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Evaluation of the imaging performance of hybrid refractive–diffractive systems using the modified phase function model

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Abstract
The imaging performance of a hybrid refractive–diffractive system is limited by the diffractive efficiency of the diffractive optical element (DOE). This phenomenon is explained in this paper by the wavefront deformation from the ideal wavefront at the exit pupil. The modified phase function model is developed to correct the wavefront deformation, and the application of the model is extended to the harmonic diffractive optical element (HOE) and the multi-layer diffractive optical element (MOE) as well. The general phase delay factor is derived for DOE, HOE and MOE. The validity of our model is verified by comparison with the weighted summation model of orders using a simple hybrid optical system example. Finally, the performance of the hybrid system with a HOE for dual waveband is evaluated.

Keywords: diffractive optical elements, phase function, hybrid refractive–diffractive system

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Diffractive optical elements have been widely used in optical systems for the visible and IR wavebands to correct the chromatic aberration, reduce the secondary spectrum and produce an arbitrary phase distribution [1–3]. A diffractive lens with a continuous surface relief can theoretically achieve 100% diffractive efficiency at nominal wavelength. However, the energy scattered into additional parasitic diffraction orders will limit the imaging performance of hybrid systems when a different wavelength is chosen. The diffractive efficiency of the prime order (the first order generally) will decrease due to the appearance of the other orders. The imaging performance of hybrid systems can be evaluated by summating all the orders with their diffractive efficiencies as the weight [4–6]. However, the calculation of diffractive efficiency and the summation is complex and impractical for all orders in this method. In this paper, the effect of diffractive efficiency is explained theoretically in terms of the wavefront deformation at the exit pupil, and a more practical and tractable model [7] is introduced to evaluate the imaging performance of hybrid refractive–diffractive systems. With this model the actual wavefront can be directly computed by the optical design software package to facilitate the evaluation of the performance of hybrid systems.

The theoretical coherence between the modified phase function model and the weighted summation model is described in section 2. The general expression of the phase delay factor for DOE, HOE and MOE is derived and a unified model is established in section 3. An illustrative example is given in section 4 to validate our approach by comparing our results with that of the weighted summation model of orders. Our conclusions and remarks are put forth in section 5.

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2. Fundamentals of the modified phase function model

Most optical design software codes model the DOE as an ideal thin phase screen over the substrate surface, and trace the ray with the grating equation. The phase function of the DOE is given by the following polynomial in common software packages:

$$\phi_0(r) = \frac{2\pi}{\lambda_0}\sum_{i=1}^{N} a_i r^{2i} = 2\pi(Ar^2 + Br^4 + \cdots), \tag{1}$$

where i is the number of the term in the polynomial, $\phi_0(r)$ is the phase at radius r for the nominal wavelength of λ_0, and the maximum value of N used in the software package is usually less than five. In equation (1), $A = a_1/\lambda_0$, $B = b_1/\lambda_0, \ldots$, and if the variable r is replaced by $\xi(r) = Ar^2 + Br^4 \cdots$, the phase function can be written as

$$\phi_0(\xi) = 2\pi\xi. \tag{2}$$

The phase function can be modulo 2π to make the diffractive lens sufficiently thin:

$$|\phi_0(\xi)|_{2\pi} = 2\pi(\xi + j), \qquad j = 0, \pm1, \pm2, \ldots, \tag{3}$$

and r_j, the radius of each zone can be obtained by setting $\xi + j = 0$. When a different wavelength λ is chosen, a phase delay, $\alpha = \lambda_0(n-1)/\lambda(n_0-1)$, will be induced, where n_0 and n are the refractive index of the optical material used for the diffractive lens at λ_0 and λ, and the actual phase function for λ becomes

