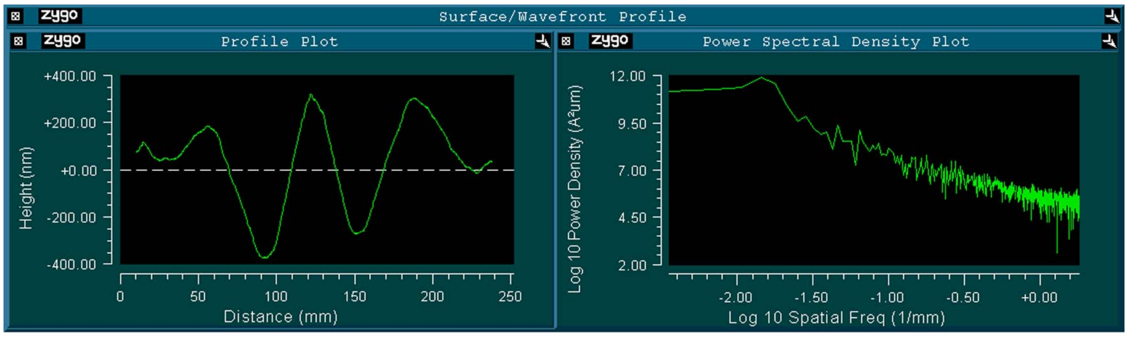
It can be explained that PV gradient, as with PV, indicates the worst zone of a phase. It could be a very small portion of the whole map and have little influence on the image to be unnoticeable. The main functions and purposes of a high-power laser system and an imaging system are totally different; it may not be certain whether the parameter PSD is suitable for an imaging system. If the optical components are of very high precision, then physical optics is going to play a more important role than geometric optics when explaining an imaging phenomenon. Components with better phase gradient perform better. It is still reasonable that phase-gradient parameters are used to specify systems of tight tolerances.

6 Conclusions

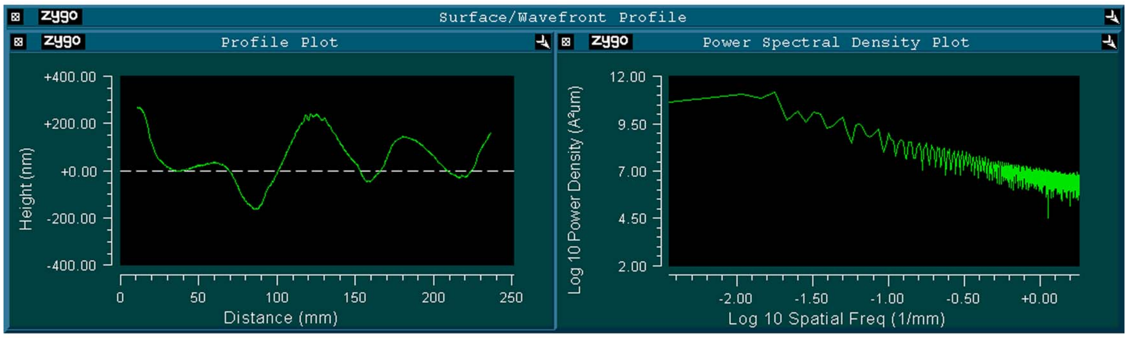
The theoretical analyses and experiment have shown that the performance of an imaging system can be predicted and controlled quite well by parameters of phase gradient all through the manufacturing process. Opticians are able to improve image quality directly by focusing on gradient parameters and following the instruction of the map of phase-gradient deviation. The gradient parameters can be a bridge crossing the gap between manufacturing process testing and final inspection. And because of the usage of PV gradient and rms gradient, the tolerances of PV and rms are not as tight as before. Therefore, when specifying an imaging system, there should be a trade-off between the parameters of phase error, phase gradient, and PSD in order to optimize optical specifications so as to maintain or even increase the imaging performances while decreasing the difficulties and cost of optical manufacturing. However, the gradient parameters are sensitive to the resolution of testing instruments and have a high requirement of the testing environment, especially vibration. There is still a long way to go for them to be recognized as a standard.

Table 1 Four sets of experimental data of a mirror.

