Birefringent filter with tilted optic axis for tuning dye lasers: theory and design

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The dependence of the tuning behavior of a birefringent filter on the orientation of the optic axis to the surface of the plate is studied. The Jones matrix representing a tilted birefringent plate is reduced to practical form, and this leads to simplifying the theoretical operation of tuning a birefringent filter in a laser resonator. The principles of proper determination of the filter parameters are given. A design method, which takes into consideration the bandwidth, rejection and tuning range of the filter, and design examples, is provided.

I. Introduction

Lyot first published the basic operation principles of the birefringent filter based on the interference of polarized light having passed through a birefringent filter.[1] Due to the need for tuning devices that can be used with cw tunable lasers, such as a dye laser, in which the tuning element does not employ spatial dispersion as the selection mechanism, Yarborough and Hobart suggested and successfully demonstrated that tilted birefringent plates can be used as highly tunable and narrowband selection devices.[2] Bloom studied the modes of a laser resonator containing this type of filter.[3] Over the past few years the tilted birefringent filter has proved an effective tuning device in tunable lasers. It is desirable to have a filter that tracks properly over several thousand angstroms, so that it could be used as a universal tuning element in any existing dye laser. For this to be accomplished, it is important to study the influence of crystal orientation on the performance of the tuning birefringent filter. In fact, the performance of a filter with a properly tilted optic axis at an angle e to the surface of the plate is considerably superior to that of a filter with the optic axis in the surface of the plate. Holton and Teschke have enumerated advantages of the tilted optic axis[4]; however, they do not point out the way to choose the tilted optic axis.

As we know, if the birefringent plates used as tuning elements within the resonator of a dye laser are tilted so that the angle of incidence is the Brewster angle, these plates act both as retarding and polarizing elements. Tuning is accomplished by rotating the plates as a whole about an axis perpendicular to the surface; such rotation which changes the rotation angle ϕ directly also changes the polar angle η and the azimuth α (see Fig. 1). The change of η affects the peak transmission wavelength of the filter, and the change of α has influence on the transmission bandwidth and the rejection of unwanted wavelengths. Therefore it is important to know the dependence among angles e, η, α, and ϕ to design a tuning birefringent filter with optimum performance. Here optimum performance refers mainly to maximum tuning range, narrow transmission bandwidth, and maximum rejection of unwanted wavelengths.

In this paper the influence of crystal orientation on the performance of the tuning birefringent filter is studied on the basis of a given relationship among these angles. The Jones matrix representing a tilted birefringent plate is reduced to practical form, and this leads to considerably simplifying the theoretical operation of a tuning birefringent filter in a laser resonator. The principles of proper determination of the filter parameters are given. A design method, which takes into consideration the bandwidth, rejection and tuning range of the filter, and design examples, is provided.

II. Tilted Birefringent Filter

A tilted birefringent plate which is used as a tuning filter for dye lasers is illustrated in Fig. 1. The plate is assumed to be cut with its optic axis at a tilt angle e to the surface of the plate. The plate is tilted so that the angle of incidence is the Brewster angle θ. The refracted ray in the crystal travels at a polar angle η with respect to the optic axis. The azimuth α is the angle between the plane of incidence and the plane containing the optic axis and the refracted ray. The rotation

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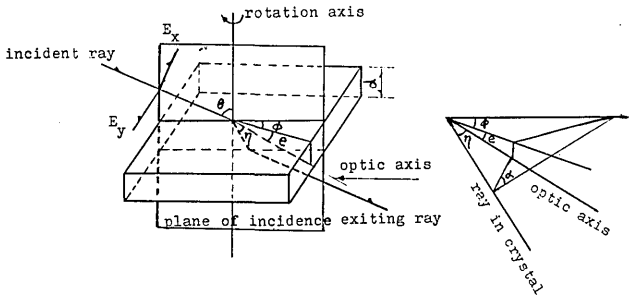

Fig. 1. Tilted birefringent filter.

angle ϕ is the angle between the plane of incidence and the plane containing the optic axis and the rotation axis perpendicular to the surface of the plate.