$$\phi(\xi) = \alpha|\phi_0(\xi)|_{2\pi} = 2\pi\alpha(\xi + j). \tag{4}$$

It is known that the pupil function of the system can be written as

$$P(x, y) = E(x, y)\exp[\mathrm{i}\phi(x, y)], \tag{5}$$

where $E(x, y)$ represents the amplitude distribution at the exit pupil, and x and y are the coordinates at pupil. $\phi(x, y)$ represents the phase function corresponding with $\phi(\xi)$, and $\xi(r)$ can be rewritten as $\xi(x, y)$ because of $r^2 = x^2 + y^2$. If the entrance pupil is illuminated with a constant amplitude light beam we will have $E(x, y) = 1$, and the actual pupil function at λ can be obtained from equations (4) and (5):

$$P(\xi) = \exp[\mathrm{i}\phi(\xi)] = \exp[\mathrm{i}2\pi\alpha(\xi + j)]. \tag{6}$$

This is a periodic function where the period is 1, so it can be expanded as a Fourier series

$$P(\xi) = \sum_{m=-\infty}^{+\infty} C_m \exp(\mathrm{i}2\pi m\xi)$$
$$C_m = \mathrm{sinc}(\alpha - m)\exp[\mathrm{i}\pi(\alpha - m)], \tag{7}$$

where m is integer; the pupil function can be decomposed to an infinite number of functions, each of them representing one diffractive order, m. The amplitude of each order is C_m and the diffractive efficiency of this order is [8]

$$\eta_m = C_m C_m^* = \mathrm{sinc}^2(\alpha - m), \tag{8}$$

where $\mathrm{sinc}(x) = \sin(\pi x)/(\pi x)$. The imaging performance of the hybrid system can be considered to be affected by all the diffractive orders, so the modulation transfer function (MTF) of the system for λ can be obtained by the summation of each order's MTF with their diffractive efficiency as the weight [4–6]:

$$\mathrm{MTF}(\lambda) = \sum_{m=-\infty}^{+\infty}(\eta_m \mathrm{MTF}_m). \tag{9}$$

The MTF for each order can be calculated through the autocorrelation of its pupil function, so equation (9) can be rewritten as

$$\mathrm{MTF}(\lambda) = \sum_{m=-\infty}^{\infty}\left[\eta_m\left|\frac{\iint_{-\infty}^{\infty} P_m(x, y)P_m^*(x + \lambda R f_x, y + \lambda R f_y)\,\mathrm{d}x\,\mathrm{d}y}{\iint_{-\infty}^{\infty}|P_m(x, y)|^2\,\mathrm{d}x\,\mathrm{d}y}\right|\right], \tag{10}$$

where P_m represents the pupil function for each order and is given by equation (7) and the diffractive efficiency is given by equation (8). Hence, the MTF can be written as

$$\begin{aligned}\mathrm{MTF}(\lambda) &= \sum_{m=-\infty}^{\infty}\left[\eta_m\left|\frac{\iint_{-\infty}^{\infty}|C_m|^2\exp(\mathrm{i}2\pi m\xi)\exp(-\mathrm{i}2\pi m\xi')\,\mathrm{d}x\,\mathrm{d}y}{\iint_{-\infty}^{\infty}|C_m|^2\,\mathrm{d}x\,\mathrm{d}y}\right|\right]\\ &= \left|\int\int_{-\infty}^{\infty}\sum_{m=-\infty}^{\infty} C_m\exp(\mathrm{i}2\pi m\xi)\right.\\ &\quad\left.\times\sum_{m=-\infty}^{\infty} C_m^*\exp(-\mathrm{i}2\pi m\xi')\,\mathrm{d}x\,\mathrm{d}y\right|,\end{aligned} \tag{11}$$

where $\xi = \xi(x, y)$, $\xi' = \xi(x + \lambda R f_x, y + \lambda R f_y)$, f_x and f_y are the spatial frequencies at the x and y directions respectively and R is the radius of the wavefront. From equation (11), the term following the symbol of summation can be transformed into a format similar to equation (4) by an inverse Fourier transformation:

