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On the Eigenvalue Problem in One-Dimensional Mesoscopic Rings with Electron-Phonon Coupling *

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By invoking the concept of displaced number state in quantum optics, the complete eigenstates in one-dimensional mesoscopic rings with electron-phonon coupling are abtained and the eignevalues followed. It is shown that the eigenvalues and persistent current depend on both coupling strength and phonon number.

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Much attention has been paid to the mesoscopic rings pierced by a magnetic flux in recent years since the famous prediction that a persistent current exists in such a system.[1-6] The effects of electron-electron coupling, spin-orbit coupling and electron-phonon coupling on persistent current have been studied by serveral authors. Interestingly, the Hamiltonian of one-dimensional mesoscopic rings with electron-phonon coupling was solved exactly by Wang *et al.*[7] and some eigenstates and the corresponding eigenvalues are obtained. The aim of this paper is to find complete eigenstates in the system by invoking the concept of displaced number states in quantum optics.

The so-called displaced number state is defined as[8]

$$|\alpha, n\rangle = D(\alpha)|n\rangle\,, \tag{1}$$

where $D(\alpha) = \exp(\alpha b^+ - \alpha^* b)$ is the displacement operator, $|n\rangle$ is the Fock state, b and b^+ are annihilation and creation operators of boson type, respectively. For $n = 0$, the displaced number state is reduced to the well-known coherent state. The displaced number state has quite different properties comparing with the coherent state. For instance, the field obeys sub-Poissonian statistics independent of n if $|\alpha|^2 < 1/2$, while the coherent field statistics is always of Poissonian type.

Considering a one-dimensional mesoscopic ring with circumference Na threaded by a magnetic flux ϕ. N is the number of lattice-sites and a is the distance between two neighbored

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lattice-sites. Within the tight-binding approximation, the Hamiltonian for the system including the electron-phonon interaction is

$$H = H_l + H_{lq}\,,$$
$$H_l = \sum_{l=1}^{N}[\varepsilon_0 - J(C_{l+1}^+ C_l + C_l^+ C_{l+1})]\,, \tag{2}$$
$$H_{lq} = \sum_q \hbar\omega_q b_q^+ b_q + \sum_{q,l} M_q[\exp(\mathrm{i}\,qla)b_q + \exp(-\mathrm{i}\,qla)b_q^+]C_l^+ C_l\,,$$

where ε_0 is the on-site energy of electron, J is the hopping integral, C_l and C_l^+ are the annihilation and creation operators of electron on site l, b_q and b_q^+ are the annihilation and creation operator of q-th phonon, respectively, M_q is the coupling strength.

First, we consider the Hamiltonian H_l which can be solved exactly when subjected to the boundary condition $C_{l+N} = C_l \exp(\mathrm{i}\,2\pi\phi/\phi_0)$.[6] Its n-th eigenfunction is

$$|\phi_n\rangle = \frac{1}{\sqrt{N}}\sum_{l=1}^{N} C_l^+ \exp\left[\mathrm{i}\,\frac{2\pi l}{N}\left(n + \frac{\phi}{\phi_0}\right)\right]|0\rangle\,, \tag{3}$$

and the corresponding n-th eigenvalue is

$$E_n = \varepsilon_0 - 2J\cos\left[\frac{2\pi}{N}(n + \frac{\phi}{\phi_0})\right]. \tag{4}$$

It is obvious that E_n is the peroidic function of magnetic flux with the period $\phi_0 = \hbar c/e$.

Next, we gear to find the eigenfunction and eigenvalue of the Hamiltonian H_{lq} and rewrite it as

$$H_{lq} = \sum_q \hbar\omega_q b_q^+ b_q + f_q b_q^+ + f_q^* b_q\,, \tag{5}$$

where

$$f_q = M_q \sum_l \exp(-\mathrm{i}\,qla)C_l^+ C_l\,,$$
$$f_q^* = M_q \sum_l \exp(\mathrm{i}\,qla)C_l^+ C_l\,. \tag{6}$$

By using the displacement operator $D_q(\alpha) = \exp(\alpha b_q^+ - \alpha^* b_q)$ and its properties, H_{lq} can be written in the following desired form for our purpose:

$$H_{lq} = \sum_q \hbar\omega_q D_q^{-1}\Big(\frac{f_q}{\hbar\omega_q}\Big) b_q^+ b_q D_q\Big(\frac{f_q}{\hbar\omega_q}\Big) - \frac{f_q f_q^*}{\hbar\omega_q}\,. \tag{7}$$

Inspecting the form of Eq. (7) and using the displaced number state, we construct the eigenstates as

$$|\phi'_n\rangle = \frac{1}{\sqrt{N}} \sum_{m=1}^{N} C_m^+ \exp\left[\mathrm{i}\,\frac{2\pi m}{N}\Big(n + \frac{\phi_0}{\phi}\Big)\right]|0\rangle \otimes \coprod_q D_q(-\alpha_q^m)|n_q\rangle\,, \tag{8}$$

where $\alpha_q^m = [M_q/(\hbar\omega_q)]\exp(-\mathrm{i}\,qma)$. It is not difficult to prove that it is the eigenstate of H_{lq} by considering the fact

$$\begin{aligned} f_q C_m^+|0\rangle &= M_q \exp(-\mathrm{i}\,qma)C_m^+|0\rangle\,, \\ D_q\Big(\frac{f_q}{\hbar\omega_q}\Big)C_m^+|0\rangle &= D_q(\alpha_q^m)C_m^+|0\rangle\,. \end{aligned} \tag{9}$$

