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Tracking and ray tracing equations for the target-aligned heliostat for solar tower power plants

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ABSTRACT

The tracking and ray tracing equations for the target-aligned heliostat for solar tower power plants have been derived in this paper. Based on the equations, a new module for analysis of the target-aligned heliostat with an asymmetric surface has been developed and incorporated in the code HFLD. To validate the tracking and ray tracing equations, a target-aligned heliostat with a toroidal surface is designed and modeled. The image of the target-aligned heliostat is calculated by the modified code HFLD and compared with that calculated by the commercial software Zemax. It is shown that the calculated results coincide with each other very well. Therefore, the correctness of the tracking and ray tracing equations for the target-aligned heliostat is proved.

1. Introduction

For solar tower power plants, the heliostats track the sun and reflect the solar radiation onto the receiver atop a tower. The heliostat field plays a pivotal role in contribution of the total cost and efficiency of solar tower power plants [1]. Traditionally, heliostats utilize spherical reflector with azimuth-elevation mount. During the sun tracking, the incident angle relative to the heliostat changes and the reflection is off-axis, which is most of the time in practical situations. For the spherical reflector, the meridian focus moves along a circle with a diameter equal to the paraxial focal length. However, the sagittal focus moves along a straight line with the increase of incident angle. The sagittal and meridian focuses never overlap so that the image spreads in the focal plane. In order to correct the astigmatism and reduce the solar image size, an asymmetric reflector was suggested to be used which means that the heliostat should have different curvature radii along the meridian and sagittal direction in heliostat plane [2,3]. However, the asymmetric reflector requires that the incident direction of sunlight remains stationary relative to the heliostat plane and the incident plane of sunlight coincides with the meridian plane of the heliostat during the sun tracking. RIES H presented a new tracking method called target-aligned mount which can be used for the asymmetrical heliostat [4].

In this work, the tracking and ray tracing equations for the target-aligned heliostat are firstly derived and given explicitly. Then based on the equations, a new module for analysis of the target-aligned heliostat with asymmetric surface is incorporated in the code HFLD [5–7]. The image of a single target-aligned heliostat with toroidal surface is calculated by using the modified code HFLD and the software Zemax respectively. Zemax is well known commercial software which is widely used for the optical design and analysis [8,9]. The correctness of the tracking and ray tracing equations is proved by comparing the calculated results.

2. Tracking equations for the target-aligned heliostat

The target-aligned heliostat has two rotation axes as shown in Fig. 1. The first axis is fixed relative to the ground and points toward the target and the second axis is perpendicular to the first axis and is located in the heliostat plane. As the sun moves, the heliostat rotates about the first axis firstly so that the incident plane of sunlight coincides with the meridian plane of the heliostat, and then the heliostat rotates about the second axis to reflect the sunlight to the target. The rotation angles of heliostat can be calculated according to the solar time, the heliostat and the target locations on earth.

For deriving the rotation angle formulas for the target-aligned heliostat, Cartesian right-handed coordinate systems are established and illustrated in Fig. 2. Ground-coordinates are defined as X_g, Y_g, Z_g, where the tower base center G is the origin, X_g is directed toward south, Y_g points toward north, and Z_g points toward the

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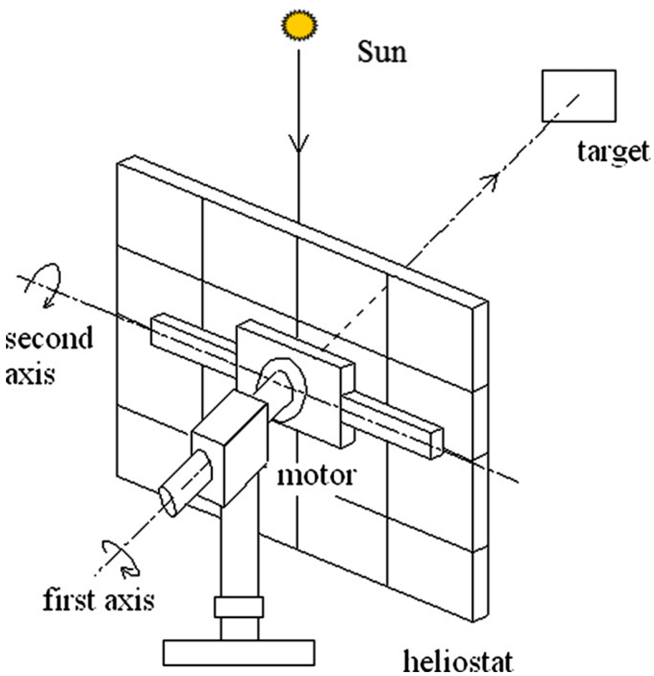

Fig. 1. The heliostat with target-aligned mount.

zenith. Reflection-normal coordinates are defined as X_r, Y_r, Z_r, where the heliostat plane center M is origin, Z_r is along reflection toward the target, X_r is horizontal and perpendicular to Z_r, and Y_r is perpendicular to X_r toward up. T denotes the target center which corresponds to the intersection point between the Z_g axis and the Z_r axis. Zero position for the target-aligned heliostat is selected as the meridian plane of the heliostat is vertical. Y_rZ_r plane corresponds to the heliostat meridian plane in zero position. Auxiliary coordinates $X_1Y_1Z_1$ correspond to the real position of the heliostat after rotations about the first axis.