$$\begin{aligned}\mathrm{MTF}(\lambda) &= \left|\int\int_{-\infty}^{\infty}\exp(\mathrm{i}2\pi\alpha\xi)\exp(-\mathrm{i}2\pi\alpha\xi')\,\mathrm{d}x\,\mathrm{d}y\right|\\ &= \left|\int\int_{-\infty}^{\infty}\exp(\mathrm{i}2\pi\xi)\exp(\mathrm{i}2\pi(\alpha-1)\xi)\right.\\ &\quad\left.\times\exp(-\mathrm{i}2\pi\xi')\exp[-\mathrm{i}2\pi(\alpha-1)\xi']\,\mathrm{d}x\,\mathrm{d}y\right|\\ &= \left|\int\int_{-\infty}^{\infty}\exp\{\mathrm{i}[\phi_0(\xi) + \Delta\phi(\xi)]\}\right.\\ &\quad\left.\times\exp\{-\mathrm{i}[\phi_0(\xi') + \Delta\phi(\xi')]\}\,\mathrm{d}x\,\mathrm{d}y\right|,\end{aligned} \tag{12}$$

where

$$\Delta\phi(\xi) = \phi(\xi) - |\phi_0(\xi)|_{2\pi} = 2\pi(\alpha - 1)(\xi + j), \tag{13}$$

and represents the additional phase difference arising from the undesired wavelength dispersion. From equation (13) the actual phase function can be modified based on the ideal phase function, and a modified phase function model can be established

$$\Delta\phi(\xi) = \phi(\xi) - |\phi_0(\xi)|_{2\pi} \qquad \phi(\xi) = \phi_0(\xi) + \Delta\phi(\xi). \tag{14}$$

Therefore, the effect of the diffraction orders and efficiency can be represented by the additional phase difference from the ideal phase function. It is known that the real wavefront (OPD) deviates from the ideal one due to the phase change according to the relationship between the OPD and phase function

$$\mathrm{OPD}(x, y) = \frac{\lambda}{2\pi}\phi(x, y). \tag{15}$$

Thus the real performance of system can be evaluated accurately by correcting the actual wavefront using the new model. The calculation of the diffractive efficiency for all the diffractive orders is not required in our method, and the modification to the phase function can be processed based upon the ideal phase function offered by the common optical design software packages, making this model more compatible with commercial optical design software packages and more computationally tractable than the weighted summation model of orders.

3. Unified model for three types of diffractive element

The formulae for the diffractive efficiency of DOE, HOE and MOE shown in figure 1 have similar form to equation (9), except for the phase delay factor, α. In this section, the phase delay factor of the double-layer diffractive element will be derived to achieve the general form of the phase delay factor for the three types of diffractive element. The ideal phase function for the double-layer diffractive element can be written as [9]

$$\phi_0(r) = \frac{2\pi}{\lambda_0}[(n_{10} - 1)d_1(r) + (n_{20} - 1)d_2(r)], \tag{16}$$

where n_{10} and n_{20} represent the refractive indices of the two materials at nominal wavelength, λ_0, and $d_1(r)$ and $d_2(r)$ represent the profile function of the two DOEs. The real phase for wavelength λ is,

$$\phi(r) = \frac{2\pi}{\lambda}[(n_1 - 1)d_1(r) + (n_2 - 1)d_2(r)], \tag{17}$$

where n_1 and n_2 are the refractive indices of two materials at nonnominal wavelength λ. From equations (16) and (17) we have

$$\phi(r) = \frac{\lambda_0}{\lambda}\frac{(n_1 - 1)d_1(r) + (n_2 - 1)d_2(r)}{(n_{10} - 1)d_1(r) + (n_{20} - 1)d_2(r)}\phi_0(r), \tag{18}$$

so the phase delay factor can be written as,

$$\alpha_{\mathrm{MOE}} = \frac{\lambda_0}{\lambda}\frac{(n_1 - 1)d_1(r) + (n_2 - 1)d_2(r)}{(n_{10} - 1)d_1(r) + (n_{20} - 1)d_2(r)}. \tag{19}$$

