Effective medium theory applied to frequency selective surfaces on periodic substrates

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We apply the effective medium theory combined with the conventional periodic method of moments (MoM) to analyze frequency selective surfaces (FSSs) on periodic and anisotropic substrates. Based on the effective medium theory, even periodic and anisotropic substrates can be considered homogeneous; thus, the Green's function can be obtained. The resulting integral equation can then be solved by the MoM using rooftop basis functions and Galerkin testing functions. We analyze an example using this technique, and the numerical results agree with Fallahi's full-vector semi-analytical method, showing an increasing difference between the results as the frequencies increase. These results show that the proposed method is effective for analyzing FSSs on periodic and anisotropic substrates.

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Frequency selective surfaces (FSSs) are a subclass of metamaterials which exhibit frequency filtering properties similar to those of frequency filters in traditional radio frequency (RF) circuits. FSSs have received immense attention due to their abilities to control the propagation of electromagnetic waves[1−4]. FSS reflectors are used to separate feeds of different bands in reflector antenna systems because of their frequency selective properties. They are also used as antenna radomes for better control of transmitted and reflected electromagnetic waves.

Many authors have demonstrated that a substrate can enhance FSS angular sensitivity. Since then, numerous studies have been made on the application of FSS on substrates. However, only few studies have been conducted on the application of FSS on anisotropic substrates, such as liquid and chiral substrates. Reports about FSS on periodic inhomogeneous substrates are much fewer, but FSS on these periodic dielectric substrates, or so-called electromagnetic crystals, can improve some characteristics of the structure. Fallahi *et al.* developed a full-vector semi-analytical method for the analysis of FSS consisting of patches printed on periodic and anisotropic substrates[5,6] because the conventional periodic method of moments (MoM) that simulates FSS with homogeneous substrates is not applicable for substrates containing periodic inhomogeneities. However, Fallahi's method is too complicated and costly. We use the effective medium theory (EMT)[7−9] to calculate the effective relative permittivity of the periodic substrates first, then use the conventional MoM to simulate the reflected and transmitted fields from the substrates. The whole procedure is very simple and clear, and our calculated results are consistent with Fallahi's. Ours is a very effective method, with the basic idea of combining periodic MoM with EMT to analyze the FSS on periodic substrates[10,11].

Figure 1 shows a FSS model with the same structure as the one mentioned in Ref. [5]. This is the unit cell of the patch layer.

Substrate thickness is 1.27 mm. The radius of the circle perforated on the substrate is 1.5 mm. Relative permittivity of the substrate is 11.7. Width and length of the cross patch printed on the substrate are 1.25 and 7.5 mm, respectively, and $D_x = D_y = 10$ mm. The simulated model assumes an infinite periodic array, whereas for practical implementation, the prototype array has an overall dimension of 190×190 (mm).

The frequency response of the FSS structures is analyzed based on the spectral domain method. The solution is obtained by modifying the integral equation corresponding to a single patch element to include contributions from an array of patches.

The scattered field from a conducting patch on the x–y plane due to an incident plane wave can be calculated from the induced current on the radiating patch. The scattered field $\mathbf{E}_\mathrm{s}$ at point $\mathbf{r}$ due to a source at point $\mathbf{r}'$ is

$$\mathbf{E}_\mathrm{s} = -\mathrm{j}\omega\mu_0\mathbf{R} + \frac{1}{\mathrm{j}\omega\varepsilon_0}\nabla(\nabla\cdot\mathbf{R}), \tag{1}$$

where

$$\mathbf{R}(\mathbf{r}) = \int G(\mathbf{r},\mathbf{r}')J(\mathbf{r}')\mathrm{d}\mathbf{r}' = G*\mathbf{J}, \tag{2}$$

$$G(\mathbf{r},\mathbf{r}') = \frac{\mathrm{e}^{-\mathrm{j}k_0|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}, \tag{3}$$

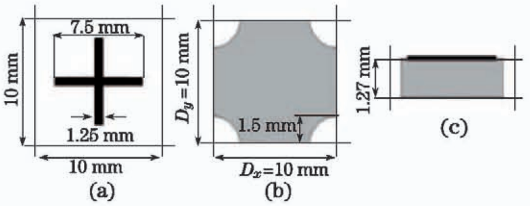

Fig. 1. FSS with periodic and isotropic substrate.

where G is the Green's function for the substrate on which the FSS is printed, $\mathbf{J}$ is the induced surface current on the conductor.

The electric field integral equation of spatial domain expression for the patches elements array is obtained as follows:

$$-\begin{bmatrix} E_x^i(x,y) \\ E_y^i(x,y) \end{bmatrix} = \frac{2\pi}{\mathrm{j}\omega\varepsilon_0 D_x D_y} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \begin{bmatrix} k_0^2-\alpha_m^2 & -\alpha_m\beta_n \\ -\alpha_m\beta_n & k_0^2-\beta_n^2 \end{bmatrix} \times \mathbf{G}(\alpha_m,\beta_n) \begin{bmatrix} \mathbf{J}_x(\alpha_m,\beta_n) \\ \mathbf{J}_y(\alpha_m,\beta_n) \end{bmatrix} \mathrm{e}^{\mathrm{j}\alpha_m x}\mathrm{e}^{\mathrm{j}\beta_n y}, \tag{4}$$

where

$$\begin{aligned} \alpha_m &= \frac{2m\pi}{D_x} + k_x^i, \\ \beta_n &= \frac{2n\pi}{D_y \sin\Omega} - \frac{2m\pi}{a}\cot\Omega + k_y^i. \end{aligned} \tag{5}$$

We replace the convolution and partial derivatives by $G*J \rightarrow G\mathbf{J}$, and $\partial\mathbf{R}/\partial x \rightarrow \mathrm{j}\alpha\mathbf{R}, \partial\mathbf{R}/\partial y \rightarrow \mathrm{j}\beta\mathbf{R}$ in the Fourier domain. The superscript i corresponds to the incident field. The skew angle Ω is 90°.

