

Merit function to design holographic gratings for moderate-resolution monochromators

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Received 15 August 2011; revised 5 October 2011; accepted 6 October 2011;
posted 7 October 2011 (Doc. ID 152896); published 15 November 2011

A merit function is proposed and applied to design holographic concave gratings for moderate-resolution monochromators. To justify the validity of the merit function, imaging properties of gratings used for the coma-correction Seya-Namioka monochromator, designed by the present authors, Noda, and Takahashi, are compared through ray tracing and their aberration-correction mechanisms are also analyzed. The capability of the merit function is well demonstrated in the design of holographic gratings for another two moderate-resolution monochromators with different requirements. All the results obtained show that the merit function is not only straight and effective but also manages to balance various aberrations of the concave holographic grating very well. © 2011 Optical Society of America

OCIS codes: 050.1950, 050.2770, 220.1010.

1. Introduction

For the advantages of holographic gratings, such as exceedingly low scattered light levels, relatively short production periods, and absolute absence of notorious ghosts, they are widely used in spectrographs and monochromators nowadays. A common way to manufacture the holographic gratings is to record, in a photosensitive resist smearing on the surface of the grating blank, the interference patterns of two beams of coherent light [1–4], in which process, by properly choosing the locations of the two coherent light sources, the aberrations of these gratings will be reduced. Although recording gratings by aspheric wavefront has been put forward [5–7], which enables sufficient variables for the optimization design of high-resolution aberration-corrected holographic gratings, here we introduce the disadvantage that the recording geometry is difficult to construct because the positions of the supplementary optical elements used to produce aspheric wavefront are not

easy to locate accurately. The aberration theory of aspheric gratings has been developed [8–11], which introduces additional variables for the optimization, but it is out of the scope of this paper.

Some types of merit function have been developed for the design of grating instruments [12], and each of them has its own advantages and drawbacks. The rms merit function mentioned in [12] is worth the whistle for its high degree of accuracy of the sophisticated grating device design and its use of gratings recorded by aspheric wavefronts [5,13], but the final formulas for the aberration reduced are extremely cumbersome and intractable for numerical optimization, so this merit function is not appropriate for applications in which the grating instruments are not sophisticated.

Although the geometric aberration theory of double-element optical systems has been put forward [14,15], the most common use in daily life is still the single-element optical systems, which necessitate only moderate resolving power. The geometric aberration theory based on the Fermat's principle for single-element optical systems has been well developed by Noda *et al.* [1] and widely used for many

design purposes for its understandability and availability. However, the disadvantages are that the aberration coefficients derived from the expansion of the light path function are minimized separately and the design process is divided into two steps: one is to seek instrumental parameters that involve integrals and derivatives, the other is to seek recording parameters and the varieties of the aberration to be reduced are limited by the number of the free variables. Obviously, precision errors of the optimum parameters will be brought out when the results of the first step are substituted into the second step and the optimum grating recording parameters may not be obtained under the predetermined optimum grating use parameters.

The purpose of this paper is to propose a new merit function that is able to avoid the disadvantage mentioned earlier and able to balance various aberrations of the grating by adopting appropriate weighting factors. To justify this merit function, we design a concave holographic grating used for the coma-correction Seya-Namioka monochromator and compare the result with the gratings designed by Noda [1] and Takahashi [16]. We also design holographic concave gratings for another two constant-deviation monochromators with different requirements to verify the merit function's capability and efficiency. All the gratings are evaluated by the degree of spread in their spot diagrams and their aberration coefficient curves.

