Design of multiplexed phase diffractive optical elements for focal depth extension

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Abstract: A more computationally tractable method to design a multiplexed phase diffractive optical element with optical design software to extend the depth of focus is proposed, through which the intensity distribution of the output beams can also be controlled with great flexibility. The design principle is explained in detail. And the feasibility of this design method is illustrated through a design example followed by computer simulation verification.

OCIS codes: (050.1970) Diffractive optics; (080.3620) Lens system design; (090.4220) Multiplex holography.

References and links

1. Introduction

Diffractive optical elements (DOEs) have been used to extend the depth of focus, such as non-uniform transmission filters [1], Fresnel zone pupil masks [2], hybrid refractive diffractive optical systems [3] and multiplexed phase diffractive optical elements (MPDOEs) [4,5]. The conventional way to design such long-focus-depth DOEs is to use various numerical optimization algorithms such as simulated annealing [6] or iterative gradient approaches [7,8]. However, these methods have all suffered from one significant deficiency: these optimization approaches are computer intensive and not intuitive, which makes the design of DOEs very complex and not compatible with the commercial optical design software packages such as ZEMAX, CODE-V.

In this paper we propose a more computationally tractable method to design multiplexed phase diffractive optical elements with optical design software to extend the depth of focus. It
is shown that the MPDOE designed by this method cannot only correct chromatic aberration and spherical aberration but also extend the focal depth. The design principle is described in Section 2. An illustrative example is given in Section 3. And our conclusions are put forward in Section 4.

2. Design principle

The MPDOE is one kind of diffractive optical elements in which several phase functions with weights are multiplexed into a single one. The design procedure of MPDOEs is as follows:

First, the separate phase function \( \exp(i\phi_n) \) should be derived through the optimization of the optical system at corresponding defocusing position \( t_n \) in an appropriate range of defocusing distances.

Then those separate phase function can be multiplexed into one single phase only function. Consider a linear combination of \( N \) phase functions \( \exp(i\phi_n) \) with real weights \( A_n \)

\[
M \exp(ia) = \frac{\sum_{n=1}^{N} A_n \exp(i\phi_n)}{M} \quad (1)
\]

For \( M \neq 0 \), the multiplexed phase function is

\[
\exp(ia) = \frac{\sum_{n=1}^{N} A_n \exp(i\phi_n)}{M} \quad (2)
\]

where

\[
M = [A_1^2 + A_2^2 + \cdots + A_N^2]^{1/2}
\]

\[
+ 2A_1A_2 \cos(\phi_1 - \phi_2) + 2A_1A_3 \cos(\phi_1 - \phi_3) + \cdots + 2A_1A_N \cos((\phi_1 - \phi_N))
\]

\[
+ 2A_2A_4 \cos((\phi_2 - \phi_4) - (\phi_1 - \phi_2)) + \cdots + 2A_2A_N \cos((\phi_2 - \phi_N) - (\phi_1 - \phi_2))
\]

\[
+ \cdots
\]

\[
+ 2A_{N-1}A_N \cos((\phi_{N-1} - \phi_N) - (\phi_1 - \phi_{N-1}))\]^{1/2} \quad (3)

Obviously the phase \( a \) derived by Eq. (2) is a discontinuous function with modular \( 2\pi \). When the multiplexed phase diffractive optical element with phase \( a \) is employed in optical systems, the output is a new linear combination for the original phase functions with new weights and other spurious terms, as proved below [9].

Since \( 1/M \) can be considered as a periodic function of \((\phi_1 - \phi_2), (\phi_1 - \phi_3), \ldots (\phi_1 - \phi_N)\) with period \( 2\pi \), this leads to a Fourier series expansion:

\[
M(\beta_1, \beta_2, \ldots, \beta_{N-1}) = \frac{1}{M} \sum_{m_1} \cdots \sum_{m_{N-1}} a_{m_1 \cdots m_{N-1}} \exp(im_1\beta_1 + im_2\beta_2 + \cdots + im_{N-1}\beta_{N-1}) \quad (4)
\]

Here \( \beta_1 = (\phi_1 - \phi_2), \beta_2 = (\phi_1 - \phi_3), \ldots, \beta_{N-1} = (\phi_1 - \phi_N) \) and \( m_1, m_2, \ldots, m_{N-1} \) are integral numbers.

