Synchronization of Chaotic Systems Modulated by Another Chaotic System in an Erbium-Doped Fiber Dual-Ring Laser System

Rong Wang and Ke Shen

***Abstract*—We show that two chaotic systems in erbium-doped fiber dual-ring lasers can be synchronized by using the output of one chaotic system (called the driving system) to modulate the parametrs of the two systems. Numerical calculation shows that when the stiffness of modulation is properly adjusted, the two systems can be synchronized. Simultaneously, we find that when the driving system is in various periodic states, the two synchronized systems can go into periodic states.**

***Index Terms*—Chaos, chaotic attractor, directional coupler, erbium-doped fiber dual-ring laser, power semiconductor diode, synchronization.**

I. Introduction

A CHAOTIC system is known to show a variety of oscillation patterns, even in a low-dimensional system. This characteristic of chaotic behavior is advantageous in applications in the fields of communications and neural networks if we can control a chaotic system and change its behavior into any desired nonperiodic or periodic behavior intrinsic to the system. In fact, there has been much research to demonstrate the change of chaotic behavior into periodic behavior [1]–[5] and the synchronization of a chaotic system with another chaotic system [6]–[10]. Recently, erbium-doped fiber lasers have received much attention in the optical communications, as their wavelength (about 1.5 μm) is in the third window of optical communications. The dynamics of the erbium-doped fiber lasers with various kinds of cavities have been investigated theoretically and experimentally [11]–[19]. By synchronizing the chaos of the laser, secure optical communictions was realized in both experiment [20], [21] and theory [22]. So far, the investigations of chaos synchronization in the erbium-doped fiber laser involve chaotic lasers with long and short cavities and modulating the loss parameter. They all use the method of master–slave synchronization.

In this paper, we use the output of a chaotic system as the driving signal to modulate the parameter of two chaotic systems and realize synchronization of the two systems

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R. Wang is with Living Quarters of the Jilin Provincial Committee, Luyuan District, Changchun 130062, China, and also with the Department of Optical Physics, Changchun Institute of Optics and Fine Mechanics, Changchun 130022, China (e-mail: wrzf@public.cc.jl.cn).

K. Shen is with the Department of Optical Physics, Changchun Institute of Optics and Fine Mechanics, Changchun 130022, China.

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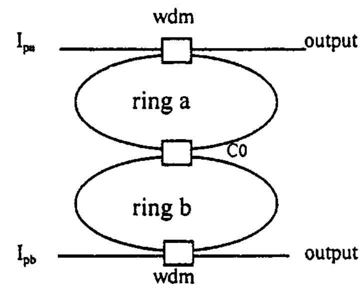

Fig. 1. Erbium-doped fiber dual-ring laser system. I_{pa}, I_{pb}: pump light. C0: coupler. WDM: wavelength division multiplexer.

II. A Scheme of Synchronization

The basic chaotic system is shown in Fig. 1 and consists of two coupled erbium-doped fiber ring lasers. This system has been shown to route to chaos [19], following a period-doubling route. To synchronize the two systems, we use the output of another chaotic system to modulate the pumping parameter of the two systems. The scheme is shown in Fig. 2.

In the scheme, S1 is the driving system, of which the output in ring a (as shown in Fig. 1) is the modulating signal; S2 and S3 are modulated systems. C is the controller, which controls the amplitude of the voltage. When the input of the controller is more than some value, it puts out a max voltage. Otherwise, it would put out some value proportional to its input. Hence, after some of the output coming from the driving system of ring a passes through the photodiode, it is turned into an electric signal. Then passing through the controller, it is put into the diode laser drivers of the two modulated systems of ring a. As a result, the pumping parameters of the two systems are modulated by the driving system. With proper modulating stiffness, the two modulated systems will synchronize.

