

# Viewing angle characteristics of a nematic liquid crystal cell with two crossed-grating surface substrates

Wenjiang Ye (叶文江)<sup>1,2</sup>, Hongyu Xing (邢红玉)<sup>1,3,4</sup>, Zhi Ren (任芝)<sup>5</sup>,  
Zhidong Zhang (张志东)<sup>1\*</sup>, Yubao Sun (孙玉宝)<sup>1</sup>, and Guoying Chen (陈国鹰)<sup>2</sup>

<sup>1</sup>School of Sciences, Hebei University of Technology, Tianjin 300401, China

<sup>2</sup>School of Information Engineering, Hebei University of Technology, Tianjin 300401, China

<sup>3</sup>Changchun Institute of Optics, Fine Mechanics and Physics, Chinese Academy of Sciences, Changchun 130033, China

<sup>4</sup>Graduate University of Chinese Academy of Sciences, Beijing 100049, China

<sup>5</sup>Department of Mathematics and Physics, North China Electric Power University, Baoding 071000, China

\*E-mail: zhidong\_zhang@eyou.com

Received June 10, 2010

Liquid crystal in a nematic liquid crystal cell surface with two crossed-grating surface substrates can be oriented along the normal grating direction with given geometric parameters of groove and anchoring strength. This display is equivalent to multi-domain vertical-alignment mode. It has a relatively wide viewing angle. In this letter, we investigate the viewing angle characteristics of this kind of cell. The viewing angle dependence of contrast ratio is obtained using the extended Jones matrix method, which also considers the flexoelectric effect. The viewing angle is dependent on the geometric parameter of grating surface, the flexoelectric coefficients, and the anchoring strength. Therefore, appropriate value for each factor needs to be selected to obtain a better viewing angle of this cell.

OCIS codes: 230.3720, 230.2090.

doi: 10.3788/COL20100812.1171.

Liquid crystal displays (LCDs) are used as interfaces between human beings and machines; thus, display characteristics, such as viewing angle, response speed, contrast ratio, etc, have always been focused. Further improvements in LCD characteristics have been achieved because of the development of fundamental research on liquid crystal and the technologies of manufacturing liquid crystal devices. Notable improvement has been observed in viewing angle characteristics. Liquid crystal devices have been applied into both spatial light modulator<sup>[1]</sup> and lens array<sup>[2,3]</sup>. For most occasions, liquid crystal devices have wide viewing angles. Therefore, several wide viewing angle techniques have been proposed by using various display modes<sup>[4–11]</sup>. The multi-domain vertical-alignment (MVA) mode is one of the most important technologies to improve molecular orientation.

The MVA mode gives each pixel two or more domains with different orientation states<sup>[7]</sup>. It uses two polymer gratings arranged orthogonal to each other to make four different domains with an applied voltage. The anchor of grating surface substrate to liquid crystal molecules is crucial. The equivalent anchoring energy formula that describes the anchor is expressed as<sup>[12]</sup>

$$f_s = \frac{1}{2}W_1(\mathbf{n} \cdot \mathbf{j})^2 + \frac{1}{2}W_2(\mathbf{n} \cdot \mathbf{i})^2, \quad (1)$$

where  $\mathbf{n}$  is the director of liquid crystal,  $\mathbf{i}$  and  $\mathbf{j}$  are the basic vectors along the  $x$ - and  $y$ -axes, respectively;  $W_1$  and  $W_2$  are the equivalent anchoring strengths that depend on the original anchoring strength and geometrical parameter of grating surface. In the following analyses, we use Eq. (1) to obtain the anchoring energy of grating surface substrate.

Our selected liquid crystal cell structure and the Cartesian coordinate system are shown in Fig. 1. A nematic

liquid crystal (NLC) is confined between two grating surface substrates whose projected planes are located in  $z=0$  and  $z=l$ , respectively, where  $z$  axis is normal to the projected plane. The groove directions of two grating surface substrates are orthogonal to each other. The director  $\mathbf{n}$  of liquid crystal can be described by the tilt angle  $\theta$  and the twist angle  $\phi$ . These two angles are only dependent on the coordinate  $z$ . An external voltage  $U$  is applied to the NLC cell, and the electric potential at the lower substrate is zero.