The incidence vector pointing toward the sun from the heliostat location in ground-coordinates is illustrated in Fig. 3.

$$\begin{bmatrix} \cos\alpha_i \\ \cos\beta_i \\ \cos\gamma_i \end{bmatrix} = \begin{bmatrix} \cos A \cos\alpha \\ \sin A \cos\alpha \\ \sin\alpha \end{bmatrix} \tag{1}$$

where $\cos\alpha_i$, $\cos\beta_i$, $\cos\gamma_i$ are the direction cosine components of the incidence vector, α is the solar altitude angle and A is the solar azimuth angle which is measured clockwise on the horizontal plane from the projection of the sun's central ray to the south-pointing coordinate axis. The angles α and A can be calculated according to the solar time and the heliostat location on earth [10].

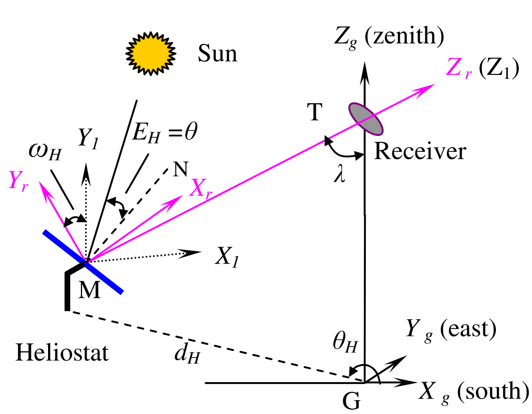

Fig. 2. Coordinate systems for deriving the rotation angle formulas for the target-aligned heliostat.

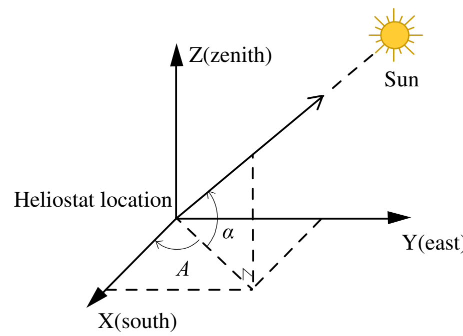

Fig. 3. Incidence vector in ground-coordinates.

Similarly, the reflection vector pointing toward the target from the heliostat center in ground-coordinates is illustrated in Fig. 4.

$$\begin{bmatrix} \cos\alpha_r \\ \cos\beta_r \\ \cos\gamma_r \end{bmatrix} = \begin{bmatrix} -\cos\theta_H \sin\lambda \\ -\sin\theta_H \sin\lambda \\ \cos\lambda \end{bmatrix} \tag{2}$$

where $\cos\alpha_r$, $\cos\beta_r$, $\cos\gamma_r$ are the direction cosine components of the reflection vector, λ is the heliostat's target angle and θ_H is the heliostat's facing angle which is measured anticlockwise on the horizontal plane from the south-pointing coordinate axis to the heliostat's position. The angles λ and θ_H depend on the heliostat and the target locations on earth.

The incident angle θ equals to the half of the angle between the incidence and reflection vectors. According to the law of vector angular cosine and using Eqs. (1) and (2), we have,

$$\cos 2\theta = \sin\alpha\cos\lambda - \cos\alpha\sin\lambda\cos(\theta_H - A) \tag{3}$$

The incident angle θ can be solved from Eq. (3) as follows,

$$\theta = \cos^{-1}\left\{\frac{\sqrt{2}}{2}[\sin\alpha\cos\lambda - \cos(\theta_H - A)\cos\alpha\sin\lambda + 1]^{1/2}\right\} \tag{4}$$

For obtaining the direction cosine of the incidence vector in auxiliary coordinates $X_1Y_1Z_1$, the coordinates need to transform from ground-coordinates to auxiliary coordinates. Firstly, ground-coordinates rotate $\theta_H - \pi/2$ about Z_g axis and then rotate λ about X_g axis so that the three axial directions of the coordinates coincide with that of reflection-normal coordinates (see Fig. 2). Then, the coordinates rotate $-\omega_H$ about Z_r axis so that the three axial directions of the coordinates coincide with that of the auxiliary coordinates. The transformation matrices are given as follows,