When a birefringent plate is rotated about an axis perpendicular to the surface, the change of the rotation angle ϕ gives rise to the changes of the polar angle η and the azimuth α, and these angles are related by following equations:

$$\cos\eta = \cos\phi \cos\theta \cos e + \sin\theta \sin e, \tag{1}$$

$$\cos\alpha = \frac{1}{\sin\eta \sin\theta} (\cos\phi \cos e - \cos\eta \cos\theta), \tag{2}$$

$$\tan\phi = \frac{\sin\alpha}{\cos\alpha \sin\theta + \cot\eta \cos\theta}, \tag{3}$$

where the value of the tilt angle e may be calculated by

$$\sin e = \cos\eta \sin\theta - \sin\eta \cos\theta \cos\alpha. \tag{4}$$

When $e = 0°$, i.e., the optic axis lies on the surface of the plate, Eqs. (1) and (2) may be reduced to

$$\cos\eta = \cos\phi \cos\theta, \tag{1a}$$

$$\tan\phi = \tan\alpha \sin\theta. \tag{2a}$$

These are the relations given in Ref. 5.

The changes which occur when light passes through a birefringent plate can be described mathematically with the aid of the Jones matrix[6] for the plate. This 2 × 2 matrix transforms the x and y components of the electric field E from one side of the plate to the other. In a rectangular coordinate system, we choose the x and y axes in the plane perpendicular to the laser beam and define the x axis to be the axis of zero loss polarization of the Brewster surface. Thus, a birefringent plate at the Brewster angle is characterized by the Jones matrix:

$$M_b = \begin{Bmatrix} \cos^2\alpha + \sin^2\alpha \exp(-i2\delta) & q \sin\alpha \cos\alpha[1 - \exp(-i2\delta)] \\ q \sin\alpha \cos\alpha[1 - \exp(-i2\delta)] & q^2[\sin^2\alpha + \cos^2\alpha \exp(-i2\delta)] \end{Bmatrix}, \tag{5}$$

where $q = 2n/(1 + n^2)$, n is the refractive index of the Brewster interface, and -2δ is the phase retardation of the birefringent plate, i.e., the phase difference between two waves propagating as ordinary and extraordinary waves. To a good approximation, this retardation is given by[7]

$$-2\delta = \frac{2\pi(n_e - n_o)d \sin^2\eta}{\lambda \sin\theta}, \tag{6}$$

where $(n_e - n_o)$ is the difference between the extraordinary and ordinary refractive index, d is the plate thickness, and λ is the wavelength.

Because δ is a function of wavelength, such a device could act as a wavelength selector for a tunable laser. In fact, at every δ for which the retardation is a full wave, the filter will have a transmission maximum. In other words, the filter transmittance is unity for TM waves of wavelength

$$\lambda = \frac{(n_e - n_o)d \sin^2\eta}{m \sin\theta}, \tag{7}$$

where m is an integer. For these waves the birefringent plate acts as a full waveplate, and the polarization is TM at two interfaces. All other waves suffer loss at the Brewster surfaces and in general the polarization changes as the waves pass through the plate. Hence the wavelength selection comes about. Tunability is achieved by rotating the plate about the plate normal because this changes polar angle η and peak transmission wavelength λ.

The special case in which the azimuth $\alpha = 45°$ is of interest. In the transmittance spectrum of the tilted birefringent filter for this value of α the rejection of unwanted wavelengths is maximum. So, for convenience sake, let us assume a general $\alpha = 45° + \Delta\alpha$; thus we can rewrite Eq. (5) as

$$M_b = \exp(-i\delta) \begin{bmatrix} \cos\delta - i \sin2\Delta\alpha \sin\delta & iq \cos2\Delta\alpha \sin\delta \\ iq \cos2\Delta\alpha \sin\delta & q^2(\cos\delta + i \sin2\Delta\alpha \sin\delta) \end{bmatrix}.$$

