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# Competition between two resonance frequency branches in a waveguide free-electron laser

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One of the problems encountered in designing a waveguide free-electron laser (FEL) is to suppress the lower-frequency branch, which may have a higher gain than the more useful higher-frequency branch. It is shown that the electron pulse length and energy spread strongly influence the competition between the gains at high and low frequency for a waveguide FEL. It is also found that the negative slippage that may take place at the lower-frequency branch results in larger reduction in gain than the positive slippage. The maximum allowable electron energy spread for both resonance frequencies is derived. © 1995 American Institute of Physics.

Free-electron lasers (FELs) in the millimeter and far-infrared wave regions usually employ waveguides to confine the electromagnetic wave to obtain a good transverse overlap with electron beams. The introduction of a waveguide can allow synchronism between the electron beam and the electromagnetic wave to occur at two different frequencies.<sup>1-3</sup> Then the competition between the higher- and lower-frequency branches will take place. Since the generation of a high-frequency output is usually preferred, it is necessary to study this competition carefully.

In this communication we first show that the negative slippage between the electron beam and the lower-frequency radiation whose group velocity is less than the axial velocity of the electrons leads to much larger decrease in gain than the positive slippage between the electron beam and the higher-frequency radiation. In the waveguide FELs with short electron pulses such as that produced by rf accelerators, the gain at the lower frequency is small. In fact, even the positive slippage results in so much reduction in the gain of the higher frequency that it becomes an obstacle to operate FELs.<sup>1,3,4</sup> In the case of waveguide FELs using long electron pulses that produced by electrostatic accelerators such as a RIKEN submillimeter FEL, the gain at lower frequency is still considerably large. It is necessary to control some factors to suppress the development of lower-frequency oscillation. We find that the electron energy spread results in larger decrease in gain at the higher frequency than the lower frequency. By controlling the electron energy spread at less than half or one-third of the allowable value in FELs, a high gain at the higher frequency can be obtained while the gain at the lower frequency remains almost constant. A big gap between the gains at two frequencies is made. The development of the lower-frequency branch can be suppressed by means such as the loss of optical wave in waveguide wall.

If the transverse cross section of the electron beam is small compared with the waveguide dimensions, only the peak electric field is seen by the electron during its motion

and the transverse geometry of the waveguide modes can be neglected. In this way the main effect of the waveguide lies in the dispersion relation. The details of the 1D time-dependent code used in this communication can be found in Ref. 3. An improvement has been made in the code by including the transverse overlap factor between the TE<sub>01</sub> mode and the electron beam with a parabolic transverse density profile,<sup>5</sup> and negative slippage. In order to describe a Gaussian distribution  $e^{-x^2/\sigma^2}/\sqrt{\pi}\sigma$  of electron energy spread, 256 electrons are used at each axial grid point.

The parameters used in the simulations are listed in Table I. Due to waveguide dispersion relation, there are two resonance frequencies of 39 and 257 GHz.

First, we study the effects of slippage on gain. The slippage length defined by the difference between the displacements of optical wavepacket and the electron in the time that the electron goes through the wiggler is<sup>1,4</sup>

$$L_s = \pm \Delta N \lambda_{\pm}, \quad (1)$$

$$\Delta = \left[ 1 - \left( \frac{\Gamma_c}{\beta \gamma_z K_w} \right)^2 \right]^{1/2}, \quad (2)$$

where  $\lambda_{\pm} = 2\pi c/\omega_{\pm}$  are the resonant wavelength,  $N$  is the number of wiggler periods,  $\Gamma_c = \omega_c/c$ ,  $\omega_c$  is the cutoff frequency,  $\gamma_z^2 = 1/(1-\beta^2)$ ,  $\beta$  is the average axial velocity of the electrons,  $K_w = 2\pi/\lambda_w$ , and  $\lambda_w$  is the wiggler period.

The slippage length is  $L_s = -31$  cm for the lower resonant frequency, 4.7 cm for the higher resonant frequency. In Fig. 1 we give the electron pulse length dependence of gain with electron energy spread of 1% from simulations. We can see that the gain at the lower frequency declines much faster than at the higher frequency as the electron pulse shortens. In Fig. 2 we show the relative gain as a function of the dimensionless slippage length  $S$  defined by  $S = L_s/L_b$ , where  $L_b$  is the length of the electron pulse. The point  $S=0$  corresponds to the steady-state limit that can be reached with a long enough electron pulse,<sup>6</sup> which is 0.25 at the higher frequency and 0.27 at the lower frequency. We find that two factors result in a larger reduction of gain at lower frequency due to slippage as shown in Fig. 2: One is the longer slippage length which means more slippage with the same electron pulse length; the other is that the negative slippage produces a larger decrease of gain than the positive slippage, which is

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TABLE I. Parameters used in the numerical simulations.

Electron beam	
Energy	0.8 MeV
Current	1 A
Emittance	2 cm mrad
Radius	0.05 cm
Wiggler	
Period	1.2 cm
Peak field strength	1.9 kG
Number of periods	50
Waveguide	
Dimensions	1.067×0.432 cm <sup>2</sup>

shown clearly by comparing the gains at same  $|S|$ . For example, the gain decreases to 20% at  $S=-1.5$ , and to 40% at  $S=1.5$ .

The gain at the lower frequency is small due to the severe negative slippage in many waveguide FELs, while it is considerable with long enough electron pulses such as with our RIKEN submillimeter FEL as discussed above. In this case, we concentrate our attention on the steady state to find a way to suppress the lower-frequency branch. As is well known, the electron energy spread is a key factor in FELs, but its effects on the competition between two frequency branches in a waveguide FEL are not clear.