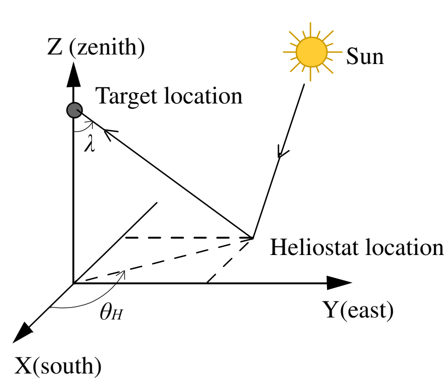

Fig. 4. Reflection vector in ground-coordinates.

$$M_1 = \begin{bmatrix} \sin\theta_H & -\cos\theta_H & 0 \\ \cos\theta_H & \sin\theta_H & 0 \\ 0 & 0 & 1 \end{bmatrix},\ M_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\lambda & \sin\lambda \\ 0 & -\sin\lambda & \cos\lambda \end{bmatrix},$$

$$M_3 = \begin{bmatrix} \cos\omega_H & -\sin\omega_H & 0 \\ \sin\omega_H & \cos\omega_H & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where ω_H is the rotation angle about the first axis (see Fig. 2). The direction cosine of the incidence vector also can be written as (0, sin2θ, cos2θ). Therefore, we have,

$$\begin{bmatrix} 0 \\ \sin 2\theta \\ \cos 2\theta \end{bmatrix} = M_3 M_2 M_1 \cdot \begin{bmatrix} \cos A \cos\alpha \\ \sin A \cos\alpha \\ \sin\alpha \end{bmatrix} \tag{5}$$

From Eq. (5), we have,

$$\cos\omega_H \cos\alpha \sin(\theta_H - A) - \sin\omega_H[\cos\lambda\cos\alpha\cos(\theta_H - A) + \sin\lambda\sin\alpha] = 0 \tag{6}$$

Solving Eq. (6), we have,

$$\omega_H = \tan^{-1}\left[\frac{\cos\alpha\sin(\theta_H - A)}{\cos\lambda\cos\alpha\cos(\theta_H - A) + \sin\lambda\sin\alpha}\right] \tag{7}$$

The tilt angle E_H of the heliostat equals to the incident angle θ, we have,

$$E_H = \theta \tag{8}$$

3. Ray tracing equations for the target-aligned heliostat

For modeling and analyzing the target-aligned heliostat with an asymmetric surface, it is necessary to derive the ray tracing equations. Cartesian right-handed coordinate systems are established and shown in Fig. 5. Incident-normal coordinates are defined as X_i, Y_i, Z_i, where the heliostat plane center M is origin, Z_i is along the incident direction toward the sun, X_i is horizontal and perpendicular to Z_i, and Y_i is perpendicular to X_i toward up. Heliostat-coordinates are defined as X_m, Y_m, Z_m, where the heliostat plane center M is origin, Z_m is along the normal direction of the heliostat toward up, X_m is along the second axis of the heliostat, Y_m is perpendicular to X_m toward up, X_m and Y_m are located in the heliostat plane. Target-coordinates are defined as X_T, Y_T, Z_T, where the receiver aperture center T is origin, Z_T is along the normal direction of the receiver aperture toward up, X_T is horizontal and perpendicular to Z_T, Y_T is perpendicular to X_T toward up, X_T and Y_T are located in the receiver aperture plane.

Considering the sun shape, the incident beam relative to a point of mirror can be regarded as a cone with a vertex angle of $\varepsilon = 9.3$mrad. The symmetric axis of the cone coincides with the Z_i axial direction. The polar coordinate system is established on the solar disc plane. The center of the solar disc is the origin. ρ_{sun}, θ_{sun} are the polar coordinate components ($0 \le \rho_{sun} \le \varepsilon/2$, $0 \le \theta_{sun} \le 2\pi$). In terms of incident-normal coordinates $X_iY_iZ_i$, the components δ_{ix}, δ_{iy}, δ_{iz} of the unit vector of incident ray can be expressed as follows,

$$\begin{bmatrix} \delta_{ix} \\ \delta_{iy} \\ \delta_{iz} \end{bmatrix} = \begin{bmatrix} \rho_{sun}\cos\theta_{sun} \\ \rho_{sun}\sin\theta_{sun} \\ \left(1 - \delta_{ix}^2 - \delta_{iy}^2\right)^{0.5} \end{bmatrix} \tag{9}$$