For a harmonic diffractive lens which has a multi-layer configuration, the nominal wavelength λ_0 should be replaced by $p\lambda_0$, where p is the resonant order, and the phase delay factor becomes

$$\alpha = \frac{p\lambda_0}{\lambda}\frac{(n_1 - 1)d_1(r) + (n_2 - 1)d_2(r)}{(n_{10} - 1)d_1(r) + (n_{20} - 1)d_2(r)}. \tag{20}$$

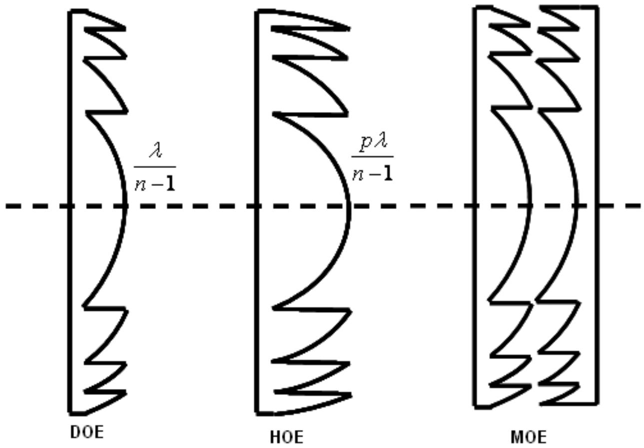

Figure 1. Three types of diffractive elements.

Equation (20) represents the phase delay of the double-layer diffractive element in general form, and if three or more layers are needed, the phase delay for a multi-layer diffractive element can be obtained by adding more $d(r)$ terms into equation (20). If $d_1(r) = 0$, or $d_2(r) = 0$, or $n_1 = n_2$, the double-layer diffractive element is simplified to a HOE and the phase delay becomes,

$$\alpha_{\mathrm{HOE}} = \frac{p\lambda_0}{\lambda}\frac{(n - 1)}{(n_0 - 1)}, \tag{21}$$

and if $p = 1$ simultaneously, the expression represents the phase delay of a DOE

$$\alpha_{\mathrm{DOE}} = \frac{\lambda_0}{\lambda}\frac{n - 1}{n_0 - 1}. \tag{22}$$

So the phase delay of three types of diffractive elements can be obtained from equation (20). Consequently, the unified form of the model for three different types of diffractive elements can be derived from equations (13) and (20) as

$$\Delta\phi(\xi) = \phi(\xi) - |\phi_0(\xi)|_{2\pi} = 2\pi\left(\frac{p\lambda_0}{\lambda}\frac{(n_1 - 1)d_1(r) + (n_2 - 1)d_2(r)}{(n_{10} - 1)d_1(r) + (n_{20} - 1)d_2(r)} - 1\right)(\xi + j). \tag{23}$$

According to equation (23), the modified phase function model can be used to evaluate the imaging performance of the hybrid refractive–diffractive system with any type of diffractive elements, including DOE, HOE and MOE, and this will be shown by the following example.

4. Example and discussions

The following example is intended to validate our approach although it may not be a really practical optical system. The specifications of the system are: effective focal length 50 mm, $F/2.5$ and nominal wavelength 2.0 μm. Only the normal incidence case is considered, so as to ignore the effect of the angle of incidence. A 17-zone diffractive lens on the back surface of a plane-parallel plate is employed to correct the

