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On the Eigenvalue Problem in One-Dimensional Mesoscopic Rings with Electron-Phonon Coupling

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(Received 23 October 1996, revision received 28 February 1997)

By invoking the concept of displaced number state in quantum optics, the complete eigenstates in one-dimensional mesoscopic rings with electron-phonon coupling are obtained and the eigenvalues followed. It is shown that the eigenvalues and persistent current depend on both coupling strength and phonon number.

PACS: 03.65. Ca, 71.38.+i

Much attention has been paid to the mesoscopic rings pierced by a magnetic flux in recent years since the famous prediction that a persistent current exists in such a system. The effects of electron-electron coupling, spin-orbit coupling and electron-phonon coupling on persistent current have been studied by several authors. Interestingly, the Hamiltonian of one-dimensional mesoscopic rings with electron-phonon coupling was solved exactly by Wang et al. and some eigenstates and the corresponding eigenvalues are obtained. The aim of this paper is to find complete eigenstates in the system by invoking the concept of displaced number states in quantum optics.

The so-called displaced number state is defined as

\[ |\alpha, n\rangle = D(\alpha)|n\rangle, \]

where \(D(\alpha) = \exp(\alpha b^+ - \alpha^*b)\) is the displacement operator, \(|n\rangle\) is the Fock state, \(b\) and \(b^+\) are annihilation and creation operators of boson type, respectively. For \(n = 0\), the displaced number state is reduced to the well-known coherent state. The displaced number state has quite different properties comparing with the coherent state. For instance, the field obeys sub-Poissonian statistics independent of \(n\) if \(|\alpha|^2 < 1/2\), while the coherent field statistics is always of Poissonian type.

Considering a one-dimensional mesoscopic ring with circumference \(Na\) threaded by a magnetic flux \(\phi\). \(N\) is the number of lattice-sites and \(a\) is the distance between two neighbored

* Supported by the Science Foundation of Jilin Education Committee for Young Scientists (No. 96-15).
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lattice-sites. Within the tight-binding approximation, the Hamiltonian for the system including the electron-phonon interaction is

\[ H = H_l + H_{iq}, \]

\[ H_l = \sum_{i=1}^{N} \left[ \varepsilon_0 - J(C^+_i C_l + C^+_l C_{i+1}) \right], \]

\[ H_{iq} = \sum_q \hbar \omega_q b_q^+ b_q + \sum_{q,l} M_q \{ \exp(i q \alpha) b_q + \exp(-i q \alpha) b_q^+ \} C^+_i C_l, \]

where \( \varepsilon_0 \) is the on-site energy of electron, \( J \) is the hopping integral, \( C_l \) and \( C^+_l \) are the annihilation and creation operators of electron on site \( l \), \( b_q \) and \( b_q^+ \) are the annihilation and creation operator of \( q \)-th phonon, respectively, \( M_q \) is the coupling strength.

First, we consider the Hamiltonian \( H_l \) which can be solved exactly when subjected to the boundary condition \( C_{l+N} = C_l \exp(i 2 \pi \phi / \phi_0) \). Its \( n \)-th eigenfunction is

\[ \langle \phi_n \rangle = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} C_l^+ \exp \left[ i \frac{2 \pi l}{N} \left( n + \frac{\phi}{\phi_0} \right) \right] |0\rangle, \]

and the corresponding \( n \)-th eigenvalue is

\[ E_n = \varepsilon_0 - 2J \cos \left[ \frac{2 \pi n}{N} + \frac{\phi}{\phi_0} \right]. \]

It is obvious that \( E_n \) is the periodic function of magnetic flux with the period \( \phi_0 = \hbar c / e \).

Next, we gear to find the eigenfunction and eigenvalue of the Hamiltonian \( H_{iq} \) and rewrite it as

\[ H_{iq} = \sum_q \hbar \omega_q b_q^+ b_q + f_q b_q^+ + f_q^* b_q, \]

where

\[ f_q = M_q \sum_i \exp(-i q \alpha) C_i^+ C_l, \]

\[ f_q^* = M_q \sum_i \exp(i q \alpha) C_i^+ C_l. \]

By using the displacement operator \( D_q(\alpha) = \exp(\alpha b_q^+ - \alpha^* b_q) \) and its properties, \( H_{iq} \) can be written in the following desired form for our purpose:

\[ H_{iq} = \sum_q \hbar \omega_q D_q^{-1} \left( \frac{f_q}{\hbar \omega_q} \right) b_q^+ b_q D_q \left( \frac{f_q}{\hbar \omega_q} \right) - \frac{f_q f_q^*}{\hbar \omega_q}. \]

Inspecting the form of Eq. (7) and using the displaced number state, we construct the eigenstates
as
\[ |\phi'_n| = \frac{1}{\sqrt{N}} \sum_{m=1}^{N} C_m^{\dagger} \exp \left[ i \frac{2\pi m}{N} \left( n + \frac{\phi_0}{\phi} \right) \right] |0\rangle \otimes \prod_{q} D_q(-\alpha_q^m)|n_q\rangle, \] (8)

where \( \alpha_q^m = [M_q/(\hbar \omega_q)] \exp(-i q M a). \) It is not difficult to prove that it is the eigenstate of \( H_{lq} \) by considering the fact

\[ f_q C_q^{\dagger} |0\rangle = M_q \exp(-i q M a) C_q^{\dagger} |0\rangle, \]
\[ D_q \left( \frac{f_q}{\hbar \omega_q} \right) C_q^{\dagger} |0\rangle = D_q(\alpha_q^m) C_q^{\dagger} |0\rangle. \] (9)

