Nuclear Instruments and Methods in Physics Research A 445 (2000) 241–246

NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH
Section A

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Laser undulator radiation

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Abstract

Various characteristics such as the number of photons, the wavelength, and the solid angle of the laser undulator radiation have been measured quantitatively. It was performed in the visible wavelength region using the interaction between a high-power pulsed CO_2 laser and a high-quality electron beam having an energy of 0.65–0.85 MeV. The experimental results were in good agreement with the theoretical calculations. A criterion to determine the limitation to the number of periods of the laser undulator was also proposed. © 2000 Elsevier Science B.V. All rights reserved.

Keywords: Undulation; Wiggler; Laser

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1. Introduction

In comparison with the conventional magnetic undulator [1], the laser undulator has a shorter pitch length, which leads to the generation of the shorter wavelength radiation.

The first trial to use the electromagnetic field as an undulator was performed to generate a visible radiation using the interaction between electron bunches of 100–150 MeV and a microwave (2856.0 MHz) in a waveguide [2,3]. The generation of the photon using the interaction between a high-power laser and a relativistic electron beam was performed in visible wavelength region to measure the energy spread of intense relativistic electron beam (IREB) [4,5]. It was given by the spectrum of the scattered photons generated by the interaction between the IREB and a high-power CO_2 laser. These were the Thomson back-scattering in the relativistic region [6]. The first experiment, which showed that a laser can operate as an optical undulator, was performed by Olshan et al. in 1987 [7]. They showed that the energy exchange occurred between the electrons and the CW CO_2 (9.3 μm) laser using the interaction between a low-energy electron beam and the CW CO_2 (10.6 μm).

There have been several trials to generate a γ-ray by an interaction between high-energy electron bunches and a CW Nd-YAG laser [8]. These experiments were based on the Compton scattering, in which the photons act as particles and the electron is scattered by the photon losing the energy. In this sense, the physical qualities of photons

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PII: S0168-9002(00)00074-7

obtained by these experiments are different from those obtained by the laser undulator.

In this paper, various characteristics such as the number of radiation photons, the wavelength, and the radiation solid angle of the laser undulator have been measured quantitatively in the visible wavelength region using the interaction between a high-power pulsed CO_2 laser and a high-quality electron beam [9]. The K value of the laser undulator are calculated and discussed. The experimental results were in good agreement with the theoretical calculations using the K value obtained in this study within the experimental errors. A criterion to determine the limitation to the number of periods of the undulator, which was determined by the electron energy and the wavelength of the pump laser, was proposed.

2. Theory

2.1. K value of the laser undulator

In a conventional magnetic undulator, the K value is calculated by

$$K = eB_m\lambda_m/2\pi mc = 93.4B_m\lambda_m \qquad (1)$$

where B_m (Wb/m^2) is the magnetic flux density and λ_m (m) is the pitch length of the undulator; e and m are the electric charge and the mass of the electron, and c is the velocity of light.

We consider the interaction between the electromagnetic field of the laser beam and the electron which is propagating against the pointing vector P of the electromagnetic radiation as shown in Fig. 1. Using the electric field E (V/m) and the magnetic field B (Wb/m^2) of the laser, the power density I (Wb/m^2) is given by

$$I = EB/\mu_0 \qquad (2)$$

where μ_0 is the permeability in vacuum.

The electromagnetic force F applied to the electron is composed of F_E and F_B as shown in Fig. 1, which have the same directions and given by

$$F_E = eE, \quad F_B = evB. \qquad (3)$$

Using Eq. (3), $\varepsilon_0 E^2 = \mu_0 H^2$, $B = \mu_0 H$ and $c = 1/(\varepsilon_0\mu_0)^{1/2}$, and $\beta = v/c$, F is calculated to be

$$F = F_B + F_E = evB(1 + 1/\beta) \qquad (4)$$

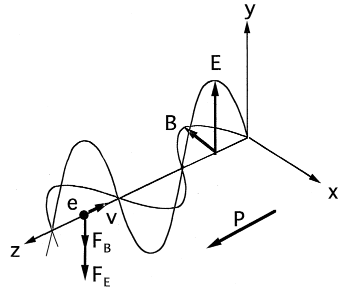

Fig. 1. Schematic diagram of the interaction between a relativistic electron and an electromagnetic wave. E and B are the electric field and the magnetic field of the laser beam, respectively. e is the electron. v is the velocity of the electron. P is the pointing vector of electromagnetic field. F_B and F_E are the electromagnetic forces induced by the electric field and the magnetic field, respectively.

where ε_0 is the dielectric constant in vacuum and c is the velocity of light.