The phase factor $\exp(-i\delta)$ may be omitted because it has unit modulus. Hence the Jones matrix for a birefringent plate at the Brewster angle may be rewritten as

$$M_b = \begin{bmatrix} \cos\delta - i \sin2\Delta\alpha \sin\delta & iq \cos2\Delta\alpha \sin\delta \\ iq \cos2\Delta\alpha \sin\delta & q^2(\cos\delta + i \sin2\Delta\alpha \sin\delta) \end{bmatrix}. \tag{5a}$$

In such a form the Jones matrix will be convenient for solving the eigenequation and make the physical significance of the obtained result even clearer (see below).

When $\alpha = 45°$, i.e., $\Delta\alpha = 0°$, the Jones matrix for a birefringent plate at the Brewster angle is

$$M_b = \begin{pmatrix} \cos\delta & iq \sin\delta \\ iq \sin\delta & q^2 \cos\delta \end{pmatrix}. \tag{5b}$$

The Jones matrix for a glass plate set at the Brewster angle is

$$M_g = \begin{pmatrix} 1 & 0 \\ 0 & q^2 \end{pmatrix}.$$

When the birefringent filter forms part of a laser cavity, the eigenmodes of the resonator by definition have polarizations that are unchanged after one round trip. The laser beam passes through the filter once per round trip for a ring resonator and twice for a Fabry-Perot resonator. To follow the changes in amplitude

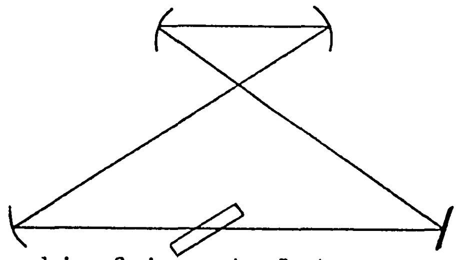

Fig. 2. Ring resonator containing a single birefringent plate tilted at the Brewster angle.

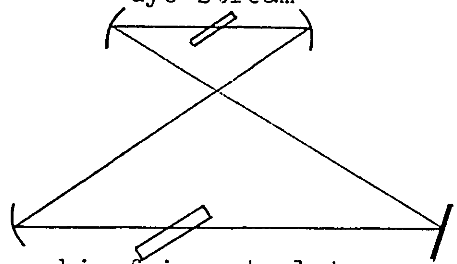

Fig. 3. Ring resonator containing a single birefringent plate plus the dye stream tilted at the Brewster angle.

and polarization that occur each round trip in the cavity, matrix M is formed by taking the product of the Jones matrices of all the elements of the filter in the order in which the beam passes through the elements; for a Fabry-Perot cavity all the elements appear twice. In terms of the matrix the condition for an eigenmode is

$$\mathrm{ME} = t\mathrm{E}, \tag{8}$$

where t is the eigenvalue and usually a complex number. In general the eigenequation (8) yields two solutions for t: the larger of the two values of $|t|^2$ is taken as transmittance T of the filter for a ring resonator. In a Fabry-Perot cavity the laser beam passes through the filter twice per round trip, and in this case we use the larger of the two values of $|t|$ as the effective single-pass transmittance T of the filter.

The simplest situation to analyze is that of a tilted birefringent plate at the Brewster angle in a ring resonator, as shown in Fig. 2. In this case, as in all other resonators, the significance of the eigenfunctions is that they represent the polarizations that are unchanged after one passage through the plate; therefore, by definition, they have to be the polarizations of the modes of the resonator. The secular equation for matrix (5a) can be solved to give the eigenvalues of Eq. (8):

$$t = \frac{-(x+iy) \pm \sqrt{(x+iy)^2 - 4q^2}}{2}, \tag{9}$$

where

$$\begin{aligned} x &= -(1+q^2)\cos\delta, \\ y &= (1-q^2)\sin 2\Delta\alpha \sin\delta. \end{aligned} \tag{10}$$

For $\Delta\alpha = 0°$, i.e., $\alpha = 45°$, we have

$$t = \frac{(1+q^2)\cos\delta \pm \sqrt{(1+q^2)^2\cos^2\delta - 4q^2}}{2}, \tag{9a}$$

and when

$$\sin\delta \geq \sin\delta_c = \frac{1-q^2}{1+q^2}, \tag{11}$$

there is only one transmittance, constant at $|t|^2 = q^2$.