And the new weights \( a_{m_1 \cdots m_{N-1}} \) can be written as

\[
a_{m_1 \cdots m_{N-1}} = \frac{1}{(2\pi)^N} \int_{0}^{2\pi} \cdots \int_{0}^{2\pi} \frac{1}{M} \exp(-im_1\beta_1 - \cdots - im_{N-1}\beta_{N-1})d\beta_1 \cdots d\beta_{N-1} \quad (5)
\]
From Eq. (2) and Eq. (4),

\[ a_i \exp\left[i\left(m_1 + \cdots + m_{N-1} + 1\right)i \phi_1 - im_1 \phi_2 \cdots \cdots - im_{N-1} \phi_N \right] + a_j \exp\left[i\left(m_1 + \cdots + m_{N-1} - 1\right)i \phi_1 - i(m_1 - 1) \phi_2 \cdots \cdots - i(m_{N-1} - 1) \phi_N \right] + \cdots \]

\[ + a_N \exp\left[i\left(m_1 + \cdots + m_{N-1} \right) \phi_1 - im_1 \phi_2 \cdots \cdots - i(m_{N-1} - 1) \phi_N \right] \quad \cdots (6) \]

Two insights can be got from the above analyses: First, as long as the phase functions \( \phi_1, \phi_2, \cdots, \phi_N \) of the diffractive optical element are separately got from the optimization at different positions within a reasonable range of defocusing distance, the phase function of the MPDOE can be calculated with the help of Eq. (2) and the depth of focus of the optical system will be extended by this multiplexed phase diffractive optical element. Second, from the relation between the input weight \( A_n \) and the output weight \( a_{m_1 \cdots m_{N-1}} \), the input weights or intensity of the phase functions can be optimized and chosen according to the wanted output intensity distribution of the MPDOE.

3. Optical system and specifications

The following optical system is intended to illustrate the design of one MPDOE with \( N \) weighted phase functions and to demonstrate its effectiveness in extending depth of focus. With the F-number (\( F^\# \)), extending focal depth (\( \Delta d \)), and the dominant wavelength (\( \lambda \)) of the optical system, the number of phase function (\( N \)) can be rounded upwards to the nearest integer as follow.

\[ N = \text{ceil}\left(\frac{\Delta d}{4\lambda(F^\#)^2}\right) \quad (7) \]

The specifications of our design example are as follows: wavelength range is 8-12um, the dominant wavelength is 10um, total Field of View is 10°, Effective Focal Length is 90mm, the F-number is 1.28, and the intended extending focal depth is 0.18mm. The focal length of conventional optical system with 1.28 F-number is about 0.06mm. With Eq. (7), the number input weights of phase functions can be determined as 3. According to Eq. (6), when \( N = 3 \), the output of the MPDOE can be written as a new linear combination of the original phase functions and spurious terms:

\[ \exp(ia) = \cdots + (a_{00}A_1 + a_{10}A_2 + a_{01}A_0)\exp(i \phi_1) + (a_{00}A_1 + a_{10}A_2 + a_{01}A_0)\exp(i \phi_2) + \cdots + (a_{00}A_1 + a_{10}A_2 + a_{01}A_0)\exp(i \phi_3) + \cdots \]

\[ = \cdots + a_i \exp(i \phi_1) + a_j \exp(i \phi_2) + a_k \exp(i \phi_3) + \cdots \quad (8) \]

Where the ellipses corresponds to the spurious terms introduced by the multiplexed procedure.