The total free energy of this NLC cell is expressed as

$$F = S \left[ \int_0^l (f_{\text{elas}} + f_{\text{diel}} + f_{\text{flexo}}) dz + f_{s1} + f_{s2} \right], \quad (2)$$

where  $S$  is the area of projected plane of grating surface substrate;  $f_{\text{elas}}$ ,  $f_{\text{diel}}$ , and  $f_{\text{flexo}}$  denote the elastic, dielectric, and flexoelectric free energy densities, respectively;  $f_{s1}$  and  $f_{s2}$  are the anchoring energy densities corresponding to the lower substrate and upper substrate, respectively. They are given by the following general formulas:

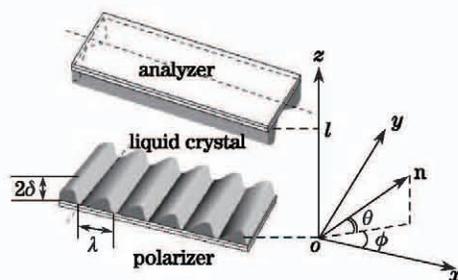


Fig. 1. Structure of liquid crystal cell and coordinate system.

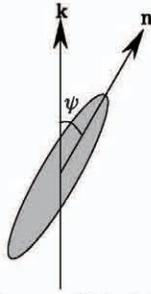


Fig. 2. Angular dependence of birefringence of liquid crystal.

$$f_{\text{elas}} = \frac{1}{2}k_{11}(\nabla \cdot \mathbf{n})^2 + \frac{1}{2}k_{22}(\mathbf{n} \cdot \nabla \times \mathbf{n})^2 + \frac{1}{2}k_{33}(\mathbf{n} \times \nabla \times \mathbf{n})^2, \quad (3)$$

$$f_{\text{diel}} = -\frac{1}{2}\mathbf{D} \cdot \mathbf{E}, \quad (4)$$

$$f_{\text{flexo}} = -\mathbf{P}_f \cdot \mathbf{E}, \quad (5)$$

$$f_{s1} = \frac{1}{2}W_{10}(\mathbf{n} \cdot \mathbf{j})^2 + \frac{1}{2}W_{20}(\mathbf{n} \cdot \mathbf{i})^2, \quad (6)$$

$$f_{s2} = \frac{1}{2}W_{1l}(\mathbf{n} \cdot \mathbf{i})^2 + \frac{1}{2}W_{2l}(\mathbf{n} \cdot \mathbf{j})^2, \quad (7)$$

where  $k_{ii}$  ( $i=1,2,3$ ) denotes the elastic constant of liquid crystal;  $\mathbf{D}$  is the dielectric displacement vector;  $\mathbf{P}_f$  is the flexoelectric polarization vector<sup>[13]</sup>;  $W_{10}$  and  $W_{20}$  are the equivalent anchoring strength for lower substrate; and  $W_{1l}$  and  $W_{2l}$  are the equivalent anchoring strength for upper substrate.

Equilibrium equations containing the bulk equation and the boundary condition can be easily derived using the variation method<sup>[14,15]</sup>:

$$\frac{1}{2} \frac{df_{\text{eff}}(\theta)}{d\theta} \theta'^2 - \frac{d}{dz} [f_{\text{eff}}(\theta) \theta'] + \frac{1}{2} \frac{dh(\theta)}{d\theta} \phi'^2 + \frac{1}{2} D_z^2 \frac{d}{d\theta} \left( \frac{1}{g(\theta)} \right) = 0, \quad (8)$$

$$\frac{d}{dz} [h(\theta) \phi'] = 0, \quad (9)$$

$$f_{\text{eff}}(\theta_0) \theta'_0 = \frac{e_{11} + e_{33}}{2} D_z \frac{\sin 2\theta_0}{g(\theta_0)} - \frac{1}{2} \sin 2\theta_0 (W_{10} \sin^2 \phi_0 + W_{20} \cos^2 \phi_0), \quad (10)$$

$$h(\theta_0) \phi'_0 = -\frac{1}{2} (W_{20} - W_{10}) \cos^2 \theta_0 \sin 2\phi_0, \quad (11)$$