2. Merit Function

To facilitate the later discussion, we introduce a rectangular coordinate system whose origin O is at the vertex of the concave grating blank shown in Fig. 1, with the x axis along the grating normal and the x - y plane as the symmetry plane of the grating system. A ray originating from a point A in the entrance slit, whose center is at point A_0 , is diffracted by a point P on the n th groove of the concave grating and

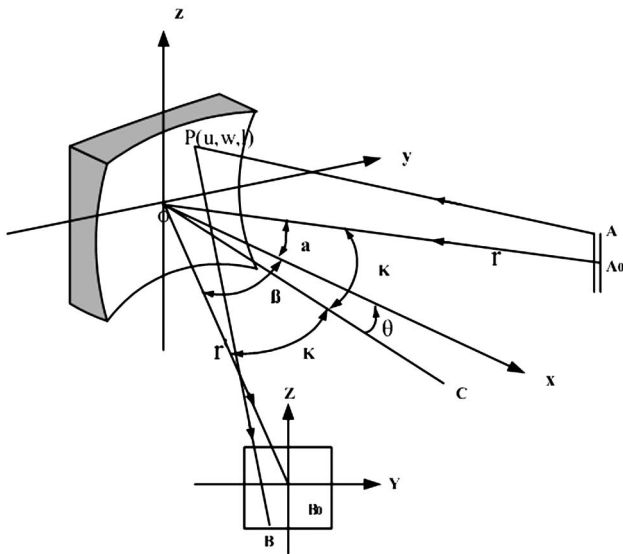


Fig. 1. Schematic diagram of monochromator.

intersects the image plane, which is assumed to be perpendicular to the principal ray OB_0 of wavelength λ in m th, at a point B . r and r' are the distances from the vertex of the grating to the center of the entrance slit and exit slit, respectively, and α and β , whose signs are opposite if the points A and B lie on the different side of the x - z plane, are the angles of incidence and diffraction, respectively. θ is the angle of grating rotation measuring from OC , which is the bisector of deviation angle $2K$, to the x axis, and has the same sign as the spectral order m . The light path function of the ray APB is defined as [17]

$$F = \langle AP \rangle + \langle PB \rangle + nm\lambda. \quad (1)$$

Applying power series expansions to expression (1), we obtain

$$\begin{aligned} F = & F_{000} + \omega F_{100} + l F_{011} + \frac{1}{2} \omega^2 F_{200} + \frac{1}{2} l^2 F_{020} \\ & + \frac{1}{2} \omega^3 F_{300} + \frac{1}{2} \omega l^2 F_{120} + \omega l F_{111} + \frac{1}{8} \omega^4 F_{400} \\ & + \frac{1}{4} \omega^2 l^2 F_{220} + \dots, \end{aligned} \quad (2)$$

where the subscripts ijk of F_{ijk} are the exponents of $w^i l^j z^k$, and the exact expressions of F_{ijk} are described in detail in [17].

The merit function is defined as

$$\begin{aligned} I = \sum_{\eta} \varepsilon(\lambda_{\eta}) [& \mu_1 F_{200}^2(\lambda_{\eta}) + \mu_2 F_{020}^2(\lambda_{\eta}) + \mu_3 F_{300}^2(\lambda_{\eta}) \\ & + \mu_4 F_{120}^2(\lambda_{\eta}) + \mu_5 F_{400}^2(\lambda_{\eta}) + \dots], \end{aligned} \quad (3)$$

where λ_{η} is one of the wavelengths chosen to be optimized and the weighting factor $\varepsilon(\lambda_{\eta})$ can be set equal to unity in most cases. μ_t ($t = 1, 2, 3, \dots$) are weighting factors for different aberration coefficients and their values are decided by the reciprocal of magnitude of their corresponding aberration coefficients, which can be estimated by experience.

3. Test of Merit Function

A. Coma-Correction Seya-Namioka Monochromator

For the purpose of comparison, it is assumed that the deviation angle $2K$ of the monochromator is 69.733° and the scanning wavelength range is 0–700 nm in the first negative order. The curvature radius of the grating is 500 mm and the effective grating constant is 1/600 mm; the effective area of the grating is $50(W) \times 30(H)$ mm² and the recording wavelength is 457.93 nm. Utilizing the merit function defined in Section 2, we can obtain the optimum grating parameters shown in Table 1 (first row), where r_C and r_D are the distances from the vertex of the grating to the two recording light sources with γ and δ as their incidence angles, respectively. The parameters of the gratings designed by Noda and by Takahashi are also shown in Table 1 (rows 2 and 3, respectively).