For obtaining the direction cosine components $\cos\alpha_I$, $\cos\beta_I$, $\cos\gamma_I$ of the incident ray in heliostat-coordinates $X_mY_mZ_m$, the coordinates need to transform from incident-normal coordinates to heliostat-coordinates. Firstly, incident-normal coordinates rotate $\alpha - \pi/2$ about X_i axis and then rotate $A - \pi/2$ about Z_i axis so that the three axial directions of the coordinates coincide with that of ground-coordinates (see Fig. 2 and Fig. 5). Secondly, the coordinates rotate $\theta_H - \pi/2$ about Z_g axis and then rotate λ about X_g axis so that the three axial directions of the coordinates coincide with that of reflection-normal coordinates. Finally, the coordinates rotate $-\omega_H$ about Z_r axis and then rotate $-\theta$ about X_r axis so that the axial directions of the coordinates coincide with that of heliostat-coordinates. The transformation matrices are given as follows,

$$M_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sin\alpha & -\cos\alpha \\ 0 & \cos\alpha & \sin\alpha \end{bmatrix},\ M_2 = \begin{bmatrix} -\sin A & -\cos A & 0 \\ \cos A & -\sin A & 0 \\ 0 & 0 & 1 \end{bmatrix},\ M_3 = \begin{bmatrix} \sin\theta_H & -\cos\theta_H & 0 \\ \cos\theta_H & \sin\theta_H & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$M_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\lambda & \sin\lambda \\ 0 & -\sin\lambda & \cos\lambda \end{bmatrix},\ M_5 = \begin{bmatrix} \cos\omega_H & -\sin\omega_H & 0 \\ \sin\omega_H & \cos\omega_H & 0 \\ 0 & 0 & 1 \end{bmatrix},\ M_6 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}.$$

So we have,

$$\begin{bmatrix} \cos\alpha_I \\ \cos\beta_I \\ \cos\gamma_I \end{bmatrix} = \Pi_{i=0(i\le 5)}^{6-i}[M_i] \cdot \begin{bmatrix} \delta_{ix} \\ \delta_{iy} \\ \delta_{iz} \end{bmatrix} = \begin{bmatrix} \delta_{ix}A_{11} + \delta_{iy}A_{12} + \delta_{iz}A_{13} \\ \delta_{ix}A_{21} + \delta_{iy}A_{22} + \delta_{iz}A_{23} \\ \delta_{ix}A_{31} + \delta_{iy}A_{32} + \delta_{iz}A_{33} \end{bmatrix} \tag{10}$$

where,

$$\begin{aligned}
A_{11} &= -\cos\omega_H\cos(\theta_H - A) - \sin\omega_H\cos\lambda\sin(\theta_H - A) \\
A_{12} &= [\sin\omega_H\cos\lambda\cos(\theta_H - A) - \cos\omega_H\sin(\theta_H - A)]\sin\alpha - \sin\omega_H\sin\lambda\cos\alpha \\
A_{13} &= [\cos\omega_H\sin(\theta_H - A) - \sin\omega_H\cos\lambda\cos(\theta_H - A)]\cos\alpha - \sin\omega_H\sin\lambda\sin\alpha \\
A_{21} &= (\cos\theta\cos\omega_H\cos\lambda + \sin\theta\sin\lambda)\sin(\theta_H - A) - \cos\theta\sin\omega_H\cos(\theta_H - A) \\
A_{22} &= (\cos\theta\cos\omega_H\sin\lambda - \sin\theta\cos\lambda)\cos\alpha - [\cos\theta\sin\omega_H\sin(\theta_H - A) + (\cos\theta\cos\omega_H\cos\lambda + \sin\theta\sin\lambda)\cos(\theta_H - A)]\sin\alpha \\
A_{23} &= [\cos\theta\sin\omega_H\sin(\theta_H - A) + (\cos\theta\cos\omega_H\cos\lambda + \sin\theta\sin\lambda)\cos(\theta_H - A)]\cos\alpha + (\cos\theta\cos\omega_H\sin\lambda - \sin\theta\cos\lambda)\sin\alpha \\
A_{31} &= (\sin\theta\cos\omega_H\cos\lambda - \cos\theta\sin\lambda)\sin(\theta_H - A) - \sin\theta\sin\omega_H\cos(\theta_H - A) \\
A_{32} &= (\sin\theta\cos\omega_H\sin\lambda + \cos\theta\cos\lambda)\cos\alpha - [\sin\theta\sin\omega_H\sin(\theta_H - A) + (\sin\theta\cos\omega_H\cos\lambda - \cos\theta\sin\lambda)\cos(\theta_H - A)]\sin\alpha \\
A_{33} &= [\sin\theta\sin\omega_H\sin(\theta_H - A) + (\sin\theta\cos\omega_H\cos\lambda - \cos\theta\sin\lambda)\cos(\theta_H - A)]\cos\alpha + (\sin\theta\cos\omega_H\sin\lambda + \cos\theta\cos\lambda)\sin\alpha
\end{aligned}$$

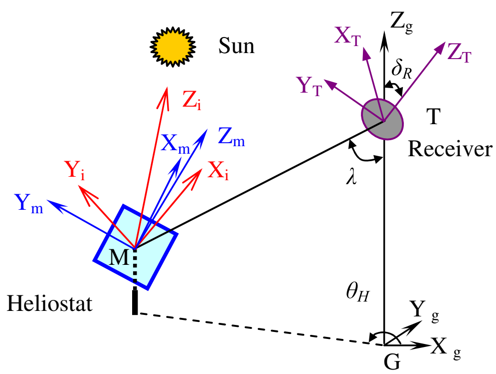

Fig. 5. Coordinate systems for deriving the ray tracing equations for the target-aligned heliostat.