For a ring resonator containing a single birefringent plate plus a dye stream at the Brewster angle, as shown in Fig. 3, we have matrix $\mathrm{M} = \mathrm{M}_b\mathrm{M}_g$. We may easily obtain the corresponding results for this case as long as all the q^2 terms in Eqs. (9)–(11) are replaced by q^4. Here we include the dye stream to provide an additional pair of Brewster surfaces having, for simplicity, the same refractive index n as the birefringent plate.

III. Determination of the Filter Parameters

The tuning range of the filter should be at least larger than the bandwidth of the gain curve of a dye medium. This allows the lasing wavelength to be tuned to one edge of the gain curve while preventing lasing from starting at the other side of the gain curve. To narrow the bandwidth of the main transmission peaks, in general we may take the tuning range of the filter equal to the free spectral range $\Delta\lambda$ of the filter.

As the filter is tuned, the form of the transmittance spectrum changes but the main transmission peaks remain equal to unity. This implies changes in the bandwidth and rejection of unwanted wavelengths. For $\alpha = 45°$, the transmission bandwidth is minimum and the rejection of unwanted wavelengths is maximum. The corresponding value of ϕ, which may be calculated by Eq. (3) for the proper value of η, should correspond to the central wavenumber of the desired tuning range. Let λ_1 and λ_2 be the edge wavelengths of the tuning range. If we make the central wavenumber, i.e., the wavelength $\lambda_0 = 2\lambda_1\lambda_2/(\lambda_1 + \lambda_2)$, correspond to $\alpha_0 = 45°$, the same rejection can be nearly obtained when the filter is tuned to λ_1 and λ_2, i.e., the corresponding α_1 and α_2 are nearly symmetrical about $\alpha_0 = 45°$. To ensure high rejection and narrow bandwidth of the filter, deviation of α from $\alpha_0 = 45°$ must be as small as possible over the whole tuning range.

When neglecting wavelength dispersion of the refractive index n, the free spectral range may be represented by

$$\Delta\lambda = \frac{\lambda^2 \sin\theta}{(n_e - n_o)d\sin^2\eta}. \tag{12}$$

Obviously, the size of $\Delta\lambda$ relates to plate thickness d and polar angle η. While from Eq. (7) we know that the peak transmission wavelengths relate also to the integer m in addition to d and η, in what follows we discuss how to choose m and η and how to determine d and e for a tuning filter.

Polar angle η should satisfy simultaneously the requirements of the free spectral range and the peak transmission wavelengths; hence from Eqs. (7) and (12) we have

$$\Delta\lambda = \lambda/m. \tag{13}$$

Thus, for the required free spectral range $\Delta\lambda$, we may choose the integer

$$m = \lambda/\Delta\lambda. \tag{13a}$$

When $\lambda/\Delta\lambda$ is not an integer, we may take the larger or less integer than $\lambda/\Delta\lambda$ as m according to the practical requirement, so that the free spectral range obtained will be slightly less than or larger than the required.

From the derivative of Eq. (7) expressing the peak transmission wavelengths with respect to the polar angle

$$\frac{d\lambda}{d\eta} = \frac{(n_e - n_o)d \sin 2\eta}{m \sin\theta},$$

we know that, when $\eta = 45°$, the rate of change of λ with η is maximum. Hence, to obtain sensitive tuning we make $\eta_0 = 45°$ correspond to the central wavenumber of the tuning range.