Table 1. Optimum Grating Parameters

	r (mm)	r' (mm)	r_C (mm)	r_D (mm)	γ (deg)	δ (deg)
1	424.088	397.267	549.450	577.834	-63.587	-38.377
2	409.659	410.870	472.903	502.063	-25.603	-44.983
3	409.607	410.959	508.634	595.645	-47.404	-27.477

To analyze their capabilities of aberration correction, we ray traced and constructed spot diagrams, as shown in Fig. 2 in three cases with gratings designed by Noda (Fig. 2(a)), Takahashi (Fig. 2(b)), and the present authors (Fig. 2(c)). To do this, the entrance slit was assumed to be 6 mm in height and infinitesimal width. The height of the entrance slit was divided into five sections equally and the point at the edge of each section was assumed to be self-luminous, and the grating was divided into 1500 sections, each of which was a 1.0 mm \times 1.0 mm mesh. The rays traced were those originating from the edge of each section in the entrance slit and diffracted at the mesh cross points on the surface of the grating. From Fig. 2, we can see that the spectral widths of the grating designed by the present authors are much better than the other two.

Now, we analyze the aberration-correction mechanism of those gratings. To do this, F_{200} curves as functions of wavelength for the three gratings are described by the solid, dashed, and dotted-dashed

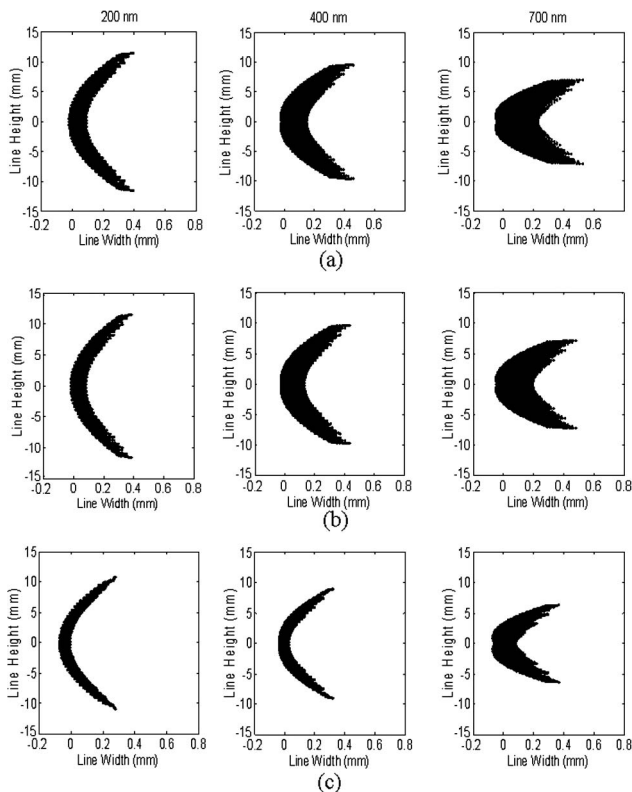


Fig. 2. Spot diagrams obtained by tracing rays through gratings, mounted in coma-correction Seya-Namioka monochromator, designed by (a) Noda, (b) Takahashi, and the (c) present authors, respectively.

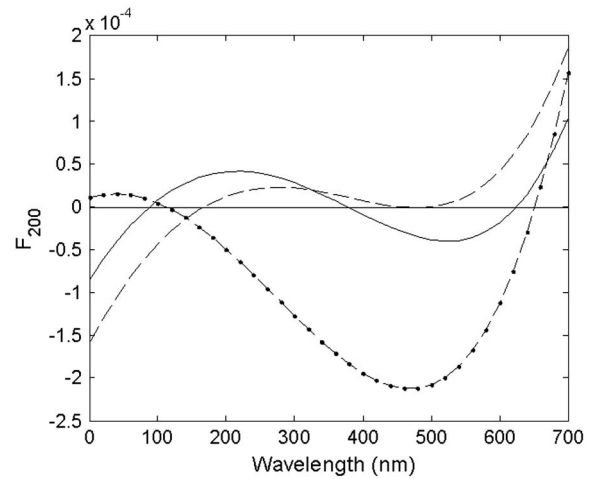


Fig. 3. Amount of defocus term F_{200} as a function of diffracted wavelength. The solid, dashed, and dotted-dashed curves describe the gratings designed by Noda, Takahashi, and the present authors, respectively.