$$f_{\text{eff}}(\theta_l) \theta'_l = \frac{e_{11} + e_{33}}{2} D_z \frac{\sin 2\theta_l}{g(\theta_l)} + \frac{1}{2} \sin 2\theta_l (W_{1l} \cos^2 \phi_l + W_{2l} \sin^2 \phi_l), \quad (12)$$

$$h(\theta_l) \phi'_l = -\frac{1}{2} (W_{2l} - W_{1l}) \cos^2 \theta_l \sin 2\phi_l, \quad (13)$$

where  $f_{\text{eff}}(\theta) = f(\theta) + \frac{(e_{11} + e_{33})^2 \sin^2(2\theta)}{4g(\theta)}$ ;  $g(\theta) = \varepsilon_{\perp} + \Delta\varepsilon \sin^2 \theta$ ;  $e_{11}$  and  $e_{33}$  are the splay and the bend flexoelectric coefficients, respectively;  $\varepsilon_{\perp}$  is the permittivity across the axis of liquid crystal;  $\Delta\varepsilon$  is the anisotropy of

the permittivity;  $f(\theta) = k_{11} \cos^2 \theta + k_{33} \sin^2 \theta$ ;  $h(\theta) = \cos^2 \theta (k_{22} \cos^2 \theta + k_{33} \sin^2 \theta)$ ;  $\theta' = d\theta/dz$ ;  $\phi' = d\phi/dz$ ;  $\theta'_0 = \frac{d\theta}{dz}|_{z=0}$ ;  $\theta'_l = \frac{d\theta}{dz}|_{z=l}$ ;  $\phi'_0 = \frac{d\phi}{dz}|_{z=0}$ ;  $\phi'_l = \frac{d\phi}{dz}|_{z=l}$ ;  $\theta_0$  and  $\theta_l$  denote the tilt angle of lower and upper substrates, respectively;  $\phi_0$  and  $\phi_l$  denote the twist angle of lower and upper substrates, respectively; and  $D_z$  is the  $z$  component of the dielectric displacement.

Viewing angle in LCDs is created by the anisotropy of refractive index of the liquid crystal<sup>[16]</sup>. The angular dependence of birefringence of liquid crystal is shown in Fig. 2, where the direction of  $\mathbf{k}$ -vector corresponds to the propagation direction of light. When the  $\mathbf{k}$ -vector of the incident light creates an angle of  $\psi$  to the director of uniaxial liquid crystal, the effective refractive indices are

$$n_o^{\text{eff}}(\psi) = n_o, \quad (14)$$

$$n_e^{\text{eff}}(\psi) = \frac{n_o n_e}{\sqrt{n_o \sin^2 \psi + n_e \cos^2 \psi}}, \quad (15)$$

where  $n_o$  and  $n_e$  are the ordinary and extraordinary refractive indices of liquid crystal, respectively. When the incident angle changes, the effective refractive index for the ordinary ray remains the same, whereas the effective refractive index for the extraordinary ray depends on the viewing angle. Therefore, the retardation of a liquid crystal layer changes with the incident angle. As a result, the birefringence-dependent transmission changes with the viewing angle.

Generally speaking, the viewing angle of liquid crystal cell is defined as a contour of isocontrast ratio<sup>[17]</sup>. For a certain observation direction, the contrast ratio of liquid crystal cell is the ratio between the maximum and minimum intensities of transmission, which correspond to the on and off states, respectively. Once the intensity of transmission with different observation directions is known, the viewing angle can be easily obtained.

The extended Jones matrix method can be used to calculate the intensity of transmission with different incident angles. The Jones matrix of the whole liquid crystal layer is related to the distribution of director of liquid crystal. Applying the finite-difference iterative method<sup>[18,19]</sup> based on the given equation and boundary condition, the distributions of the tilt angle and the twist angle in the NLC cell can be obtained by computer simulation. In addition to the liquid crystal layer, the intensity of transmission remains controlled by the polarization direction of the polarizer and the analyzer. Therefore, assuming the polarization directions of the polarizer and the analyzer along some specific direction is necessary.