Once m and η corresponding to the central wavenumber are chosen, we can determine the plate thickness with the relation obtained from Eq. (7):

$$d = \frac{m\lambda \sin\theta}{(n_e - n_o)\sin^2\eta}. \tag{7a}$$

Then using Eq. (12) we find the corresponding free spectral range.

For optimum performance of the filter, we must properly choose tilt angle e of the optic axis to the surface of the plate. It is important to select the proper e because it is closely related to the tuning range, transmission bandwidth, and rejection of the filter. To do this, we make $\alpha_0 = 45°$ and $\eta_0 = 45°$ correspond to the central wavenumber, i.e., the wavelength λ_0, then using Eq. (4) determines the proper tilt angle e.

After tilt angle e and plate thickness d are chosen, basic construction parameters of a simple tuning birefringent filter are determined. To analyze the performance of the tuning filter, we first find out the value of ϕ_0 corresponding to the central wavenumber by Eq. (3) for the chosen α_0 and η_0. Then, for every given value of ϕ around ϕ_0, using Eq. (1) we ascertain the corresponding value of η, and using Eq. (2) we calculate the corresponding value of α. The peak transition wavelengths may be obtained from Eq. (7). Substituting the obtained values of η and α corresponding to the given value of ϕ in the expression of eigenvalues of Eq. (8), i.e., in Eq. (9), gives the transmittance spectrum of the simple birefringent filter.

In practice, when a birefringent filter is used as the tuning device for dye lasers, the tuning filter often consists of several birefringent plates of different thickness in integer ratios to narrow the spectral width of the laser output. These plates are all tilted so that their surfaces are at the Brewster angle to the laser beam and so that their optic axes are all oriented in the same way, i.e., their angles e are the same and their optic axes and hence their surfaces are parallel. The free spectral range of the combination birefringent filter equals that of the thinnest plate; the bandwidth of the transmission peaks is, however, mainly determined by the thickest plate. The best thickness for the thickest plate depends on the allowable transmission near the main passband.

This type of filter is a variation of the Lyot filter, which has separate retarders and polarizers. Incomplete polarized action of the Brewster surfaces of the birefringent plates leads to secondary peaks occurring between adjacent main peaks. For a high gain laser the secondary peaks of the filter must be suppressed considerably to prevent the lasing wavelength jumping from a main peak to a secondary peak as the filter is tuned. This can usually be achieved by adding glass plates,[4] but this increases the bandwidth and in practice adds to the overall losses of the system through scattering; the number of glass plates should be kept to a minimum. Obviously, it is just as important to choose the proper construction parameters e and d of each plate constituting a combination filter to achieve optimization of the performance of the filter in the larger tuning range.

IV. Design Examples

We first consider a single birefringent plate set at the Brewster angle in a ring resonator, as shown in Fig. 2. Here, we want to design a simple tuning birefringent filter with a crystalline quartz plate. Let $\lambda_1 = 0.55\ \mu$m, $\lambda_2 = 0.65\ \mu$m be the edge wavelengths of the tuning range. Then the required free spectral range is $\Delta\lambda = \lambda_2 - \lambda_1 = 0.1\ \mu$m, and the wavelength corresponding to the central wavenumber is $\lambda_0 = 2\lambda_1\lambda_2/(\lambda_1 + \lambda_2) = 0.59583\ \mu$m. If we take approximately $n_o = 1.5443$ and $n_e = 1.5534$, the average value of the refractive indices is $n = (n_o + n_e)/2 = 1.54885$, $q = \sim 2n/(1 + n^2)$–0.911, and the Brewster angle $\theta = \tan^{-1} n = 57°9'$. For comparison sake, we give the design results of two possible orientations of the optic axis of birefringent plates: one is a birefringent filter with tilted optic axis ($e \neq 0°$) and the other is that with an optic axis ($e = 0°$) parallel to the surface of the plate.

A. Birefringent Filter with Tilted Optic Axis

First we determine the plate thickness d. Corresponding to the central wavenumber of the tuning range, from Eq. (13a) we have $m = \lambda_0/\Delta\lambda \doteq 6$, and choosing $\eta_0 = 45°$ using Eq. (7a) gives $d = \sim 660.08$–660 μm.