curves in Fig. 3, respectively. It is easy to see that the defoci of the gratings designed by Noda and by Takahashi are well balanced along with the wavelength in question. However, the defocus of the grating designed by the present authors departs from the ideal condition $F_{200} = 0$ largely in the middle of the wavelength range. Figure 4 indicates F_{300} curves as functions of wavelength for these three gratings by solid, dashed, and dotted-dashed curves, respectively. It is easy to see that the coma aberrations of the gratings designed by Noda and by Takahashi grow rapidly with the wavelength increasing.

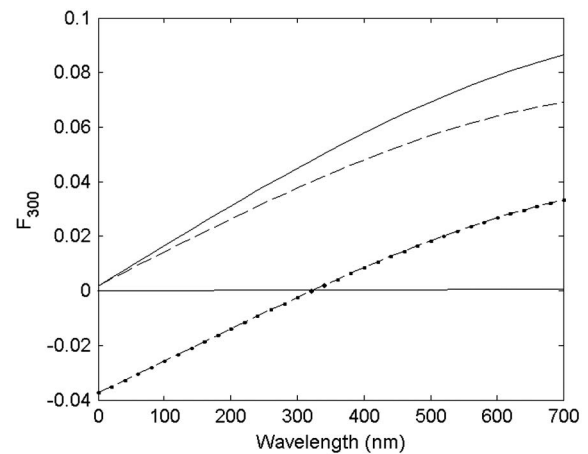


Fig. 4. Amount of coma term F_{300} as a function of diffracted wavelength. The solid, dashed, and dotted-dashed curves describe the gratings designed by Noda, Takahashi, and the present authors, respectively.

Table 2. Optimum Results for the Two Constant-Deviation Monochromators

	R (mm)	$2K$ (deg)	r_C (mm)	r_D (mm)	γ (deg)	δ (deg)
1	84.091	47.578	76.356	84.318	-35.363	-18.306
2	63.53	12.000	168.470	253.741	22.586	37.196

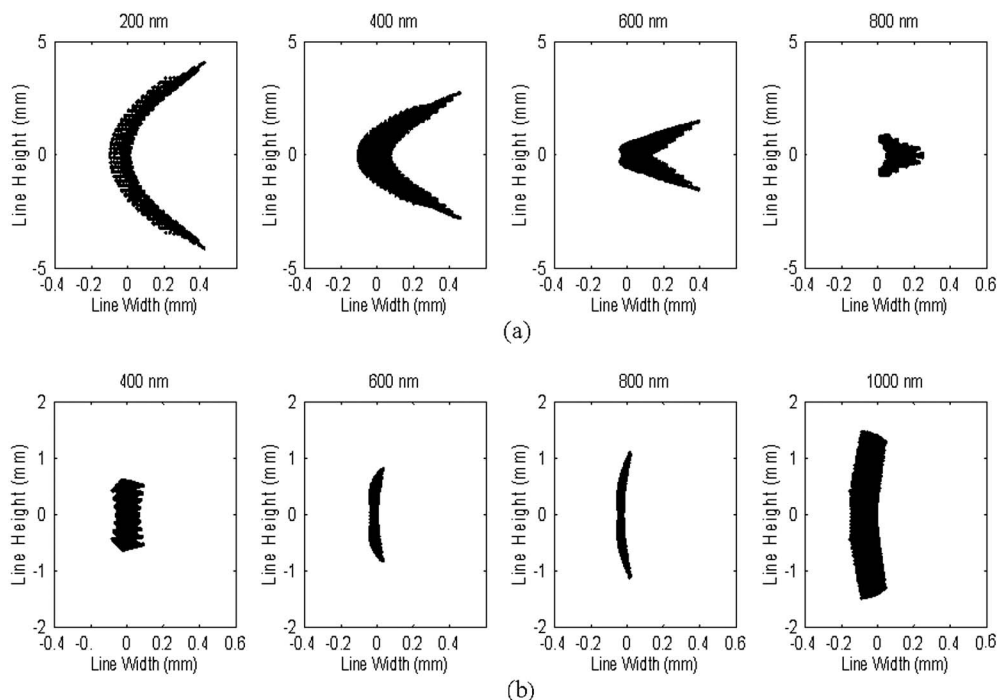


Fig. 5. Spot diagrams obtained by tracing rays through gratings mounted in (a) constant-deviation monochromators 1 and 2.