On the other hand, using $\varepsilon_0 E^2 = \mu_0 H^2$, $B = \mu_0 H$ and Eq. (2), the magnetic flux density of the electromagnetic field is expressed by the power density of the laser as shown by

$$B = (I\mu_0/c)^{1/2}. \qquad (5)$$

Taking the electric force into account and using Eq. (4), the effective value of B of the laser undulator is expressed by

$$B_{\mathrm{eff}} = (1 + 1/\beta)I^{1/2}. \qquad (6)$$

The effective pitch length of the laser undulator is written as

$$\lambda_{\mathrm{eff}} = \lambda_0(1 + \beta) \qquad (7)$$

where λ_0 is the wavelength of the laser.

From Eqs. (1), (6) and (7), the effective K value for the laser undulator is given by

$$K_{\mathrm{eff}} = 93.4 \times B_{\mathrm{eff}}\lambda_{\mathrm{eff}} = 6.04 \times 10^{-6} I^{1/2}\lambda_0/\beta. \qquad (8)$$

For $\gamma \gg 1$, K_{eff} is given by

$$K_{\mathrm{eff}} = 6.04 \times 10^{-6} \times I^{1/2}\lambda_0. \qquad (9)$$

2.2. Number of periods of the laser undulator

When the electron interacts with the pump laser having the wavelength λ_0 and generates the undulator radiation of the wavelength λ_s as shown in

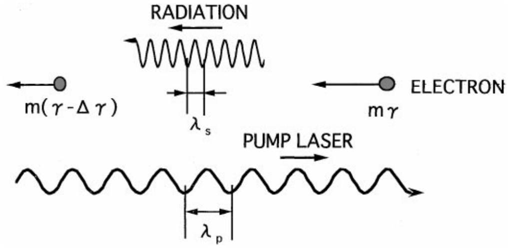

Fig. 2. Loss in the electron energy through the interaction with the pump laser during the period of N_{eff}.

Fig. 2, the electron loses the energy of $\Delta\gamma$, which is calculated to be

$$\Delta\gamma\ mc^2 = hc/\lambda_s \tag{10}$$

according to the energy conservation law. Here, h is Plank's constant. For $\gamma \gg 1$, λ_s is calculated to be

$$\lambda_s \sim \lambda_0/(4\gamma^2). \tag{11}$$

On the other hand, it is well known that the energy spread of the electron $\Delta\gamma$ must satisfy

$$\Delta\gamma/\gamma < 1/N \tag{12}$$

to obtain a high brilliance in the undulator operation. From Eqs. (10)–(12), the number of periods of laser undulator must satisfy the following equation:

$$N < mc\lambda_0/(4\gamma h). \tag{13}$$

This formula is written in a simpler style using the Compton wavelength $\lambda_c (\lambda_c = h/mc = 2.42 \times 10^{-12}\,\mathrm{m})$ as follows:

$$N < (\lambda_0/\lambda_c)/4\gamma. \tag{14}$$

This limitation to the number of the undulator pitch will be convenient in the design of the laser undulator.

3. Experimental setup

The experimental setup is shown in Fig. 3. A DISKTRON electrostatic accelerator [10] generates a high voltage of about 0.65–0.85 MeV which is applied on an aluminum photocathode. The fourth higher harmonic (266 nm) of a Nd-YAG laser is irradiated on this cathode to generate a pulse electron bunch [9], which is guided and focused into the interaction region with a high-power CO_2 laser. The pulse duration of the electron bunch, which is determined by the duration of the fourth higher harmonic of the Nd-YAG laser, is 10 ns. The pulse duration of the CO_2 laser is 10 ns. These two laser pulses are synchronized within a jitter of about 1 ns. Radiation photons generated by the interaction between the electron bunch and the CO_2 laser pulse are reflected by a beam splitter and guided into a spectrometer. The resolution of the spectrometer was about 11 nm, which was mainly determined by the slit width. The number of the photons is measured by a photo-multiplier. The signals are integrated by a boxcar integrator to increase S/N (signal-to-noise ratio). The diameter and the length of the interaction region was estimated to be about $d = 0.3\,\mathrm{mm}$ in diameter and $l = 8\,\mathrm{mm}$, respectively. The power density of the CO_2 laser and the electron density in the interaction region are estimated to be about $10\,\mathrm{GW/cm^2}$ and $1.5 \times 10^{10}/\mathrm{cm^3}$, respectively. The solid angle of the detection of the radiation was 1.6×10^{-3} s.r. The wavelength of the CO_2 laser was 10.6 μm, which corresponds to the oscillation of P(20) line. Careful attention had been paid to eliminate various kinds of optical and high-energy radiation noises. Especially, hard X-rays generated in the dumping material of the electron beam and the visible radiation from the beam splitter irradiated by the CO_2 laser were serious noise sources.