Suppose that the polarization directions of the polarizer and the analyzer are along the groove direction of two grating surface substrates (i.e.,  $\varphi_{\text{ent}}=0^\circ$  and  $\varphi_{\text{exit}}=90^\circ$ ) and a beam of light with 550-nm wavelength irradiates the liquid crystal cell along different polar angles from  $0^\circ$  to  $80^\circ$  and azimuthal angles from  $0^\circ$  to  $360^\circ$ . The ordinary and extraordinary refractive indices of liquid crystal are  $n_e=1.6$  and  $n_o=1.5$ , respectively. The entire liquid crystal layer is divided into  $N$  sub-layers. The director of liquid crystal in a sub-layer is uniform.

As indicated in Ref. [15], the geometric parameter of grating surface substrate for realizing the vertical display corresponding to different anchoring strengths

$A_1=1 \times 10^{-4}$  and  $1 \times 10^{-3} \text{ J/m}^2$  are also  $\delta/\lambda \leq 0.2439$ . As a result, we consider two kinds of grating surface substrates in calculation: a)  $\delta/\lambda=0.05$ ; b)  $\delta/\lambda=0.2$ . The pitch of these grating surface substrates is  $\lambda=1 \mu\text{m}$ , and the ratio of anchoring strengths  $A_2$  to  $A_1$  is 0.7. The material parameters of liquid crystal are  $k_{11}=16.7 \text{ pN}$ ,  $k_{22}=8.0 \text{ pN}$ ,  $k_{33}=15.6 \text{ pN}$ , and  $\Delta\varepsilon=-4.2$ <sup>[20]</sup>. The thickness of liquid crystal cell is  $l=4.7 \mu\text{m}$ . The splay and the bend flexoelectric coefficients of liquid crystal can be supposed as  $e_{11} = \pm 2.0 \times 10^{-11} \text{ C/m}$  and  $e_{33} = \pm 2.1 \times 10^{-11} \text{ C/m}$ , respectively. Viewing angle characteristics of liquid crystal cell with two grating surface substrates for

different cases are shown in Figs. 3–6.

When the contrast ratio is over 10, the viewing angle of liquid crystal cell with two grating surface substrates for each case is basically uniform. The polar angle nearly reaches  $30^\circ$  for all azimuthal angles from  $0^\circ$  to  $360^\circ$ . The contrast ratio is relatively greater near the horizontal and the vertical directions. However, small changes remain obvious from the above figures. Viewing angle of this cell varies with the different geometrical parameters of grating surface substrate and anchoring strength, and the flexoelectric effect has an influence on some cases. Comparing with these figures, the influence of flexoelectric

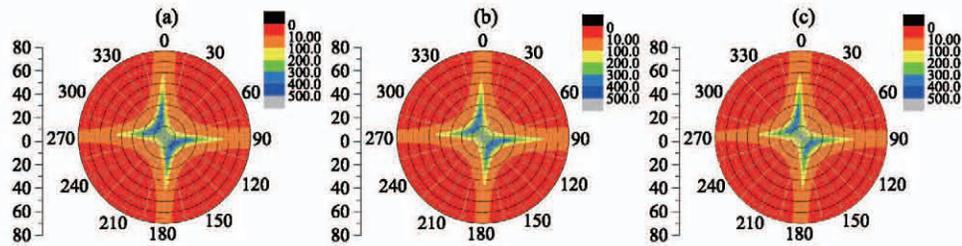


Fig. 3. Viewing angle characteristics of liquid cell with two grating surface substrates with  $\delta/\lambda=0.05$  and  $A_1 = 1 \times 10^{-3} \text{ J/m}^2$  for ignoring and considering the flexoelectric effect. (a) Flexoelectric effect is neglected; (b)  $e_{11} = 2.0 \times 10^{-11} \text{ C/m}$  and  $e_{33}=2.1 \times 10^{-11} \text{ C/m}$ ; (c)  $e_{11} = -2.0 \times 10^{-11} \text{ C/m}$  and  $e_{33}=-2.1 \times 10^{-11} \text{ C/m}$ .