Since the lasing fields in the two rings of one system are frenquency locked through the directional coupler C0 with a phase change of $\pi/2$ from one to the other [23], [24] we could express the dynamic equations [19] with real and imaginary parts. In this case, the dynamic equations of all the systems in our scheme can be expressed as follows:

$$\dot{E}_{1ar} = -\kappa_{1a}(E_{1ar} + \eta_0 E_{1bi}) + g_{1a}E_{1ar}D_{1a} \tag{1}$$
$$\dot{E}_{1ai} = -\kappa_{1a}(E_{1ai} - \eta_0 E_{1br}) + g_{1a}E_{1ai}D_{1a} \tag{2}$$
$$\dot{E}_{1br} = -\kappa_{1b}(E_{1br} + \eta_0 E_{1ai}) + g_{1b}E_{1br}D_{1b} \tag{3}$$
$$\dot{E}_{1bi} = -\kappa_{1b}(E_{1bi} - \eta_0 E_{1ar}) + g_{1b}E_{1bi}D_{1b} \tag{4}$$

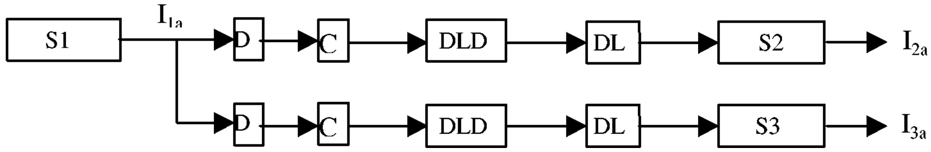

Fig. 2. Synchronization scheme: I_{2a} will synchronize to I_{3a}. S1, S2, S3: chaotic erbium-doped fiber dual-ring laser. C: controller D: photodiode. DLD: diode laser driver. DL: diode laser.

$$\dot{D}_{1a} = -(1 + I_{pa} + |E_{1a}|^2)D_{1a} + I_{pa} - 1 \tag{5}$$

$$\dot{D}_{1b} = -(1 + I_{pb} + |E_{1b}|^2)D_{1b} + I_{pb} - 1 \tag{6}$$

$$\dot{E}_{mar} = -\kappa_{ma}(E_{mar} + \eta_0 E_{mbi}) + g_{ma}E_{mar}D_{ma} \tag{7}$$

$$\dot{E}_{mai} = -\kappa_{ma}(E_{mai} - \eta_0 E_{mbr}) + g_{ma}E_{mai}D_{ma} \tag{8}$$

$$\dot{E}_{mbr} = -\kappa_{mb}(E_{mbr} + \eta_0 E_{mai}) + g_{mb}E_{mbr}D_{mb} \tag{9}$$

$$\dot{E}_{mbi} = -\kappa_{mb}(E_{mbi} - \eta_0 E_{mar}) + g_{mb}E_{mbi}D_{mb} \tag{10}$$

$$\begin{aligned}\dot{D}_{ma} = &-[1 + (I_{pa} + \Delta I_p) + |E_{ma}|^2]D_{ma} \\ &+ (I_{pa} + \Delta I_p) - 1\end{aligned} \tag{11}$$

$$\dot{D}_{mb} = -(1 + I_{pb} + |E_{mb}|^2)D_{mb} + I_{pb} - 1 \tag{12}$$

$$\Delta I_p = \begin{cases} \gamma|E_{1a}|^2, & \text{if } ,\gamma|E_{1a}|^2 \le 6I_{\text{path}} \\ 6I_{\text{path}}, & \text{otherwise} \end{cases} \tag{13}$$

where

$m = 2, 3$	two modulated systems S2, S3, respectively;
ΔI_p	change of the pump intensity of the modulated systems in ring a;
γ	modulating stiffness of the driving system
I_{path}	threshold of the pump parameters of the modulated systems in ring a
E_{1ar}, E_{1ai}	real and imaginary parts of the lasing field of S1 in ring a, respectively;
E_{1br}, E_{1bi}	real and imaginary parts of the lasing field of S1 in ring b, respectively;
E_{mar}, E_{mai}	real and imaginary parts of the lasing field of S2 or S3 in ring a, respectively
E_{mbr}, E_{mbi}	real and imaginary parts of the lasing field of S2 or S3 in ring b, respectively;
D_{1a}, D_{1b}	population inversions of S1 in ring a and b, respectively;
D_{ma}, D_{mb}	population inversions of S2 or S3 in ring a and b, respectively;
κ_{1a}, κ_{1b}	products of τ_2 and the decay rate of S1 in ring a and b, respectively;
κ_{ma}, κ_{mb}	products of τ_2 and the decay rate of S2 or S3 in ring a and b, respectively;
g_{1a}, g_{1b}	products of τ_2 and the gain coefficient of S1 in ring a and b, respectively;
g_{ma}, g_{mb}	products of τ_2 and the gain coefficient of S2 or S3 in ring a and b, respectively;
τ_2	lifetime of the population of the lasing upper level in the erbium;
I_{pa}, I_{pb}	pump intensities in the respective fiber ring laser;
η_0	coupling coefficient of the coupler C0.