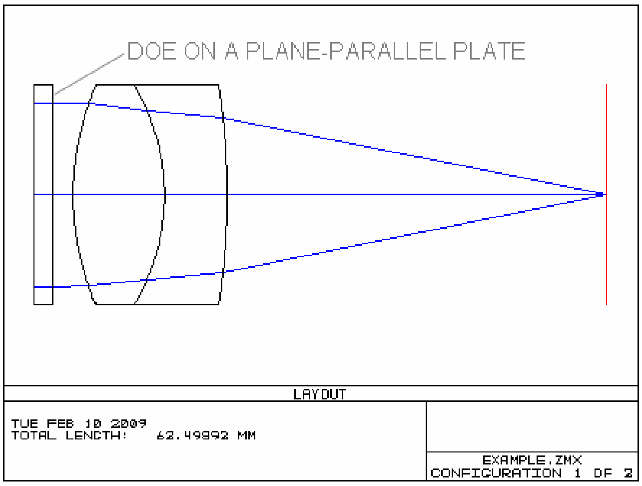

Figure 2. Layout of the hybrid system.

combined aberrations and the material of the diffractive lens is chosen as BK7. The system layout is shown in figure 2. Here the optical system design program ZEMAX[4] is used to model and analyze this system.

The models of the DOE, HOE and MOE are established with this system. According to equation (23), the additional phase departure $\Delta\phi$ is computed by three models, and the actual phase is obtained by adding the departure to the ideal phase offered by ZEMAX. In the DOE model the actual phase of two nonnominal wavelengths of 1.8 and 2.2 μm are computed, and only the former is computed in the MOE model. A double waveband system is designed to model the HOE, and the additional phase departure for 2.0 μm is computed from the ideal phase of 0.8 μm with their orders as 2 and 5. Figure 3 shows the curves of the actual phase with two zones at the center part of the lens, and figure 4 shows the curves of the optical path difference in the radial direction as a function of the pupil coordinate. From figure 3, the additional phase departure of the HOE and MOE are much smaller than that of the DOE. This is because the HOE and MOE can reduce their phase delay due to their own characteristics. It is known that the diffractive efficiency can be increased by the HOE at the resonant wavelength [10] $\lambda = p\lambda_0/m$, and by the MOE for the wide waveband [9], where m is integer. It can be explained that the HOE and MOE can reduce the actual wavefront deformation at the nonnominal wavelength, as shown in figure 4, because of their smaller phase delay and smaller additional phase difference in comparison with the DOE.

[4] ZEMAX is a trademark of Zemax Development Corporation, Bellevue, WA 98004, USA.

The MTF curves on axis are given as a function of frequency in figures 5–7 for the three types of diffractive elements. For comparison purposes these figures show the MTF of the system calculated from the different models: the conventional calculation with only one order (with 100% efficiency at all wavelengths, as assumed by most commercial optical design software), the weighted summation model of orders (five orders taken symmetrically around the prime order), and our modified model. From these figures the validity of our method for all three types of diffractive element is clearly shown in comparison with other models.

In the model of the HOE, two different back focal lengths of 39.77 and 41.12 mm are selected for two wavebands, and there exists a $\Delta f = 1.35$ mm between two bands. This discrepancy is inevitable for the double waveband hybrid system although it can be reduced by the optimization of the optical design software. It can be seen from figure 6 that the difference between the modified MTF curve and the ideal one is very small, that is to say, the effect of the modified operation is very weak. So the chromatic aberration of the system, Δf, should not be considered as caused by the dispersion of the HOE. The focal length of the HOE can be written as

$$f_2 = \frac{p\lambda_1}{m\lambda_2} f_1, \tag{24}$$

where f_1 and f_2 are the focal length of HOE for the nominal wavelength of the two wavebands, λ_1 and λ_2. In this example, $p = 5$, $\lambda_1 = 0.8$ μm, $m = 2$, $\lambda_2 = 2.0$ μm, and $f_1 = f_2$ can be achieved, which indicates that the HOE can be confocal at two nominal wavelengths. Therefore, the actual aberration Δf can only be caused by the material dispersion of the refractive elements in the hybrid system. In order to verify this point, the system is rebuilt to be a pure refractive system by deleting the diffractive surface, and the focal length is kept unchanged