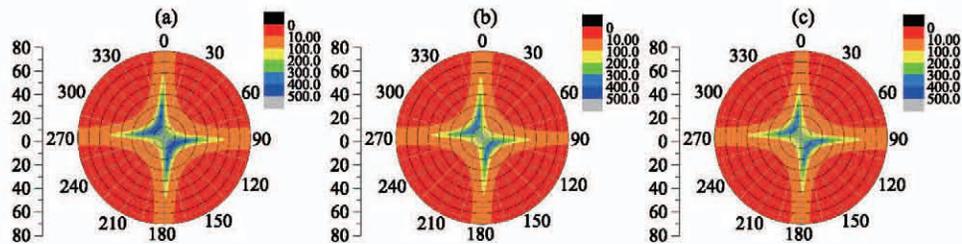


Fig. 4. Viewing angle characteristics of liquid cell with two grating surface substrates with  $\delta/\lambda=0.05$  and  $A_1 = 1 \times 10^{-4} \text{ J/m}^2$  for ignoring and considering the flexoelectric effect. (a) Flexoelectric effect is neglected; (b)  $e_{11} = 2.0 \times 10^{-11} \text{ C/m}$  and  $e_{33}=2.1 \times 10^{-11} \text{ C/m}$ ; (c)  $e_{11} = -2.0 \times 10^{-11} \text{ C/m}$  and  $e_{33}=-2.1 \times 10^{-11} \text{ C/m}$ .

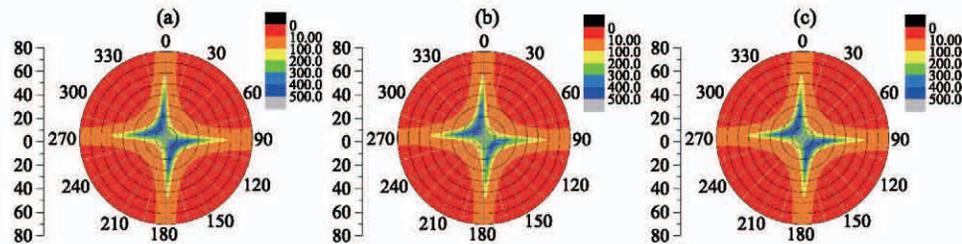


Fig. 5. Viewing angle characteristics of liquid cell with two grating surface substrates with  $\delta/\lambda=0.2$  and  $A_1 = 1 \times 10^{-3} \text{ J/m}^2$  for ignoring and considering the flexoelectric effect. (a) Flexoelectric effect is neglected; (b)  $e_{11} = 2.0 \times 10^{-11} \text{ C/m}$  and  $e_{33} = 2.1 \times 10^{-11} \text{ C/m}$ ; (c)  $e_{11} = -2.0 \times 10^{-11} \text{ C/m}$  and  $e_{33} = -2.1 \times 10^{-11} \text{ C/m}$ .

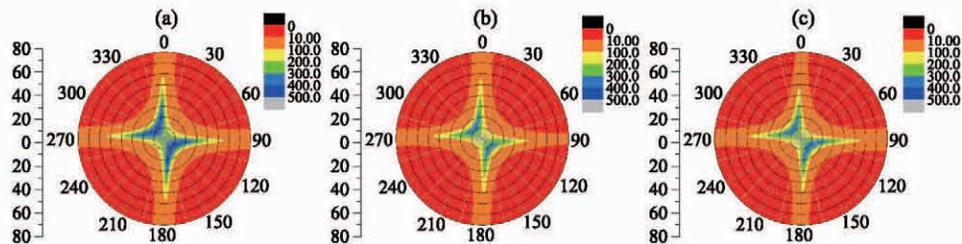


Fig. 6. Viewing angle characteristics of liquid cell with two grating surface substrates with  $\delta/\lambda=0.2$  and  $A_1 = 1 \times 10^{-4} \text{ J/m}^2$  for ignoring and considering the flexoelectric effect. (a) Flexoelectric effect is neglected; (b)  $e_{11} = 2.0 \times 10^{-11} \text{ C/m}$  and  $e_{33} = 2.1 \times 10^{-11} \text{ C/m}$ ; (c)  $e_{11} = -2.0 \times 10^{-11} \text{ C/m}$  and  $e_{33} = -2.1 \times 10^{-11} \text{ C/m}$ .

effect on viewing angle decreases to a minimum for the bigger value of  $\delta/\lambda$  with the increase of anchoring strength  $A_1$ , in which they are almost consistent with the case without considering flexoelectric effect. This means that, for this kind of liquid crystal cell with two grating surface substrates with relatively strong anchoring strength, the flexoelectric effect may be neglected in the deep groove. On the contrary, it must be considered in relatively weak anchoring and shallow groove cases. Flexoelectric effect may be induced by the influence of flexoelectric effect on the effective anchoring energy of grating surface substrate.