However, the coma aberration of the grating designed by the present authors balanced along with the wavelength very well. From the earlier discussions, we can see that the merit function defined in this paper has the capability to balance different aberrations for a grating.

B. Moderate-Resolution Constant-Deviation Monochromators

To illustrate the capability of the merit function, we utilize it to design gratings for another two moderate-resolution constant-deviation monochromators: (1) the entrance path length is 80 mm and the exit path length is 75 mm with the deviation angle $2K \geq 34.7^\circ$; the required wavelength range is from 200 to 800 nm in the first negative order with the

effective grating constant confined to $1/600$ mm. (2) Both the entrance and the exit path length are 64 mm with the deviation angle $2K$ confined to be 12° ; the required wavelength range is from 400 to 1000 nm in the first positive order and the effective grating constant is required to be $1/500$ mm. The optimum results are listed in Table 2 (row 1) and Table 2 (row 2), where R is the curvature of radius of the grating blank. We did ray racing and constructed spot diagrams for the two gratings on the assumption that both of the entrance slit heights are 1 mm with infinitesimal widths, and the ruled areas are $30(W) \text{ mm} \times 30(H) \text{ mm}$. Spot diagrams for both gratings are shown in Figs. 5(a) and 5(b). F_{200} and F_{300} curves of the two gratings are shown Figs. 6 and 7 by solid and dashed curves, respectively. We can see that the defocus and coma of the two gratings

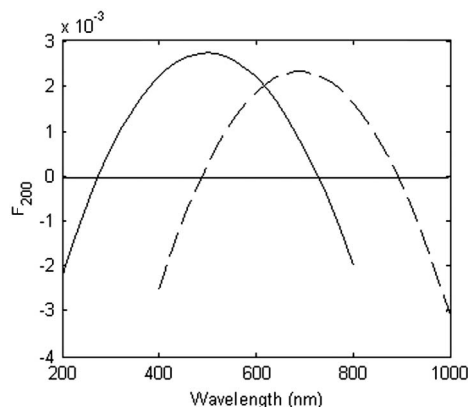


Fig. 6. Amount of defocus term F_{200} as a function of diffracted wavelength. The solid and dashed curves describe the gratings designed for monochromators 1 and 2, respectively.

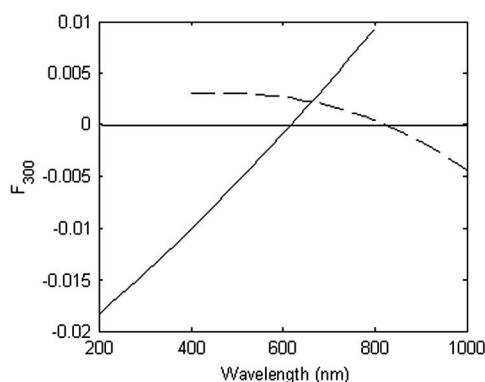


Fig. 7. Amount of coma term F_{300} as a function of diffracted wavelength. The solid and dashed curves describe the gratings designed for monochromators 1 and 2, respectively.

are well corrected for the wavelength range of interest.

4. Conclusion

We proposed a merit function, consisting of the squares of aberration coefficients derived from the expansion of the light path function with weighting factors, to design holographic concave gratings for moderate-resolution constant-deviation monochromators. The validity of the merit function is verified by comparisons between the gratings designed by the present authors and the former authors, and the capability of the merit function is illustrated by the design of gratings for another two constant-deviation monochromators with different requirements. In the view of these results, it is certain that the merit function proposed here is able to afford an effective method for moderate-resolution holographic grating design with an exceptional ability to balance different aberrations.

The authors acknowledge support from the Chinese Scientific Technologies Department for the Special Innovative Program (grant 2008IM040700).

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