

# Experimental Study of Small Signal Amplification in Bragg Acousto-optic Bistable System

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## ABSTRACT

*The external small signal amplification ability of the Bragg acousto-optic system has been studied in this paper. It has been proven by experiment that there are saturation and resonance phenomena in the amplification. Bifurcation parameters at the bifurcation points are decreased by external simple harmonic signals. At the period-2 bifurcation point, small signal amplification energy is mainly from the period-2 component. External signals have the ability of frequency pulling and synchronising period-2 frequency. These phenomena have great significance concerning bifurcation and chaos. © 1998 Elsevier Science Ltd.*

## 1 INTRODUCTION

It was put forward theoretically, by Wiesenfeld and McNamara in 1985, that non-linear dynamic systems, which are in a bifurcation state, have external small signal amplification abilities.<sup>1</sup> In the same year the small signal amplification experiment was successfully carried out using a ruby laser.<sup>2</sup> Since then there has been theoretical analysis.<sup>3–8</sup> A Bragg acousto-optic bistable system with delayed feedback was set up in our laboratory. Bifurcation and chaos were observed, and bifurcation parameters decreased by external simple harmonic signal were also observed. When the system was near the onset of period-2 bifurcation, it had a remarkable ability to amplify small signals.

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## 2 OPERATION PRINCIPLE

The relation between first-order diffraction intensity  $I_1$ , and voltage  $U_m$  applied to modulator M can be described by the following equation:

$$I_1 = I_L \sin^2(aU_m), \quad (1)$$

where  $a$  is parameter decided by the structure of the modulator and driver, and  $I_L$  is the intensity of the input laser light. It is easily obtained that

$$\frac{d^2I_1}{dU_m^2} = 2a^2I_L \sin(2aU_m).$$

Let  $d^2L/dU_m^2 = 0$ : then  $U_m = \pi/4a$ .  $dI_1/dU_m$  has different signs, on both sides of this point. Therefore, this point is the inflection point of the curve. This is a necessary condition to make bistability. The relation between  $U_m$  and the output voltage  $U_D$  of an amplifier can be described by the following equations:

$$U_m = \beta U_D + U_o \quad (2)$$

$$U_D = \kappa I_1 \quad (3)$$

where  $\kappa$  is light-voltage conversion of the photodiode,  $\beta$  is the gain of the amplifier,  $U_o$  is the bias voltage. Let  $Tr = I_1/I_L$ , which represents the transparency of the acoustooptic cell. The modulation curve of Bragg acoustooptic bistability was obtained from eqn (1)

$$Tr = \sin^2(aU_m). \quad (4)$$

The feedback curve of Bragg acoustooptic bistability was obtained from eqns (2) and (3)

$$Tr = \frac{U_m - U_D}{\beta\kappa I_L}. \quad (5)$$

The transient behaviour of an acoustooptic system can be described by following differential equation:

$$\frac{\tau dX(t)}{dt} + X(t) = \pi \{A - \mu \sin^2[X(t - T) - X_B]\} \quad (6)$$

where  $X(t)$  is the normalized voltage at the input driver, and  $\tau$  is the time delay.

The system described by formula eqn (6) exhibits double-bifurcation behaviour. The Bragg acoustooptic bistable system in an unstable state, has high-order modes. However, near the period-2 frequency and the period-4 frequency, the high-order modes have greater resistance than the base mode.

Therefore, the high-order modes can be neglected. The amplification times for a small signal,  $M_o$ , can be solved by the Green function<sup>3</sup>

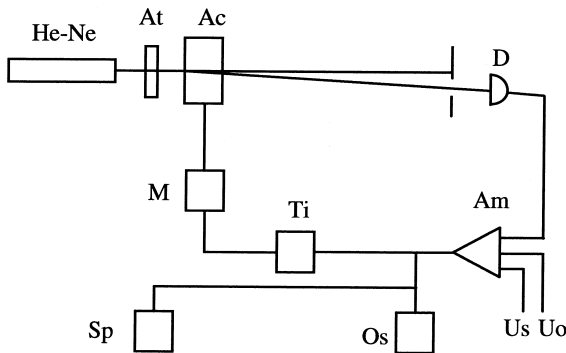
$$M_o = \frac{\{[1 + 0.5\pi^2 q^2(1 + 2q + 2q^2)]^2 + q^2\pi^2\}^{1/2}}{0.5q^2\pi^2(1 + q)(1 + q^2\pi^2)} \quad (7)$$

$$q = \frac{T}{\tau}. \quad (8)$$

Bifurcation parameters at bifurcation points can be calculated by computer. The period-2 bifurcation parameter is 0.368, and the period-4 bifurcation parameter is 0.624. The system will be in chaos as the bifurcation parameter is increased. When external signals are added, these bifurcation parameters at bifurcation points will decrease. The bifurcation parameter is given by  $\mu = I_L K_o/b$ , where  $K_o$  is the total gain; and  $b$  is the constant related to the structure.