The above relations can be obtained by using the anti-commutator $[C_l^+, C_m]_+ = \delta_l^m$ repeatly. The eigenequation is

$$H_{lq}|\phi'_n\rangle = \Big(\sum_q n_q\hbar\omega_q - \frac{M_q^2}{\hbar\omega_q}\Big)|\phi'_n\rangle\,, \tag{10}$$

Therefore, the eigenequation for the total Hamiltonian $H = H_l + H_{lq}$ is

$$H|\phi'_n\rangle = \Big[\varepsilon_0 - J\sum_{l=1}^{N}(C_{l+1}^+C_l + C_l^+C_{l+1}) + \sum_q(n_q\hbar\omega_q - \frac{M_q^2}{\hbar\omega_q})\Big]|\phi'_n\rangle = E'_n|\phi'_n\rangle\,. \tag{11}$$

Operating both sides of Eq. (11) with $\langle\phi'_n|$, we obtain

$$E'_n = \varepsilon_0 - 2J\langle\phi'_n|\sum_{l=1}^{N} C_{l+1}^+C_l + C_l^+C_{l+1}|\phi'_n\rangle + \sum_q n_q\hbar\omega_q - \sum_q \frac{M_q^2}{\hbar\omega_q}\,. \tag{12}$$

It is not difficult to obtain that

$$\langle\phi'_n|\sum_{l=1}^{N} C_{l+1}^+C_l|\phi'_n\rangle = \frac{1}{N}\exp\Big[-\mathrm{i}\,\frac{2\pi}{N}\Big(n + \frac{\phi}{\phi_0}\Big)\Big]\sum_{m=1}^{N}\prod_q\langle n_q|D_q(\alpha_q^{m+1})D_q(-\alpha_q^m)|n_q\rangle\,. \tag{13}$$

From the fact that

$$D(\alpha)D(\beta) = D(\alpha+\beta)\exp\Big[\frac{1}{2}(\alpha\beta^* - \alpha^*\beta)\Big]\,, \tag{14}$$

Eq. (13) can be written as

$$\begin{aligned} \langle\phi'_n|\sum_{l=1}^{N} C_{l+1}^+C_l|\phi'_n\rangle =& \frac{1}{N}\exp\Big[-\mathrm{i}\,\frac{2\pi}{N}\Big(n + \frac{\phi}{\phi_0}\Big)\Big]\exp\Big[\mathrm{i}\sum_q \sin(qa)\Big(\frac{M_q}{\hbar\omega_q}\Big)^2\Big] \\ &\cdot\sum_{m=1}^{N}\prod_q\langle n_q|D_q(\alpha_q^{m+1} - \alpha_q^m)|n_q\rangle\,. \end{aligned} \tag{15}$$

Using the property of q distribution $(-\frac{\pi}{a} \le q < \frac{\pi}{a})$, we have

$$\exp\Big[\mathrm{i}\sum_q \sin(qa)\Big(\frac{M_q}{\hbar\omega_q}\Big)^2\Big] = 1\,. \tag{16}$$

Because the term $\langle n_q|D_q(\alpha_q^{m+1}-\alpha_q^m)|n_q\rangle$ does not depend on m, from Eqs. (15) and (16) we obtain

$$\langle\phi'_n|\sum_{l=1}^{N}C_{l+1}^{+}C_l|\phi'_n\rangle=\exp\Big[-\mathrm{i}\,\frac{2\pi}{N}\Big(n+\frac{\phi}{\phi_0}\Big)\Big]\exp\Big[-\sum_q\Big(\frac{M_q}{\hbar\omega_q}\Big)^2(1-\cos qa)\Big]$$
$$\cdot\prod_q\sum_{k=0}^{n_q}\frac{n_q![2[\cos(qa)-1][M_q/(\hbar\omega_q)]^2]^{n_q-k}}{k!(n_q-k)!(n_q-k)!}. \quad (17)$$

From Eqs. (12) and (17), the eigenvalue for the Hamiltonian H is

$$E'_n=\varepsilon_0-2J\cos\Big[\frac{2\pi}{N}\Big(n+\frac{\phi}{\phi_0}\Big)\Big]\exp\Big[-\sum_q\Big(\frac{M_q}{\hbar\omega_q}\Big)^2(1-\cos qa)\Big]$$
$$\cdot\prod_q\sum_{k=0}^{n_q}\frac{n_q!\{2[\cos(qa)-1][M_q/(\hbar\omega_q)]^2\}^{n_q-k}}{k!(n_q-k)!(n_q-k)!}+\sum_q n_q\hbar\omega_q-\sum_q\frac{M_q^2}{\hbar\omega_q}. \quad (18)$$

For $n_q=0$, E'_n reduces to

$$E'_n=\varepsilon_0-2J\cos\Big[\frac{2\pi}{N}\Big(n+\frac{\phi}{\phi_0}\Big)\Big]\exp\Big[-\sum_q\Big(\frac{M_q}{\hbar\omega_q}\Big)^2(1-\cos qa)\Big]-\sum_q\frac{M_q^2}{\hbar\omega_q}, \quad (19)$$

it just the result obtained as in Ref. 7.

In conclusion, the eigenvalue problem in one-dimensional mesoscopic rings with electron-phonon coupling is solved completely. The results show that the eigenvalues depend on both coupling strength M_q and phonon number n_q. According to the defination of persistent current $I_n=-C_n(\partial E'_n/\partial\phi)$, it not only depends on coupling strength but also on the phonon number.

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