$|E_{1a}|^2, |E_{1b}|^2, |E_{ma}|^2$, and $|E_{mb}|^2$ are the lasing intensities $(I_{1a}, I_{1b}, I_{ma}, I_{mb})$ of S1, S2, or S3, respectively. The time is normalized to the lifetime of the lasing upper level.

III. Numerical Simulation

To eclucidate the synchronization of chaos in the scheme, on the one hand, we consider the case that the driving system and the two modulated systems are identical and all in chaos. In the numerical simulation, the parameters used are [19]: $\kappa_{1a}, \kappa_{1b}, \kappa_{ma}, \kappa_{mb} = 1000$; $g_{1a}, g_{ma} = 10\,500$; $g_{1b}, g_{mb} = 4800$; $\eta_0 = 0.2$; $I_{pa}, I_{pb} = 4$; $\tau_2 = 10$ ms. The numerical result shows that the two modulated systems can synchronize as the modulating stiffness γ increases beyond 0.031. It is remarkable that the temporal evolutions of the output intensities in modulated systems are identical but different from the driving system. This is shown in Fig. 3(a). The regime of synchronized chaos between the modulated systems or nonsynchronization between the driving and modulated systems can be seen [Fig. 3(b) and (c)] in the projection of the flow onto I_{2a}-I_{3a} and I_{2a}-I_{1a}. At the same time, the intensities I_{2b} and I_{3b} could synchronize to each other while I_{2b} (or I_{3b}) and I_{1b} could not synchronize. In the I_{2b}–I_{3b} plane, it is clear that the motion is along a strange attractor [Fig. 3(d)]. Fig. 3 is shown at the modulating stiffness $\gamma = 0.06$.

On the other hand, we investigate the synchronization under the conditions that the driving and modulated systems are in different states and that the modulated sytems are still in chaos. Here, we make the driving system in different states by changing the gain coefficient of the driving system in ring b. Similarly, we find that the modulated systems can synchronize each other with proper modulating stiffness while the driving system still cannot synchronize to the modulated systems. Moreover, it is remarkable that the synchronized modulated systems are in different states as the driving system is in different state, as shown in Fig. 4, in which $\gamma = 0.06$. Fig. 4(a) shows that the state of the driving system changes via the gain coefficient of ring b. Fig. 4(b) shows that the states of the synchronized modulated systems change accordingly.

Compared with the method of mutual coupling, the method of unidirectional driving has the advantage that it may realize the synchronization of several systems. This is significant in realizing secure communications by chaos synchronization. Here, we present it as an example. In this case, there are three modulated systems besides the driving system. In our simulation, the parameters used are the same as above. The numerical results show that the three modulated systems can synchronize with proper modulating stiffness. Under these conditions, the regimes of synchronization or nonsynchronization are the same as is the case with two modulated systems. This demonstrates that we can make several systems synchronized by this method. Simultaneously, the synchronized systems can go into periodic or chaotic states as the driving state changes. This result has significant practical applications.