### 3 EXPERIMENTAL SET-UP

Figure 1 shows the experiment set-up. The acoustooptic cell is driven by a radio frequency generator which creates an acoustic grating. The He-Ne laser light enters the crystal at the Bragg angle to obtain maximum first-order diffraction. The first diffraction light is received by a photodiode D. The signal received by D, is amplified by the amplifier  $A_m$ , before being fed back to the modulator M. The output voltage of the driver (100 kHz) is linearly modulated by the output voltage of the amplifier. There is an adder in the amplifier, which can add bias voltage and small signal to the signals in the system. The oscilloscope Os and Spectrum analyzer Sp are used to observe the



**Fig. 1.** Experimental set-up of small signal amplification in a Bragg bistable system. At: attenuator; Ac: acoustooptic cell; M: driver; Am: amplifier and adder; D: photodiode; Sp: spectrum analyzer; Os: oscilloscope.

waves and the frequency spectrum. The acoustooptic cell is mainly used in laser printing, its maximum first diffraction efficiency being 0.9. Due to its small size, it has a short response time. The coaxial cable  $T_i$  is used to delay time, and  $A_t$  is an attenuator.

## 4 EXPERIMENTAL RESULT

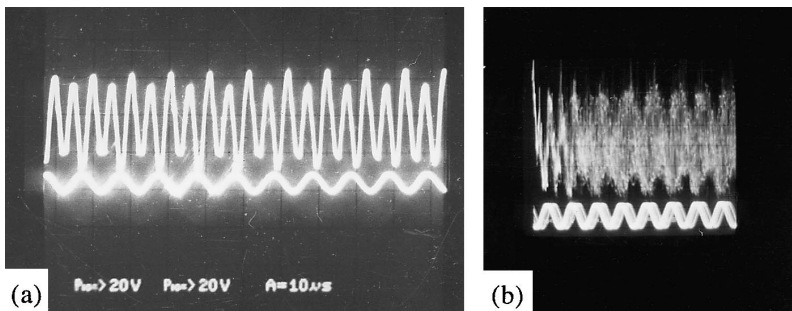
### 4.1 Small signal effect on Bragg acoustooptic bistable system

If the Bragg acoustooptic bistable system is in an unstable state its bifurcation parameter is less than the period-2 bifurcation parameter. When the small signal is put into the system and it is gradually made stronger, it is observed that the system goes into period-2, and period-4 bifurcation and then chaos. If the system is in chaos, it is possible to remove the system from chaos by introducing a very strong harmonic signal. This phenomenon illustrates that a harmonic small signal is able to decrease the bifurcation parameter at bifurcation points (Fig. 2).

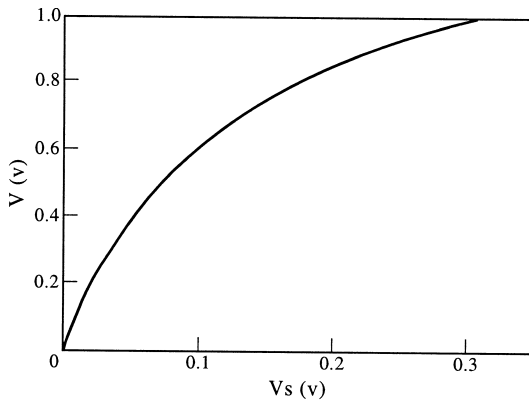
When the Bragg acoustooptic bistable system is in an unstable region and it is near the onset of period-2 bifurcation, it has a remarkable ability to amplify a small signal.

### 4.2 Amplitude response of small signal amplification

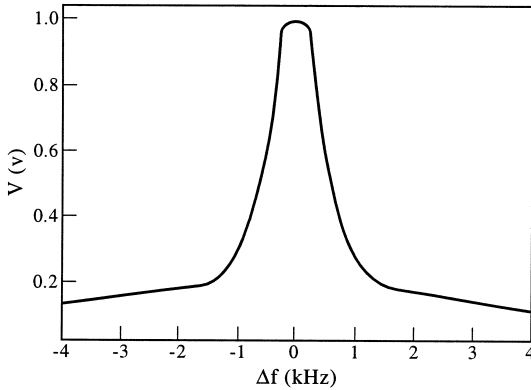
Let  $\Delta f = f_s - (f_o/2)$ , where  $f_s$  is the small signal frequency,  $f_o$  is the self-pulse frequency. When  $\Delta f = 0$ , the small signal is in resonance with the period-2 frequency. Under this condition the Bragg acoustooptic bistable system has a maximum 10-fold amplification ability. The amplitude of the small signal  $V$  turns to saturation, as small signal amplitude  $V_s$  increases (Fig. 3).