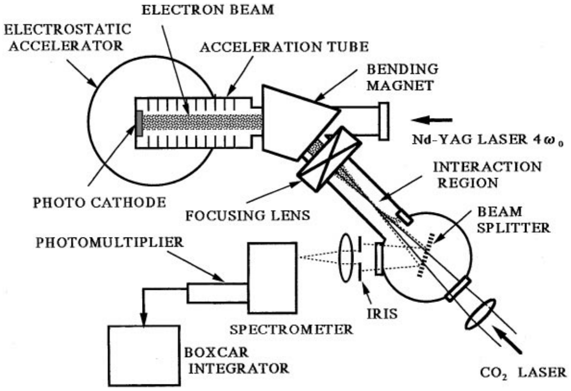

Fig. 3. Experimental setup for the measurement of the laser undulator radiation using a high-power CO_2 laser and a pulsed high-quality electron bunch.

It had already been confirmed that the energy spread of the laser-induced photoelectrons, which was used in this experiment was less than a few electron volts [11], which corresponds to $\Delta\gamma \sim 3\times10^{-5}$. Using $l = 8\,\text{mm}$, $\lambda_0 = 10.6\,\mu\text{m}$, and Eq. (7), the number of periods in this experiment is estimated to be $N = 2\times10^3$. These experimental parameters satisfy Eq. (14), therefore this experiment is in the regime of the laser undulator operation.

4. Experimental results and discussions

Fig. 4 shows a typical signal of the radiation photons as a function of the delay time between the CO_2 laser pulse and the electron bunch. The signal can be observed during a delay time of about 10 ns, which is equivalent to the pulse duration of the CO_2 laser pulse and the electron bunch. The following experiments were done under the condition of the delay time, in which the maximum radiation signal was obtained.

Fig. 5 shows the photon numbers of the undulator radiation as a function of the detecting solid angle. Open circles represent the experimental results.

In the conventional magnetic undulator, the following equation is known to be a simple and convenient formula to calculate the number of photons induced by a single electron into a full solid angle (4π s.r.) through the interaction with the undulator [12]

$$n_s = (2\pi/3)\alpha N K^2/(1 + K^2) \qquad (15)$$

where n_s α, N and K are the number of photons, the fine structure constant, the number of periods of the undulator and the K value of the undulator, respectively. The fine structure constant α is defined by $2\pi e/hc$ and was calculated to be $\frac{1}{137}$.

In this experiment, the effective K value of the laser undulator K_{eff}, which was introduced by Eq. (8) in Section 2, was applied to K in Eq. (15). N is calculated by $N = l/\lambda_{\text{eff}}$, where l is the length of the interaction region and λ_{eff} is the effective pitch length of the laser undulator, which was defined by Eq. (7). These values were applied to Eq. (15) to calculate the total number of photon radiations obtained in this experiment. The dotted line represents the theoretical calculations of the number of photons as a function of the solid angle of the radiation, under the assumption that the radiation photons distribute uniformly within the solid angle of $\Omega = \pi(1/\gamma)^2$. The total number of photons is proportional to the solid angle, and the experimental results were in agreement with the theoretical calculations obtained by Eq. (15) within the experimental errors.

In this calculation, the optical loss from the interaction region to the photo-multiplier is not included, and it was also difficult to estimate the length and the diameter of the interaction region exactly. The sensitivity of the photo-multiplier used in this calculation was the catalog value. The difference between the experimental results and the calculation is considered to be due to the above three factors.