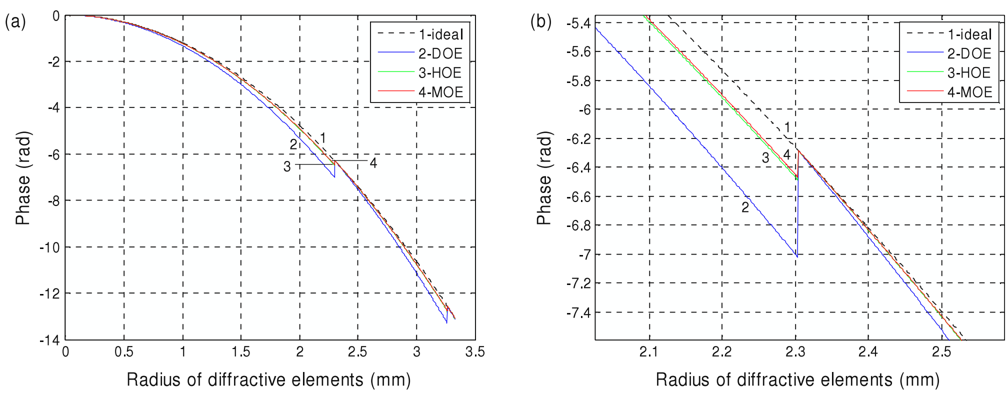

Figure 3. Phase plot versus radius of the DOE. Here only the first two zones of the 17-zone diffractive lens are shown. (b) is the magnified view of (a).

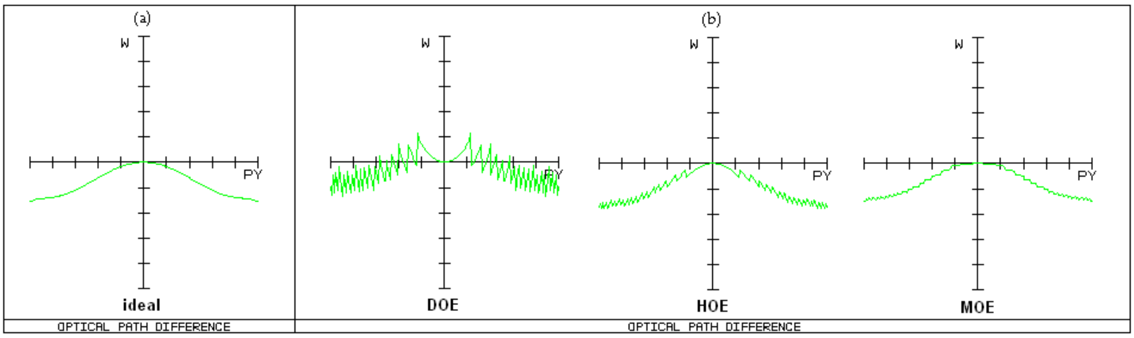

Figure 4. Optical path differences as a function of pupil coordinate for a wavelength of 1.8 μm. (a) Original plot, (b) modified plots.

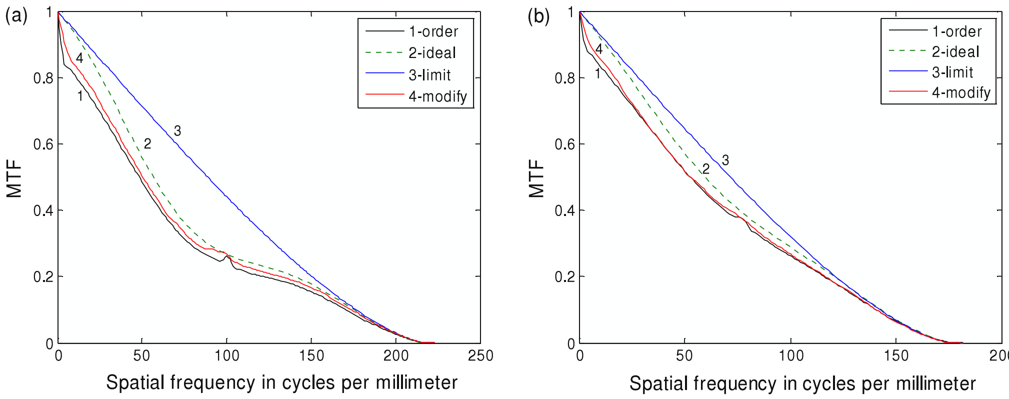

Figure 5. MTF of the hybrid system with DOE for (a) 1.8 μm and (b) 2.2 μm.