In conclusion, comparing with the previous work<sup>[6-9]</sup> on vertically aligned NLC displays, we theoretically analyze the viewing angle of the NLC cell. Analysis is conducted with two crossed-grating surface substrates based on the equivalent anchoring energy formula of grating surface substrate and the simulation results on the viewing angle of the cell. Analysis result realizes the vertical display of a number of given geometric parameters of groove and anchoring strength obtained using the extended Jones matrix method with flexoelectric effect considered. The viewing angle of this cell is smaller than those in display; however, it has a relatively wider viewing angle. If the compensation films are designed in this device, the viewing angle will become much wider. As a result, to obtain a better viewing angle of this cell, appropriate value should be selected for these factors.

This research was supported by the Natural Science Foundation of Hebei Province (No. A2010000004), the National Natural Science Foundation of China (Nos. 10704022 and 60736042), and the Key Subject Construction Project of Hebei Province University.

#### References

1. T. Shi, Z. Huang, and C. Zhang, Chinese J. Lasers (in Chinese) **36**, 1999 (2009).
2. H. Li, K. Liu, X. Zhang, T. Zhang, and X. Shen, Acta Opt. Sin. (in Chinese) **8**, 218 (2010).
3. H. Li, K. Liu, X. Zhang, T. Zhang, and X. B. Shen, Chin. Opt. Lett. **8**, 329 (2010).
4. H. Seki, Y. Masuda, and T. Uchida, Liq. Cryst. **14**, 279 (1993).
5. S. H. Hong, H. Y. Kim, M.-H. Lee, and S. H. Lee, Liq. Cryst. **29**, 315 (2002).
6. S. H. Lee, H. Y. Kim, Y. H. Lee, I. C. Park, B. G. Rho, H. G. Galabova, and D. W. Allender, Appl. Phys. Lett. **73**, 470 (1998).
7. J.-H. Park, Y. Choi, T.-Y. Yoon, and S.-D. Lee, J. SID **11**, 283 (2003).
8. S. H. Lee, S. H. Park, M.-H. Lee, S. T. Oh, and G.-D. Lee, Appl. Phys. Lett. **86**, 031108 (2005).
9. H. Lin, Appl. Phys. Lett. **90**, 151112 (2007).
10. Z. B. Ge, R. Lu, T. X. Wu, S.-T. Wu, C.-L. Lin, N.-C. Hsu, W.-Y. Li, and C.-K. Wei, Opt. Express **16**, 3120 (2008).
11. Z. B. Ge, S.-T. Wu, and S. H. Lee, Opt. Lett. **33**, 2623 (2008).
12. W. J. Ye, H. Y. Xing, and G. C. Yang, Chin. Phys. **16**, 493 (2007).
13. R. B. Meyer, Phys. Rev. Lett. **22**, 918 (1969).
14. W. J. Ye, H. Y. Xing, and G. C. Yang, Chinese J. Comput. Phys. **25**, 591 (2008).
15. W. J. Ye, H. Y. Xing, G. C. Yang, and M. Y. Yuan, Chin. Phys. B **18**, 238 (2009).
16. W. M. Liu, "Characterization of some wide viewing angle liquid crystal devices" PhD. Thesis (Kent State University, 2000).
17. D.-K. Yang and S.-T. Wu, *Fundamentals of Liquid Crystal Devices* (John Wiley and Sons Ltd., Chichester, 2006).
18. Q. Wang and S. L. He, Acta Phys. Sin. (in Chinese) **50**, 926 (2001).
19. Z. D. Zhang, W. J. Ye, and H. Y. Xing, Chinese J. Comput. Phys. (in Chinese) **21**, 156 (2004).
20. G. P. Bryan-Brown, C. V. Brown, I. C. Sage, and V. C. Hui, Nature **392**, 365 (1998).