The expression for the steady-state gain in the small-signal low-gain regime can be written in form<sup>1,3</sup>

$$G = \frac{\pi e J \mu^2 a_w^2 K_s^2}{4 m c^3 \beta^5 \gamma^5 K_z} \lambda_w^3 N^3 g(\theta), \quad (3)$$

where  $a_w = e B_w / K_w m c^2$ ,  $B_w$  is the strength of the wiggler field on axis,  $\mu^2 = 1 + \frac{1}{2} a_w^2$  is the mass shift,  $J$  the current density of the electron beam,  $\gamma$  the electron energy, and  $K_{z,s}$  the optical wave number in waveguide and in vacuum.  $g(\theta)$  is the usual gain line-shape function,

$$g(\theta) = -\frac{d}{d\theta} \left( \frac{\sin \theta}{\theta} \right)^2, \quad (4)$$

where

$$\theta = \frac{1}{2} (K_w + K_z - K_s / \beta) N \lambda_w. \quad (5)$$

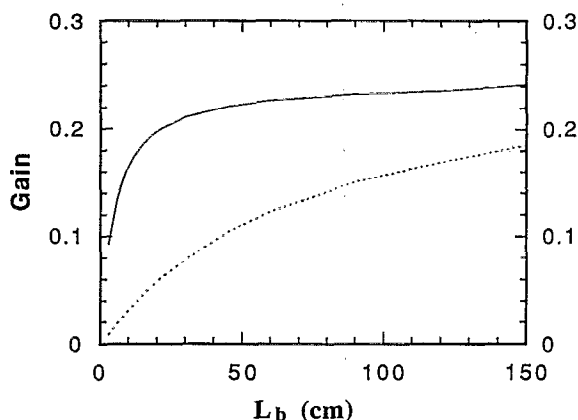


FIG. 1. Gain as a function of electron pulse length. Solid line: higher-frequency branch; dotted line: lower-frequency branch.

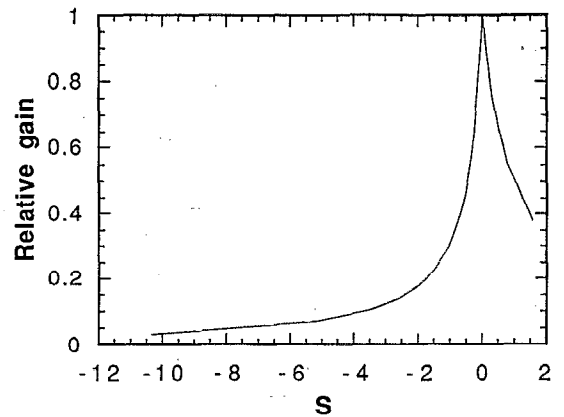


FIG. 2. Gain as a function of dimensionless slippage length.  $S < 0$ : lower-frequency branch;  $S > 0$ : higher-frequency branch. Normalized to steady-state gain.

The electron energy spread  $\sigma$  will lead to a spread in  $\beta$  and  $\theta$ . From Eq. (5) and the relative  $\beta^2 = 1 - \mu^2 / \gamma^2$ , we have

$$\theta = \theta_0 + \pi N \frac{K_s}{K_w} \frac{\mu^2}{\beta^3 \gamma^2} \sigma. \quad (6)$$

$g(\theta)$  has a maximum value of 0.54 when  $\theta_0 = 1.3$ , and  $g(\theta) > 0$  when  $0 < \theta < \pi$ . From Eq. (6), the requirement for energy spread is

$$\sigma < \frac{1}{2N} f, \quad (7a)$$

$$f = \frac{2 \gamma^2 K_w}{\mu^2 K_s}. \quad (7b)$$

With the FEL resonance relative in vacuum,  $f = 1$ ,  $\sigma < 1 / 2N$ , we obtain the well-known results.<sup>7</sup> In waveguide FELs,  $f > 1$ , FELs are less sensitive to the energy spread, which has been indicated in some numerical simulations.<sup>1,3</sup> From Eq. (8),  $f$  is inversely proportional to the resonant frequency, the allowable energy spread is larger for the lower frequency than the higher frequency. The gain decreases slowly at the lower frequency than the higher frequency.

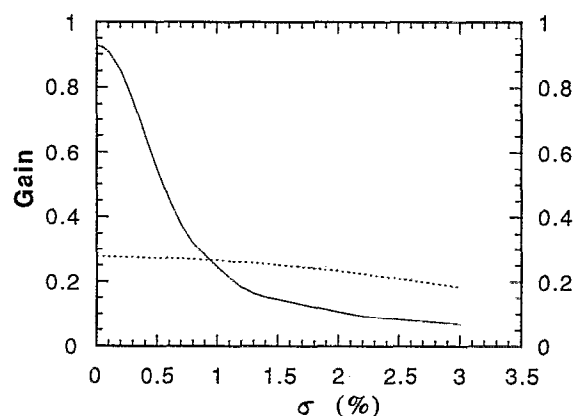


FIG. 3. Gain as a function of electron energy spread. Solid line: higher-frequency branch; dotted line: lower-frequency branch.

In Fig. 3 we shows the results from numerical simulations. The curve for the higher frequency is similar to common results without a waveguide due to optical wavelength close to the value in vacuum, while the gain at the lower frequency has little change because the energy spread is still much smaller than the maximum allowable value of about 7% from Eq. (7). So it is important to control the energy spread as small as possible to obtain large gain at the higher frequency and suppress the development of the lower-frequency branch. An energy spread less than 0.5% can be obtained in RIKEN FEL device quite easily,<sup>8</sup> which provides a good basic condition for the successful FEL operation.

In summary, the significant influences of the electron pulse length and the energy spread on the competition between the gains at two frequencies in a waveguide FEL have

been studied. The maximum allowable electron energy spread for both higher- and lower-frequency branches is given.

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