Next we determine tilt angle e of the optic axis with respect to the surface of the plate. Choosing $\alpha_0 = 45°$ and $\eta_0 = 45°$ corresponding to the central wavenumber, using Eq. (4) we may obtain $e = \sim 18.835$–$18°50'$.

We then use Eq. (3) to find out the value of ϕ_0 corresponding to $\alpha_0 = 45°$ and $\eta_0 = 45°$ and Eq. (7) to ascertain the values of η_1 and η_2 corresponding to the edge wavelengths of the tuning range λ_1 and λ_2, respectively. Then the values of ϕ_1 and ϕ_2 corresponding to η_1 and η_2 are obtained with Eq. (1). Finally, the corre-

Table I. Birefringent Filter with $e = 18°50'$ and $d = 660\ \mu m$

$\lambda(\mu m)$	η (deg)	ϕ (deg)	α (deg)	$\Delta\alpha$ (deg)
0.55	42.794	25.702	37.171	−7.829
0.59583	45	31.890	45	0
0.65	47.608	38.283	52.556	7.556

Table II. Birefringent Filter with $e = 0°$ and $d = 399\ \mu m$

$\lambda(\mu m)$	η (deg)	ϕ (deg)	α (deg)	$\Delta\alpha$ (deg)
0.55	60.925	26.374	30.549	−14.451
0.59583	65.461	40.034	45	0
0.65	71.828	54.902	59.443	14.443

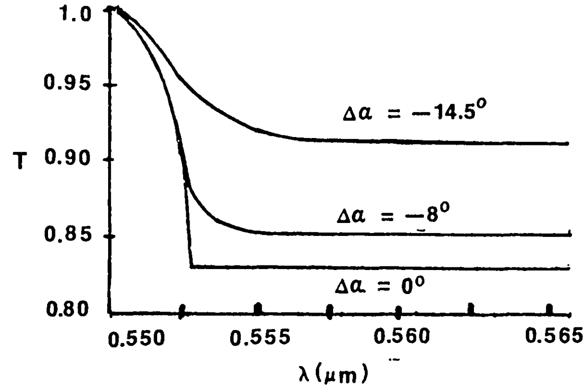

Fig. 4. Transmittance for the eigenmode of the resonator in Fig. 2 for three different filter designs.

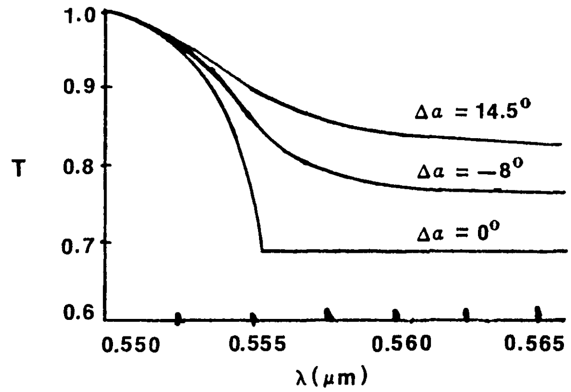

Fig. 5. Transmittance for the eigenmode of the resonator in Fig. 3 for three different filter designs.

sponding values of α_1 and α_2 are obtained with Eq. (2). The results obtained are given in Table I, from which one may see that the values of α_1 and α_2 corresponding to edge wavelengths λ_1 and λ_2 about 45° are nearly symmetrical by reason of choosing $\alpha_0 = 45°$ corresponding to the central wavenumber, i.e., λ_0.

When the filter is tuned to the edge wavelengths, rejection of the unwanted wavelengths is worse. Curve $\Delta\alpha = -8°$ in Fig. 4 shows the transmittance spectrum of the birefringent filter with tilted optic axis ($e = 18°50'$, $d = 660\ \mu m$) for tuning at the short edge $\lambda_1 = 0.55\ \mu m$. In this case the transmittance at λ_0 is T = 0.85.