This process is the conventional MoM method, which is only applicable to homogeneous substrates. However, we can use the EMT to calculate the effective relative permittivity of periodic substrates so that they can be considered homogeneous. The effective permittivity is given by

$$\varepsilon_{\text{eff}} = \varepsilon_{\text{s}} f + \varepsilon_{\text{i}}(1-f), \tag{6}$$

where f is the duty cycle of medium whose permittivity is ε_{s}, 1–f is the duty cycle of medium whose permittivity is ε_{i}. We omit the high-order term because of the size of the microwave's wavelength.

The element in the rectangular frame shown in Fig. 1(a) is selected as the unit cell for the structure, which is discretized to employ the rooftop domain basis function. We can then use Eq. (4) to solve the unknown surface current distribution on the patch of the FSS screen with the MoM. Next, we can determine the transmission coefficients.

According to Eq. (6), ε_{eff} can be calculated as 10.94 with f=92.92%. We can then treat it as an isotropic substrate and use the conventional MoM based on the spectral-domain method to calculate the transmission coefficient versus frequency. Figure 2 shows the results of combining MoM with EMT under the perpendicular incidence.

Our results agree with Fallahi's results, but a few discrepancies should be noted at higher frequencies. These discrepancies are due to the increase in the period of substrate wavelength ratio as the frequency becomes higher. The effective permittivity obtained from EMT then becomes more inaccurate.

A comparison of the transmittance versus frequency in the case of an incidence angle of 20° with a TE polarized plane wave is shown in Fig. 3.

Even at an oblique incidence angle of 20°, the two results still agree at a low frequency range. However, at a higher frequency range, the difference between the two results is more obvious due to the inaccuracy of effective permittivity. These results show that as the angle of the plane wave becomes larger, the effective permittivity obtained from EMT becomes more inaccurate at higher frequencies than when the plane waves are at normal incidence.

For some applications of FSS, the phase response to the incident plane waves is very important. Figures 4 and 5 illustrate the resulting phase variations of the reflected electric field at the patch surface versus frequency.

The phase changes from –180° to 180°. The phase-jump occurs at the resonant frequency; thus, more jump points can be seen at the 20° incidence angle.

In conclusion, using the EMT, we can treat the periodic substrate as an equivalent isotropic dielectric and then utilize the conventional MoM in spectral domain.

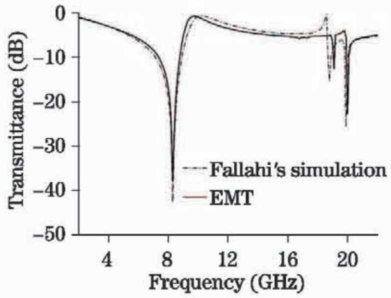

Fig. 2. Comparison of the EMT and Fallahi's results for normal incidence of the plane wave.

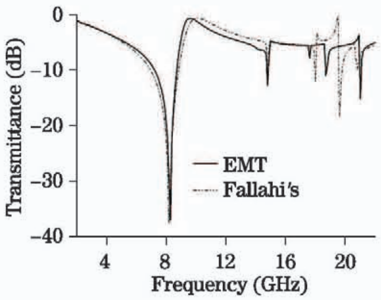

Fig. 3. Comparison of the EMT and Fallahi's results for a TE-polarized plane wave with a 20° incidence angle.

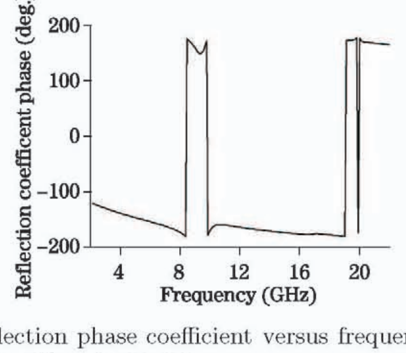

Fig. 4. Reflection phase coefficient versus frequency for normal incidence of a plane wave.

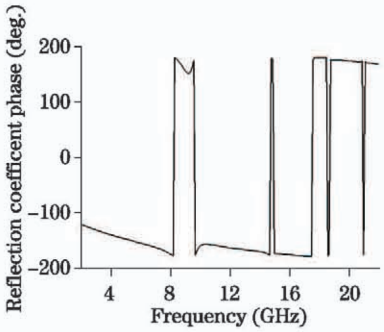

Fig. 5. Reflection phase coefficient versus frequency for a TE-polarized plane wave with a 20° incidence angle.

Our numerical calculations show that the using results EMT agree with Fallahi's full-vector semi-analytical data. There are only some minute differences between the two results at a high frequency range because of the inaccurate effective relative permittivity of the periodic substrate obtained from the EMT. This shows that the numerical procedure is not only applicable for the analysis of such planar structures, but computationally more efficient as well.

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