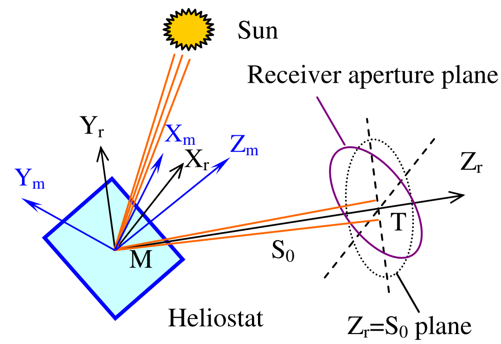

Fig. 6. Coordinate transformation from heliostat coordinates to reflection-normal coordinates.

The coordinates of a point of mirror are denoted by X_m, Y_m, Z_m. In terms of heliostat-coordinates, the components δ_{nx}, δ_{ny}, δ_{nz} of the unit normal vector of a point on the mirror surface can be expressed as follows,

$$\begin{bmatrix} \delta_{nx} \\ \delta_{ny} \\ \delta_{nz} \end{bmatrix} = \begin{bmatrix} -\delta_{X_m} + \delta_{nwx} \\ -\delta_{Y_m} + \delta_{nwy} \\ \left(1 - \delta_{nx}^2 - \delta_{ny}^2\right)^{0.5} \end{bmatrix} \tag{11}$$

where, δ_{X_m} and δ_{Y_m} are angular components along X_m and Y_m axes respectively due to the mirror surface shape, δ_{nwx} and δ_{nwy} are angular components along X_m and Y_m axes respectively due to the shape errors. The direction cosine components $\cos\alpha_N$, $\cos\beta_N$, $\cos\gamma_N$ of the normal vector of a point can be written as follows,

$$\begin{bmatrix} \cos\alpha_N \\ \cos\beta_N \\ \cos\gamma_N \end{bmatrix} = \begin{bmatrix} \delta_{nx} \\ \delta_{ny} \\ \left(1 - \delta_{nx}^2 - \delta_{ny}^2\right)^{0.5} \end{bmatrix} \tag{12}$$

According to the law of vector angular cosine, the incident angle θ_m of all traced rays relative to the mirror surface can be written as follows,

$$\cos\theta_m = \cos\alpha_I\cos\alpha_N + \cos\beta_I\cos\beta_N + \cos\gamma_I\cos\gamma_N \tag{13}$$

According to the Snell law, the direction cosine components $\cos\alpha_{mr}$, $\cos\beta_{mr}$, $\cos\gamma_{mr}$ of reflection rays in terms of heliostat-coordinates are written as follows,

$$\begin{bmatrix} \cos\alpha_{mr} \\ \cos\beta_{mr} \\ \cos\gamma_{mr} \end{bmatrix} = \begin{bmatrix} 2\cos\theta_m\cos\alpha_N - \cos\alpha_I \\ 2\cos\theta_m\cos\beta_N - \cos\beta_I \\ 2\cos\theta_m\cos\lambda_N - \cos\gamma_I \end{bmatrix} \tag{14}$$

For obtaining the direction cosine components $\cos\alpha_r$, $\cos\beta_r$, $\cos\gamma_r$ of the reflection ray and the coordinate components X_{mr}, Y_{mr}, Z_{mr} of a point of mirror in reflection-normal coordinates $X_rY_rZ_r$, the coordinates need to transform from heliostat-coordinates to reflection-normal coordinates. Therefore, heliostat-coordinates rotate θ about X_m axis and then rotate ω_H about Z_m axis so that the axial directions of the coordinates coincide with that of reflection-normal coordinates (see Fig. 6). The transformation matrices are given as follows,

$$M_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}, \quad M_2 = \begin{bmatrix} \cos\omega_H & \sin\omega_H & 0 \\ -\sin\omega_H & \cos\omega_H & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

So we have,

$$\begin{bmatrix} \cos\alpha_r \\ \cos\beta_r \\ \cos\gamma_r \end{bmatrix} = M_2M_1 \begin{bmatrix} \cos\alpha_{mr} \\ \cos\beta_{mr} \\ \cos\gamma_{mr} \end{bmatrix} = \begin{bmatrix} \cos\alpha_{mr}\cos\omega_H + \cos\beta_{mr}\sin\omega_H\cos\theta + \cos\gamma_{mr}\sin\omega_H\sin\theta \\ -\cos\alpha_{mr}\sin\omega_H + \cos\beta_{mr}\cos\omega_H\cos\theta + \cos\gamma_{mr}\cos\omega_H\sin\theta \\ -\cos\beta_{mr}\sin\theta + \cos\gamma_{mr}\cos\theta \end{bmatrix} \tag{15}$$

$$\begin{bmatrix} X_{mr} \\ Y_{mr} \\ Z_{mr} \end{bmatrix} = M_2M_1 \begin{bmatrix} X_m \\ Y_m \\ Z_m \end{bmatrix} = \begin{bmatrix} X_m\cos\omega_H + Y_m\sin\omega_H\cos\theta + Z_m\sin\omega_H\sin\theta \\ -X_m\sin\omega_H + Y_m\cos\omega_H\cos\theta + Z_m\cos\omega_H\sin\theta \\ -Y_m\sin\theta + Z_m\cos\theta \end{bmatrix} \tag{16}$$