B. Birefringent Filter with a Parallel Optic Axis

In this configuration the optic axis is required to lie in the plane of the plate ($e = 0°$). Using the same method as above we take $m = 6$ corresponding to λ_0. Then, choosing $\alpha_0 = 45°$ corresponding to λ_0, from Eq. (2a) we have $\phi_0 = 40.034°$, from Eq. (1a) we have $\eta_0 = 65.461°$, and from Eq. (7a) we obtain $d = 398.84$–$399\ \mu m$. Using Eqs. (7), (1a), and (2a) gives the values of η, ϕ, and α corresponding to λ_1 and λ_2; the results obtained are presented in Table II. Curve $\Delta\alpha = -14.5°$ in Fig. 4 shows the transmittance spectrum at the short edge $\lambda_1 = 0.55\ \mu m$ of the tuning range for the design with $e = 0°$ (upper curve). In this case the transmittance at λ_0 is $T = 0.91$. For comparison, the transmittance spectrum at $\lambda_1 = 0.55\ \mu m$ as the center of the tuning range, i.e., for the design with $\alpha_1 = 45°$ corresponding to $\lambda_1 = 0.55\ \mu m$, is also shown in Fig. 4 (lower curve). In this case the transmittance at λ_0 is $T = 0.83$.

Figure 5 shows the transmittance spectra for a ring resonator containing a single birefringent plate plus a dye stream at the Brewster angle (see Fig. 3). Curves $\Delta\alpha = 0°$, $-8°$, and $-14.5°$ show transmittance spectra at $\lambda_1 = 0.55\ \mu m$ as the center of the tuning range (lower curve), as one edge of the tuning range for the design with $e = 18°50'$ (middle curve), and as one edge of the tuning range for the design with $e = 0°$ (upper curve), respectively. Obviously, adding additional Brewster surfaces increases the s-plarization losses. As a result, transmittance T in region $\delta \geq \delta_c$ is decreased and the bandwidth is increased, all in accord with the analysis of the simple case.

V. Discussion

By comparison of the results in Tables I and II, as will be readily seen, being tuned to the edge wavelengths the values of $|\Delta\alpha|$ for the design with $e = 18°50'$ is <8°, while the corresponding $|\Delta\alpha|$ for the design with $e = 0°$ is ~14.5°. To tune across the entire peak separation, the rotation angle ϕ changes only 13° for $e = 18°50'$, while the change of ϕ is larger than 28° for $e = 0°$.

It can be seen that, from the transmittance spectra tuned to the short edge λ_1 for the three different values of $\Delta\alpha$ shown in Fig. 4, at the central wavenumber (i.e., λ_0) the transmittance of the filter with $e = 18°50'$ and $\Delta\alpha = -8°$ differs from that of the filter with $\Delta\alpha = 0°$ by 0.02, but the transmittance of the filter with $e = 0°$ and $\Delta\alpha = -14.5°$ differs from that of the filter with $\Delta\alpha = 0°$ by 0.08, the former is only one-fourth of the latter. As shown, the rejection of the filter with a tilted optic axis is considerably superior to that of the filter with a parallel optic axis, so that the former will give a larger tuning range. Obviously, this is important in practice.

VI. Summary

Typical combination birefringent filters containing three plates with thicknesses in the different integer ratios, for example, 1:4:16, 1:2:9, etc., have been used as tuning devices for dye lasers. Use of the present design method of the filter with tilted optic axis to design combination filters will give optimum performance. In addition, Hodgkinson and Vukusic theoretically discussed the properties of the periodic birefringent filter constructed from a number of identical, similarly oriented plates and gave an example for $e = 25°$.[8] It should be pointed out that we will obtain some new performances of the filters if we use $e = 18°50'$ but not 25° in the examples of Refs. 4 and 8. Here, these problems are not discussed in detail.

I am indebted to the reviewers for helpful suggestions.

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