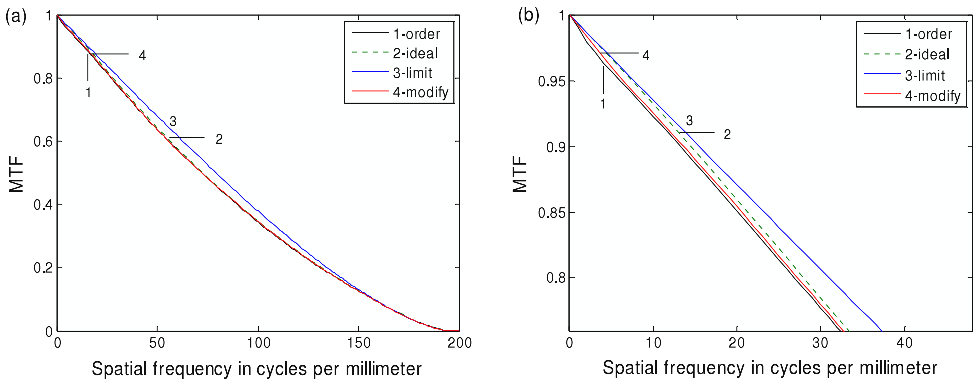

Figure 6. MTF of the hybrid system with the HOE for 2.0 μm. (b) is the magnified view of (a).

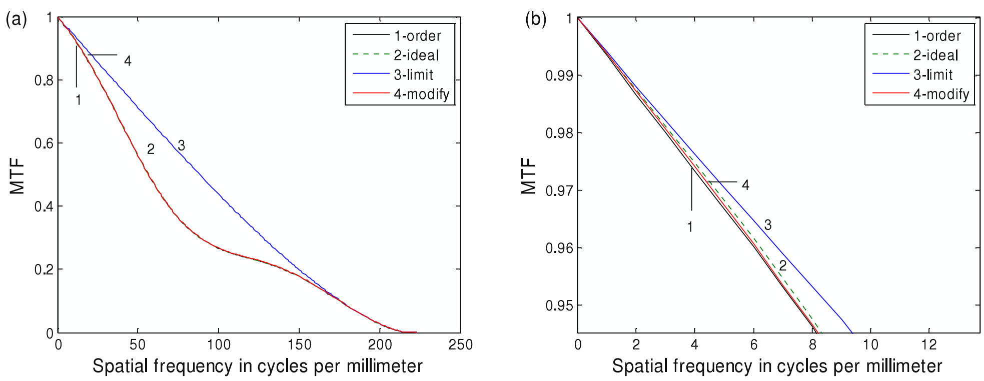

Figure 7. MTF of the hybrid system with the MOE for 1.8 μm. (b) is the magnified view of (a).

by introducing an ideal thin lens, which does not cause the aberration. The back focal length of refractive system for two wavebands are 40.57 mm and 41.94 mm respectively, and $\Delta f = 1.37$ mm. This value is close to that of original system, and it is concluded that the chromatic aberration of the hybrid system should be attributed to the material dispersion of the refractive elements in the system.

5. Conclusion

A modified phase model has been developed in this paper which is theoretically coherent with the weighted summation model of orders. The modified model has been verified to be applicable for three different types of diffractive element. The advantage of our model over other existing models, such as the weighted summation model, is that the calculation of the diffractive efficiency of the diffraction orders is not needed, making our model simpler and more practical. Furthermore, our method is more compatible with the common optical design software due to the operation of our approach being ultimately based upon the optical design software package, ZEMAX.

Acknowledgments

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