Given the requirement of the intensity distribution along the axis, the weight \( a_1, a_2, a_3 \) of the output phase functions can be got and then the input weights \( A_1, A_2, A_3 \) of the phase functions can be determined according to Eq. (5) and Eq. (6). For example, when more energy is distributed to the outer regions of the extending focal depth, in order to improve corresponding image quality, the input weights \( A_1, A_2 \) of the MPDOE should be larger than \( A_3 \). On the contrary, when more energy is distributed to the central regions of the extending focal depth, in order to improve corresponding image quality, the input weights \( A_2 \) should be larger than \( A_1, A_3 \). For simplicity, we assume the input intensity of three phase function is equal, which results the input weights:
The layout of the optical system is shown in Fig. 1. The optical system consists of two positive lenses made of Germanium. The diffractive surface is on the convex surface of the first lens. With the help of the optical design program ZEMAX, this system is set up with three configurations: they have the same structure with the same parameters except the distance \( t \) between the window and the image plane and the phase coefficients of the diffractive surface.

![Fig. 1. Layout of the optical system](image)

All diffractive surfaces in ZEMAX, for example binary 2, bend rays according to grating equation:

\[ n_2 \sin \theta_2 - n_1 \sin \theta_1 = \frac{m\lambda}{d} = m\lambda T \]  

(9)

Where \( n_1 \) is the index of the material before the diffractive surface, \( n_2 \) is the index of the material after the diffractive surface, \( \theta_1 \) is the angle of incidence, \( \theta_2 \) is the angle of exitance, \( m \) is the diffraction order, \( \lambda \) is the wavelength and \( T \) is the grating period (inverse of the line spacing \( d \)). The equation above is Snell's law for refraction, plus an additional ray bending term representing diffraction. The Binary 2 surface allows the grating period to vary as a rotationally symmetric polynomial. Zemax uses the phase advance or delay represented by the binary 2 surface locally to change the direction of the propagation of the ray. The binary 2 surface adds phase to the ray according to the following equation no matter what the wavelength is:

\[ \phi_{con}(r) = \sum_{i=0}^{n} B_i r^{2i} \]  

(10)

Where \( \phi_{con}(r) \) is the phase in periods at radius \( r \), \( con \) is the configuration number, \( n \) is the number of the polynomial coefficients in the series, \( B_i \) is the coefficient on the 2\( i \)th power of \( r \), which is the normalized radial aperture coordinate, \( m \) is the diffraction order and the maximum value of \( n \) used in this system is three.

The coefficient \( B_1 \) of Eq. (10) is the determinant of the diffractive surface optical power when \( m = 1 \):
\[ \Phi_D (r) = \frac{1}{f} = \lambda B \]

Where \( \lambda \) is the wavelength. It means that different wavelengths will give different optical powers. The coefficients dispersion of the diffractive surface is determined by Eq. (11):

\[ V_D = \frac{-\lambda_0}{\lambda_{\text{max}} - \lambda_{\text{min}}} \]

Where \( \lambda_{\text{max}} \) is the shortest wavelength, \( \lambda_{\text{max}} \) is the shortest wavelength, \( \lambda_0 \) is the dominant wavelength. It is contrary to the coefficients of dispersion of most lens materials. So diffractive-refractive hybrid can correct chromatic aberration of optical system [10–12].

The parameters of this optical system are shown in Table 1. The distance \( t \) between the window and the image plane is 0mm, 0.06mm, and \(-0.06\)mm in three configurations respectively, so the depth of focus is 0.18mm. When the diffractive order \( m = 1 \) is chosen, optimizing the phase coefficients \( B_1, B_2, \) and \( B_3 \) of the diffractive surface for each configuration. Because only the material Germanium is used in the optical system, the diffractive lens plays a key role in correcting chromatic aberration. Meanwhile the diffractive lens also correct some spherical aberration introduced by the two positive lenses.
Table 1. Parameters of optical system