The equation of the reflection ray is written as follows,

$$\frac{x_r - X_{mr}}{\cos\alpha_r} = \frac{y_r - Y_{mr}}{\cos\beta_r} = \frac{z_r - Z_{mr}}{\cos\gamma_r} \tag{17}$$

The coordinates of intersection point between the reflection ray and the plane $Z_r = S_0$ (see Fig. 6) are solved as follows,

$$\begin{bmatrix} X_r \\ Y_r \\ Z_r \end{bmatrix} = \begin{bmatrix} (S_0 - Z_{mr})\cdot\cos\alpha_r/\cos\gamma_r + X_{mr} \\ (S_0 - Z_{mr})\cdot\cos\beta_r/\cos\gamma_r + Y_{mr} \\ S_0 \end{bmatrix} \tag{18}$$

where S_0 is the distance between the heliostat center and the receiver aperture center (see Fig. 6).

A new reflection auxiliary coordinates $X_{r1}Y_{r1}Z_{r1}$ are obtained by translating the reflection-normal coordinate S_0 along Z_r axis (see Fig. 7). In terms of the new coordinates, the coordinates of intersection point between the reflection ray and the plane $Z_r = S_0$ can be written as X_r, Y_r, 0.

For obtaining the direction cosine components $\cos\alpha_{tr}$, $\cos\beta_{tr}$, $\cos\gamma_{tr}$ of the reflection ray and the coordinate components X_{t0}, Y_{t0}, Z_{t0} of the intersection point in target-coordinates $X_TY_TZ_T$, the coordinates need to transform from reflection auxiliary coordinates to target-coordinates. Firstly, the coordinates rotate $-\lambda$ about X_{r1} axis and then rotate $\pi/2-\theta_H$ about Z_{r1} axis so that the axial directions of the coordinates coincide with that of ground-coordinates (see Fig. 7). Then, the coordinates rotate δ_R about Y_g axial direction and then rotate $\pi/2$ about Z_g axial direction so that the axial directions of the coordinates coincide with that of the target-coordinates. The transformation matrices are given as follows,

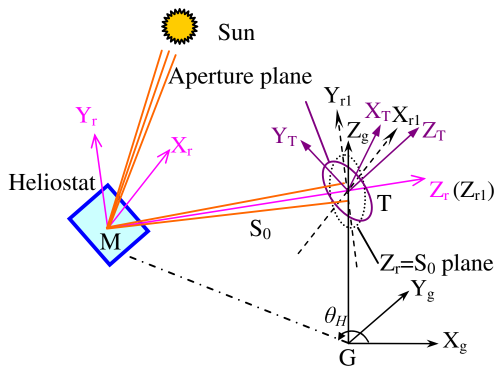

Fig. 7. Coordinate transformation from reflection auxiliary coordinates to target-coordinates.

$$M_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\lambda & -\sin\lambda \\ 0 & \sin\lambda & \cos\lambda \end{bmatrix}, \quad M_2 = \begin{bmatrix} \sin\theta_H & \cos\theta_H & 0 \\ -\cos\theta_H & \sin\theta_H & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$M_3 = \begin{bmatrix} \cos\delta_R & 0 & -\sin\delta_R \\ 0 & 1 & 0 \\ \sin\delta_R & 0 & \cos\delta_R \end{bmatrix}, \quad M_4 = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

So we have,

$$\begin{bmatrix} X_{t0} \\ Y_{t0} \\ Z_{t0} \end{bmatrix} = \begin{bmatrix} -X_r\cos\theta_H + Y_r\sin\theta_H\cos\lambda \\ -X_r\sin\theta_H\cos\delta_R - Y_r(\cos\theta_H\cos\delta_R\cos\lambda - \sin\delta_R\sin\lambda) \\ X_r\sin\theta_H\sin\delta_R + Y_r(\cos\theta_H\sin\delta_R\cos\lambda + \cos\delta_R\sin\lambda) \end{bmatrix} \tag{19}$$

$$\begin{aligned} \cos\alpha_{tr} &= -\cos\alpha_r\cos\theta_H + \cos\beta_r\sin\theta_H\cos\lambda - \cos\gamma_r\sin\theta_H\sin\lambda \\ \cos\beta_{tr} &= -\cos\alpha_r\sin\theta_H\cos\delta_R - \cos\beta_r(\cos\theta_H\cos\delta_R\cos\lambda - \sin\delta_R\sin\lambda) + \cos\gamma_r(\cos\theta_H\cos\delta_R\sin\lambda + \sin\delta_R\cos\lambda) \\ \cos\gamma_{tr} &= \cos\alpha_r\sin\theta_H\sin\delta_R + \cos\beta_r(\cos\theta_H\sin\delta_R\cos\lambda + \cos\delta_R\sin\lambda) - \cos\gamma_r(\cos\theta_H\sin\delta_R\sin\lambda - \cos\delta_R\cos\lambda) \end{aligned} \tag{20}$$

where δ_R is the tilt angle of the receiver aperture (see Fig. 5).