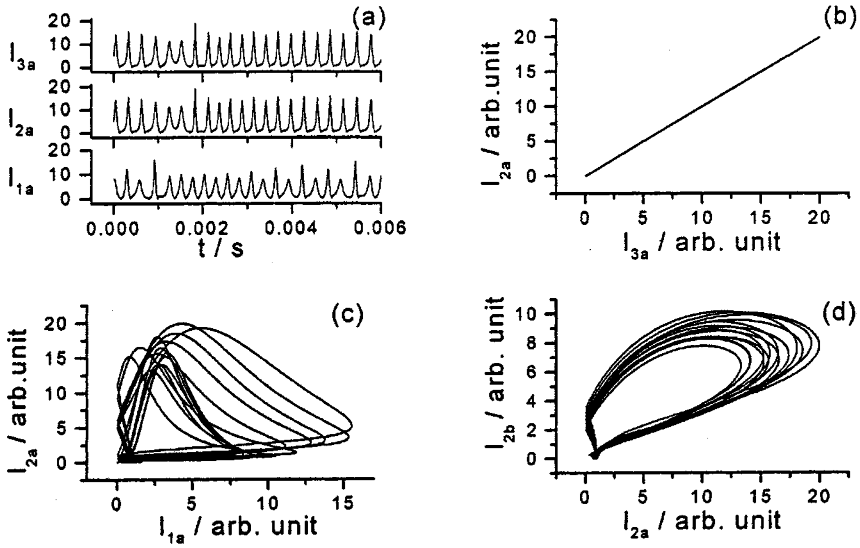

Fig. 3. Synchronized chaos for $\gamma = 0.06$. (a) Time series for the intensities I_{1a}, I_{2a}, and I_{3a}. (b) Projection of the flow onto the I_{2a}-I_{3a} plane. (c) Projection of the flow onto the I_{2a}-I_{1a} plane. (d) The strange attractor in the I_{2a}-I_{2b} plane.

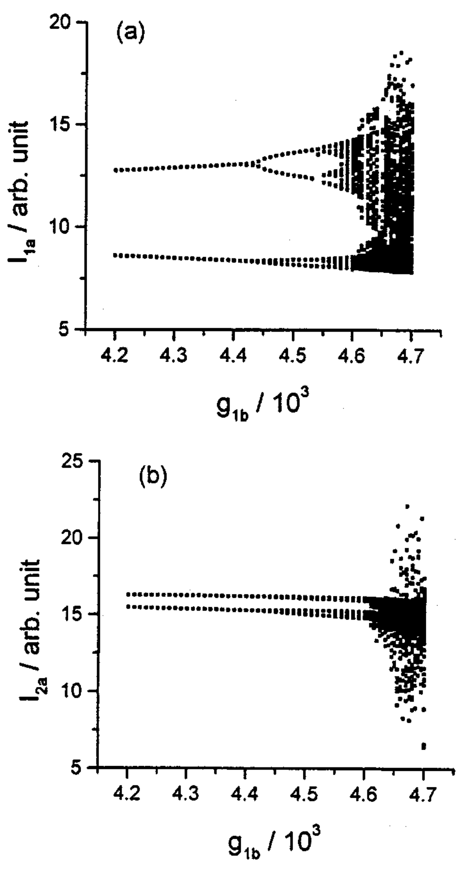

Fig. 4. Bifurcation diagram. (a) I_{2a}-g_{1b}. (b) I_{2a}-g_{1b}.

IV. Conclusion

We have demonstrated a new method of chaostic synchronization by chaotic modulation. This method has the following advantages: 1) it may make several systems synchronized and 2) by changing the state of the driving system, we could obtain periodic or chaotic synchronized systems. With this method, we present a scheme of chaotic synchronization in erbium-doped fiber dual-ring lasers. Numerical results have verified these conclusions. At the same time, we give the bifurcation diagram of the synchronized systems via the driving system. These results are significant in practice.

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Rong Wang received the Bachelors degree in 1989 and the Masters degree in physics in 1997, both from Jilin University of Technology, Changchun, China. She is currently working toward the Doctorate degree in optical engineering at Changchun Institute of Optics and Fine Mechanics, Changchun, China.

She worked in the Physics Laboratory, Jilin University of Technology, before beginning her doctoral studies. Her reseach intrests include erbium-doped fiber lasers and chaos.

Ke Shen graduated with a degree in physics from Jilin University, Changchun, China, in 1958.

He is currently a Professor and Tutor for students who are advancing to the doctoral degree in Changchun Institute of Optics and Fine Mechanics, Changchun, China. He has been involved in the research of laser physics and quantum optics and has published books and papers on these topics. Currently, his main research involves the theory of chaos generation, control, synchronization, and its application.