<table>
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<tr>
<th>Surf Type</th>
<th>Radius (mm)</th>
<th>Thinkness (mm)</th>
<th>Glass</th>
<th>Conic</th>
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<td>7.67</td>
<td>Germanium</td>
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<tr>
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</table>

Three phase functions corresponding to three configurations are as follows:

\[
\phi_i(r) = -0.007829 r^2 + 1.021439 \times 10^{-6} r^4 - 4.263861 \times 10^{-10} r^6 \quad (13)
\]

\[
\phi_2(r) = -0.006565 r^2 + 1.381937 \times 10^{-6} r^4 - 6.007342 \times 10^{-10} r^6 \quad (14)
\]

\[
\phi_3(r) = -0.008575 r^2 - 1.249763 \times 10^{-7} r^4 + 5.872878 \times 10^{-12} r^6 \quad (15)
\]

In order to increase the depth of focus, the above three phase functions with equal amplitude need to be multiplexed into one diffractive lens. According to Eq. (2), the multiplexed phase function of the MPDOE is

\[
\exp(i\alpha) = \frac{1}{M} \left\{ \frac{\sqrt{3}}{3} \exp(i\phi_1) + \frac{\sqrt{3}}{3} \exp(i\phi_2) + \frac{\sqrt{3}}{3} \exp(i\phi_3) \right\} \quad (16)
\]

And the discontinuous phase function of the MPDOE with modular $2\pi$ is sketched in Fig. 2. Through phase unwrapping, the continuous phase function is given in Fig. 3. The minimum zone spacing of the MPDOE is about 3.7mm, so this MPDOE can be fabricated easily by diamond turning technique.
Fig. 4. The modulus transfer function of optical system using multiplexed phase DOE (a) with −0.15 mm defocusing length (b) with 0 mm defocusing length (c) with 0.15 mm defocusing length.

Fig. 5. The point spread function of optical system using multiplexed phase DOE (a) with −0.15 mm defocusing length (b) with 0 mm defocusing length (c) with 0.15 mm defocusing length.
Fig. 6. The modulus transfer function of optical system without using multiplexed phase DOE
(a) with −0.15mm defocusing length (b) with 0mm defocusing length (c) with 0.15mm defocusing length

Fig. 7. PSF of optical system without MPDOE (a) with −0.15mm defocusing length (b) with 0mm defocusing length (c) with 0.15mm defocusing length
For the second configuration, the diffractive lens surface is replaced by the MPDOE with multiplexed phase functions into the optical system in optical design software ZEMAX. The polychromatic MTF curves and PSF curves of the optical system with the MPDOE are shown in Fig. 4 and Fig. 5 at different defocusing location, respectively. It can be observed that MTF and PSF are almost unchanged at the three positions along the axis and it can be regarded that the depth of focus of this system is extended to 0.18mm. For the purpose of comparisons, the polychromatic MTF curves and PSF curves of the optical system with the traditional diffractive lens at the second configuration are also given in Fig. 6 and Fig. 7 respectively. It is clear that good imaging performance is only achieved at the nominal focal plane and the imaging quality is greatly deteriorated in defocusing positions. It should be noted that the peak intensity is normalized to the peak of the unaberrated PSF in Fig. 5 and Fig. 7. The simulated results confirm that the MPDOE can efficiently extend the depth of focus of the optical system.

4. Conclusion

Based on the relationship between the multiplexed phase functions and the output phase functions of the MPDOE, a more computationally tractable method of designing multiplexed phase diffractive optical elements is proposed to increase the depth of focus of the optical systems. The simulation results of the design example with the help of the optical design software ZEMAX confirm that the MPDOE does extend the depth of focus by keeping a stable imaging quality within a wide range of the defocusing distance.

Although a host of questions concerning this new design method such as the number of weighted phase functions, the determination of the optimal weights of the multiplexed phase functions and maximizing the diffractive efficiency, our design experience will reveal a good foreground by using MPDOEs in the application of extending the depth of focus of the optical systems.

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