The coordinates of intersection point between the reflection ray and the receiver aperture plane $z_T = 0$ are derived as follows,

$$\begin{bmatrix} x_T \\ y_T \\ z_T \end{bmatrix} = \begin{bmatrix} -Z_{t0}\cos\alpha_{tr}/\cos\gamma_{tr} + X_{t0} \\ -Z_{t0}\cos\beta_{tr}/\cos\gamma_{tr} + Y_{t0} \\ 0 \end{bmatrix} \tag{21}$$

According to the equation (21), the coordinates of intersection points between all traced rays and the receiver aperture can be calculated.

4. Validation of the tracking and ray tracing equations

Based on the tracking and ray tracing equations, a new module for the analysis of the target-aligned heliostats with an asymmetric surface is developed and incorporated in the code HFLD. A target-aligned heliostat with a toroidal surface [11] is designed. The parameters of the target-aligned heliostat are given in Table 1.

Table 1
Parameters of the single heliostat.

Latitude	40.4°
Heliostat size	5 m × 5 m
Height of the pedestal	2.5 m
Location (x_g,y_g)	(−31.058 m, 0)
Facing angle	180°
Tracking method	Target-aligned tracking
Target distance	36.94 m
Tilt angle of target plane	30°

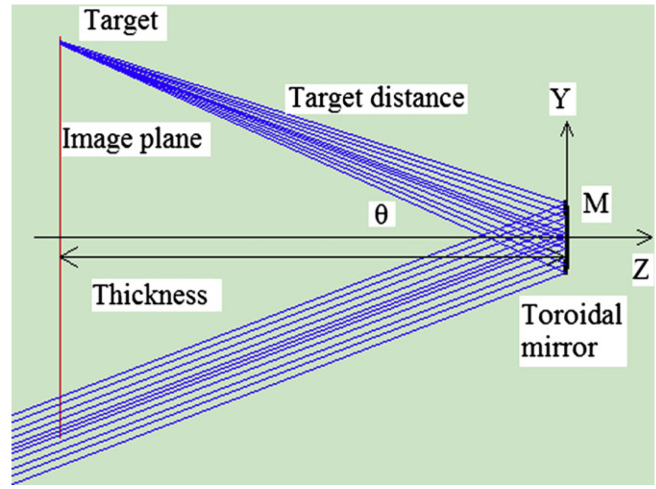

Fig. 8. Optical model of the target-aligned mirror with the toroidal surface in Zemax software.

According to the locations of the heliostat and the target on the earth, the range of incident angle θ relative to the heliostat center in a year can be calculated. Assuming the working hours of the heliostat are from solar time 8:00 a.m. through 4:00 p.m. every day, the range of incident angle θ is $0° \sim 35°$. The toroidal surface equation in the form $z = f(x,y)$ is given as follows,

$$z = R_s - \sqrt{\left[R_s - \frac{1 - \sqrt{1-(1+K)C^2y^2}}{(1+K)C}\right]^2 - x^2} \tag{22}$$

where $C = 1/R_t$, R_s and R_t are curvature radii along the meridian and sagittal direction, K is the cone factor.

The size and tilt angles of a mirror and the surface shape parameters including R_s, R_t and K can be input into Zemax so that the optical model of the target-aligned mirror with a toroidal surface can be easily created (see Fig. 8).

The field of view along the Y direction of the mirror equals to the incident angle θ and the field of view along the X direction equals to 0. The distance between the mirror center and the image plane equals to the product of the target distance and the cosine of the incident angle. As the incident angle changes, the corresponding tilt angles of the toroidal mirror about X, Y and Z axes are inputted in order to reflect light into a defined direction. An optimal toroidal surface for the target-aligned heliostat is designed by using Zemax. The parameters of the toroidal surface are given in Table 2.

Table 2
Parameters of the toroidal surface.