The above relations can be obtained by using the anti-commutator \([C_q^{\dagger}, C_q]_+ = \delta_q^m \) repeatedly. The eigenequation is

\[ H_{lq} |\phi'_n\rangle = \left( \sum_q n_q \hbar \omega_q - \frac{M_q^2}{\hbar \omega_q} \right) |\phi'_n\rangle. \] (10)

Therefore, the eigenequation for the total Hamiltonian \( H = H_l + H_{lq} \) is

\[ H |\phi'_n\rangle = \left[ \varepsilon_0 - J \sum_{i=1}^{N} (C_{i+1} C_i + C_{i+1}^{\dagger} C_i) + \sum_q (n_q \hbar \omega_q - \frac{M_q^2}{\hbar \omega_q}) \right] |\phi'_n\rangle = E'_n |\phi'_n\rangle. \] (11)

Operating both sides of Eq. (11) with \( \langle \phi'_n| \), we obtain

\[ E'_n = \varepsilon_0 - 2J \sum_{i=1}^{N} C_{i+1} C_i + \sum_q n_q \hbar \omega_q - \sum_q \frac{M_q^2}{\hbar \omega_q}. \] (12)

It is not difficult to obtain that

\[ \langle \phi'_n| \sum_{i=1}^{N} C_{i+1} C_i |\phi'_n\rangle = \frac{1}{N} \exp \left[ - i \frac{2\pi}{N} \left( n + \frac{\phi_0}{\phi} \right) \right] \sum_{m=1}^{N} \prod_q (n_q |D_q(\alpha_q^{m+1})D_q(-\alpha_q^m)|n_q). \] (13)

From the fact that

\[ D(\alpha)D(\beta) = D(\alpha + \beta) \exp \left[ \frac{1}{2} (\alpha \beta^* - \alpha^* \beta) \right], \] (14)

Eq. (13) can be written as

\[ \langle \phi'_n| \sum_{i=1}^{N} C_{i+1} C_i |\phi'_n\rangle = \frac{1}{N} \exp \left[ - i \frac{2\pi}{N} \left( n + \frac{\phi_0}{\phi} \right) \right] \exp \left[ i \sum_q \sin(qa) \left( \frac{M_q}{\hbar \omega_q} \right)^2 \right] \]
\[ \cdot \sum_{m=1}^{N} \prod_q (n_q |D_q(\alpha_q^{m+1} - \alpha_q^m)|n_q). \] (15)

Using the property of \( q \) distribution \((-\pi/a \leq q < \pi/a), \) we have

\[ \exp \left[ i \sum_q \sin(qa) \left( \frac{M_q}{\hbar \omega_q} \right)^2 \right] = 1. \] (16)
Because the term \( \langle n_q | D_q (\alpha_q^{m+1} - \alpha_q^m) | n_q \rangle \) does not depend on \( m \), from Eqs. (15) and (16) we obtain

\[
\langle \phi'_n | \sum_{l=1}^{N} C_{l+1}^l C_l | \phi'_n \rangle = \exp \left[ - i \frac{2\pi}{N} \left( n + \frac{\phi}{\phi_0} \right) \right] \exp \left[ - \sum_q \left( \frac{M_q}{\hbar \omega_q} \right)^2 (1 - \cos qa) \right] \cdot \prod_{q \neq k=0}^{n_q} \frac{n_q! [2\cos(qa) - 1] [M_q/(\hbar \omega_q)]^2 n_q - k}{k!(n_q - k)!(n_q - k)!}.
\]

(17)

From Eqs. (12) and (17), the eigenvalue for the Hamiltonian \( H \) is

\[
E'_n = \epsilon_0 - 2J \cos \left[ \frac{2\pi}{N} \left( n + \frac{\phi}{\phi_0} \right) \right] \exp \left[ - \sum_q \left( \frac{M_q}{\hbar \omega_q} \right)^2 (1 - \cos qa) \right] \cdot \prod_{q \neq k=0}^{n_q} \frac{n_q! [2\cos(qa) - 1] [M_q/(\hbar \omega_q)]^2 n_q - k}{k!(n_q - k)!(n_q - k)!} + \sum_q n_q \hbar \omega_q - \sum_q \frac{M_q^2}{\hbar \omega_q}.
\]

(18)

For \( n_q = 0 \), \( E'_n \) reduces to

\[
E'_n = \epsilon_0 - 2J \cos \left[ \frac{2\pi}{N} \left( n + \frac{\phi}{\phi_0} \right) \right] \exp \left[ - \sum_q \left( \frac{M_q}{\hbar \omega_q} \right)^2 (1 - \cos qa) \right] - \sum_q \frac{M_q^2}{\hbar \omega_q},
\]

(19)

it just the result obtained as in Ref. 7.

In conclusion, the eigenvalue problem in one-dimensional mesoscopic rings with electron-phonon coupling is solved completely. The results show that the eigenvalues depend on both coupling strength \( M_q \) and phonon number \( n_q \). According to the definition of persistent current \( I_n = -C_n (\partial E'_n/\partial \phi) \), it not only depends on coupling strength but also on the phonon number.

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