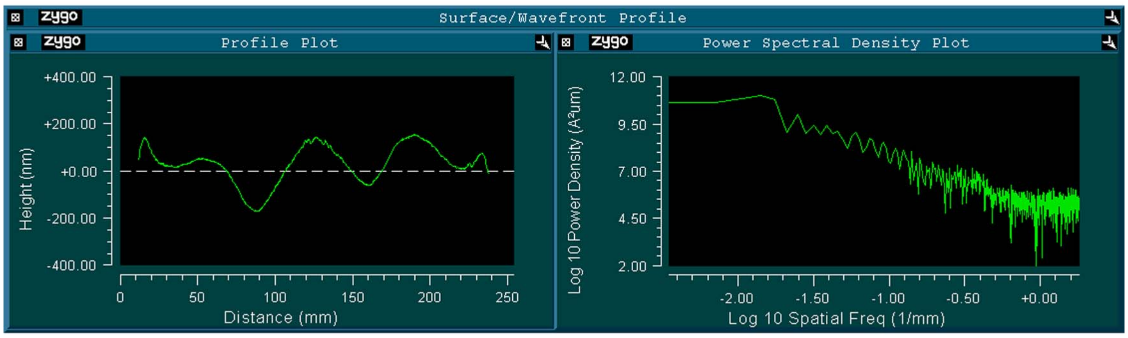
	PV (nm)	rms (nm)	PVG (nm/mm)	RMSG (nm/mm)	Spot diameter (mm)	PSD (Fig. 10)
1	763.79	150.61	22.34	3.10	0.085	(a)
2	470.17	87.33	21.71	2.02	0.04	(b)
3	461.31	74.67	15.31	1.33	0.025	(c)
4	124.66	13.29	4.49	0.30	0.0025	(d)



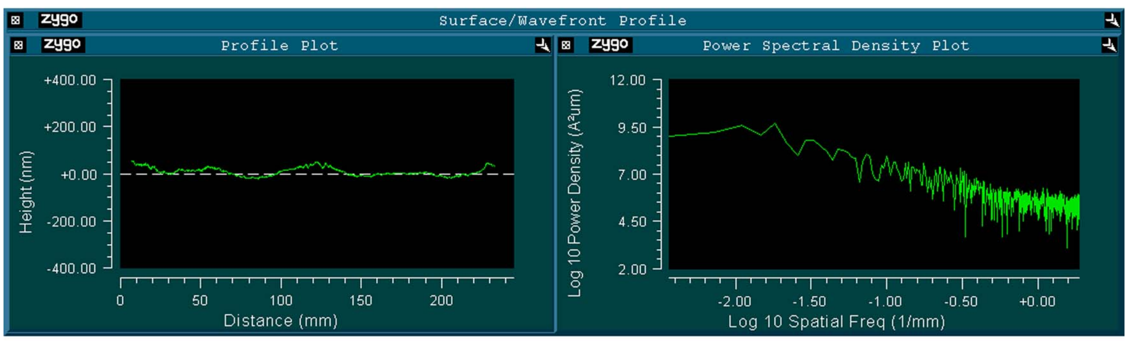
(a)



(b)



(c)



(d)

Fig. 10 Four PSD data of profiles in Table 1. The abscissas and ordinates are of the same units and scales.

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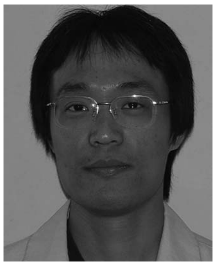

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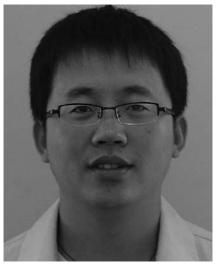

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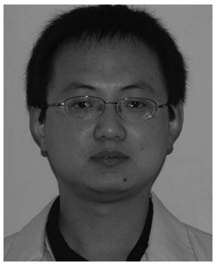

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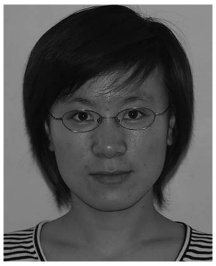

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