Range of incident angle	$0° \sim 35°$
Target distance	36.94 m
Curvature radius along meridian direction (R_t)	81.40 m
Cone factor (K)	−19.32
Curvature radius along sagittal direction (R_s)	68.00 m

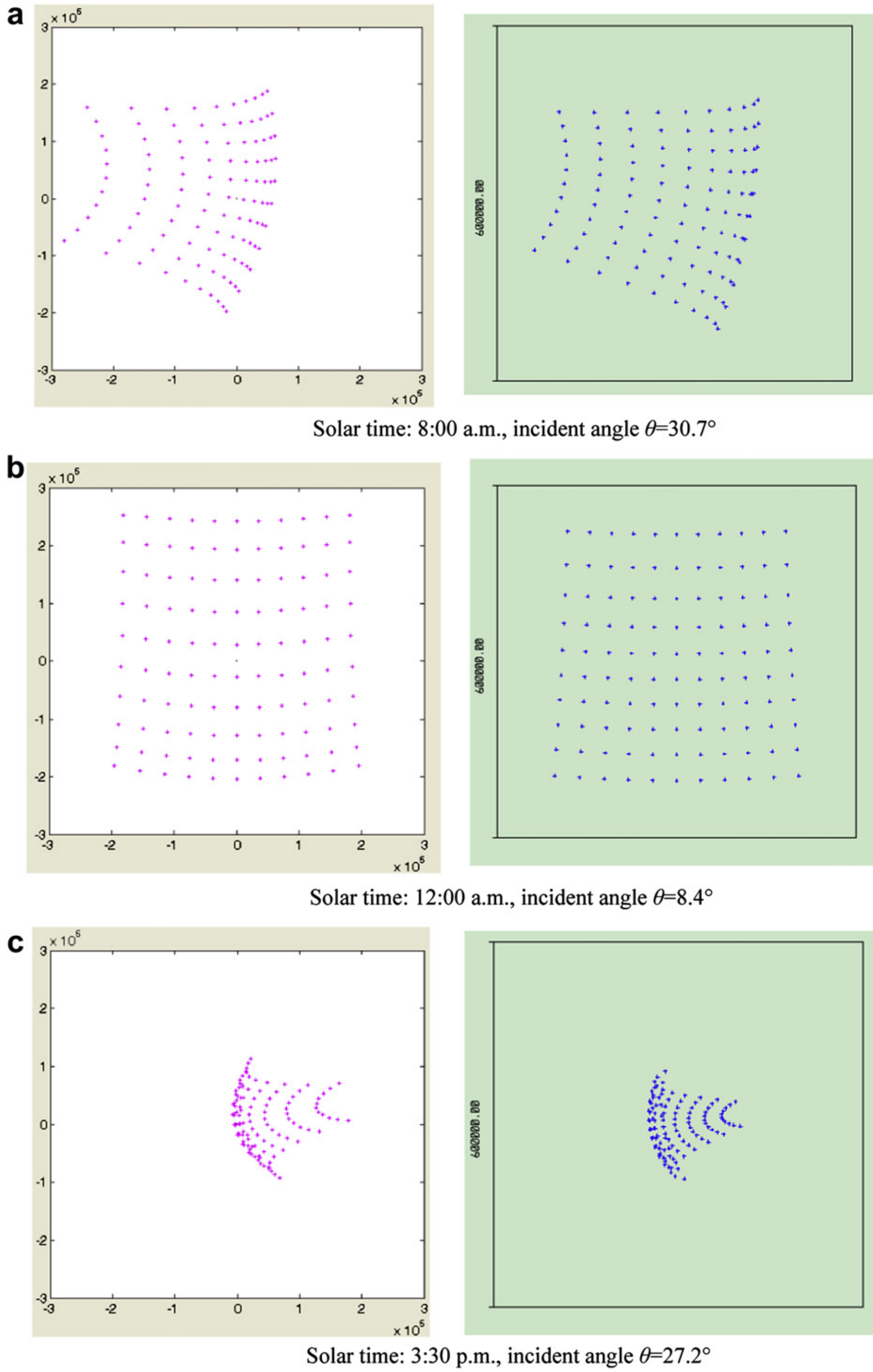

Fig. 9. The comparison of image for the target-aligned heliostat 15 with toroidal surface at different times on the vernal equinox, the dimensions shown are in micrometers, where left image is calculated by the HFLD and right image is calculated by Zemax.

According to the parameters of the heliostat as listed in Table 1 and Table 2, the target-aligned heliostat with the toroidal surface is modeled and the image of the heliostat is calculated by using the code HFLD and the software Zemax respectively. The solar shape is not considered in the calculation. The comparisons of the results are shown in Fig. 9. It can be seen that the results coincide with each other very well. Therefore, the correctness of the tracking and ray tracing equations for the target-aligned heliostat is proved.

5. Conclusions

In this work, the tracking and ray tracing equations for the target-aligned heliostat for solar tower power plants are derived and given explicitly. With the equations, a new module for the analysis of the target-aligned heliostat with asymmetric surface is incorporated in the code HFLD. To validate the correctness of the tracking and ray tracing equations, a target-aligned heliostat with a toroidal surface is designed and modeled. The image of the target-aligned heliostat is calculated by the code HFLD and the software Zemax respectively. The calculated results coincide with each other and thus the correctness of the equations is proved.

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