**Fig. 2.** The system is put into the period-2 bifurcation state (a), and chaos (b) from self-pulse state by external signals. Lower waves are external signals.



**Fig. 3.** Amplitude response of small signal amplification.  $V$ : intensity of amplified small signal;  $V_s$ : intensity of input small signal. Electrical levels used are peak-peak value.

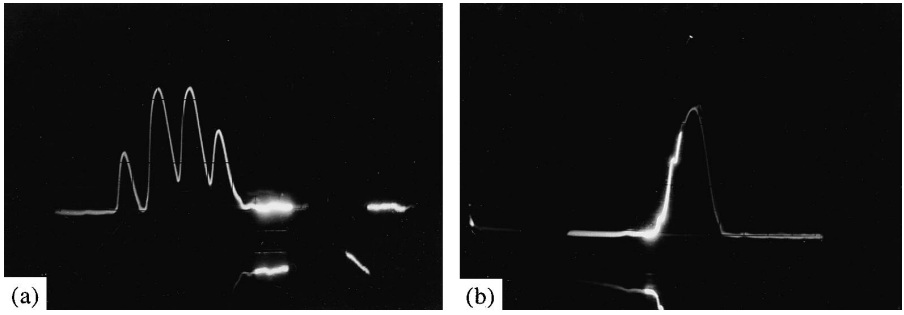


**Fig. 4.** Frequency response of small signal amplification.  $V_s = 0.25V$ .  $f_s$  is the frequency of small signal;  $f_o/2$  is the period-2 frequency.

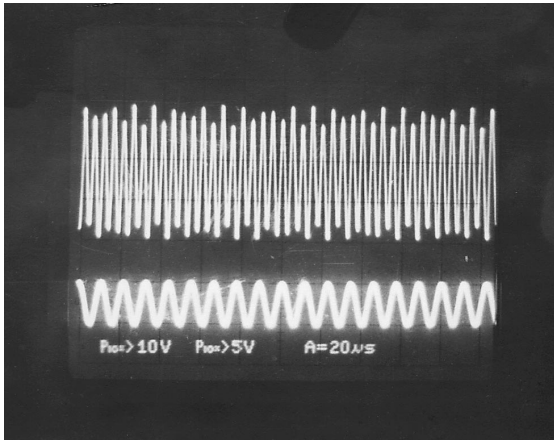
### 4.3 Frequency response of small signal amplification

The intensity of the amplified small signal changes remarkably as the small signal frequency is changed. Only in the frequency difference  $\Delta f = \pm 1$  kHz does the system have amplification abilities. Small signal amplification cannot be observed out of this scope. Therefore, it is seen that the small signal frequency is strongly in resonance with the period-2 frequency (Fig. 4).

Figure 5(a) shows the frequency spectrum near the small signal frequency. The second peak from the right represents the intensity of the period-2 component. When the small signal frequency equals the period-2 frequency ( $\Delta f = 0$ ), the two peaks merge into one [Fig. 5(b)].



**Fig. 5.** Spectrum near the small signal frequency. (a) The first peak from the right represents the intensity of the small signal. The second peak from right represents the intensity of the period-2 component,  $\Delta f = 2$  kHz; (b)  $\Delta f = 0$  two peaks merge into one.



**Fig. 6.** Beat frequency  $f_s - f_o/2$ .

From these pictures we can see that the energy of the amplified small signal is mainly from the energy of period-2 produced by the decrease of the bifurcation parameter at the period-2 bifurcation point.

#### 4.4 Beat frequency, frequency pulling effect and synchronization

The beat frequency between the period-2 frequency and the small signal frequency is observed when the input signal is small (Fig. 6). There are two groups of sinusoidal envelope curves on the top and the bottom of self-pulse waves, and there are two envelope curves in the same group. One of them is in opposite phase to the other. When the small signal frequency equals the

period-2 frequency the envelope curves become two straight lines. This can be used to make the small signal frequency equal the period-2 frequency. If the external signal is small it has less effect on the period-2 frequency. It is observed that the period-2 frequency approaches external signal frequency as external signal is made stronger. This is frequency pulling (Fig. 6). When the external signal is strong enough, the period-2 frequency is locked to the external signal.

## 5 CONCLUSION

A Bragg acoustooptic bistable system with an external delay near the onset of bifurcation exhibits a small signal amplification ability. There are saturation, remarkable resonance phenomena in this amplification. If the external simple harmonic signal is strong, it can pull the period-2 frequency so that they become synchronized.

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