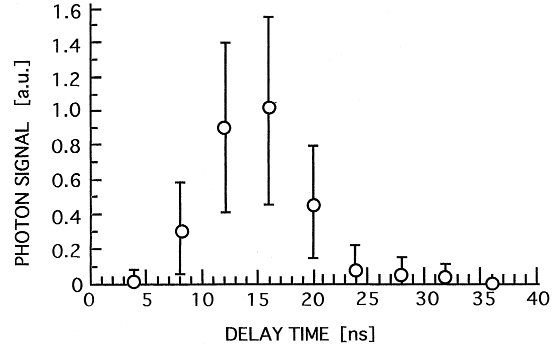

Fig. 4. Radiation photon signal as a function of the delay time between the CO_2 laser pulse and the electron bunch.

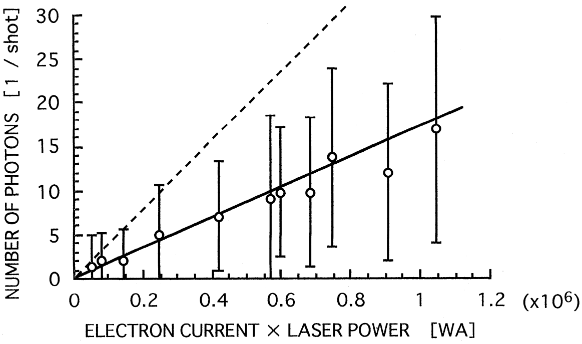

Fig. 5. Photon numbers of the undulator radiation as a function of the detecting solid angle.

Fig. 6 shows the number of radiation photons as a function of the product of the electron beam current and the CO_2 laser power. The solid angle of the detection was 1.55×10^{-3} s.r. and the wavelength of the radiation was 500 nm. The number of radiation photons is proportional to the product of the electron beam current and the CO_2 laser power. Theoretical calculation obtained by Eq. (15) is represented by a dotted line.

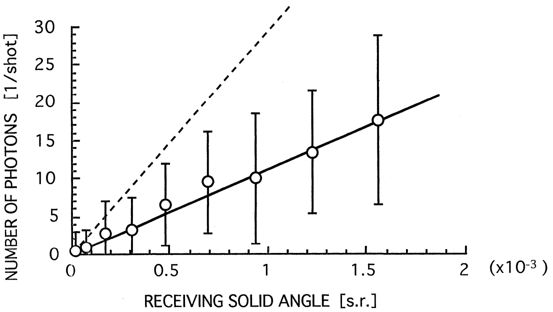

Fig. 6. Radiation photons as a function of the product of the electron beam current and CO_2 laser power.

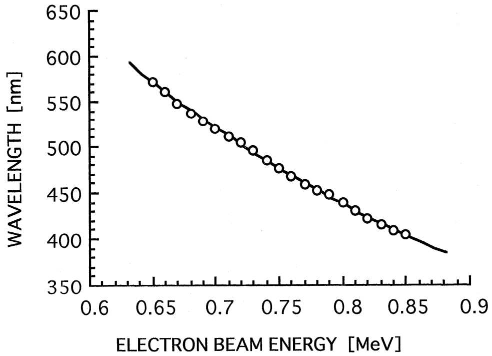

Fig. 7. Wavelength of the radiation as a function of the electron beam acceleration energy.

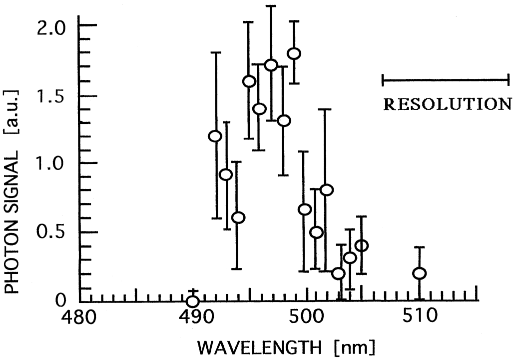

Fig. 8. The spectrum width of the laser undulator radiation. The resolution of the spectrometer was about 11 nm.

Fig. 7 shows the wavelength of the radiation as a function of the electron beam accelerating energy. Open circles represents the experimental results. The solid line represents the theoretical calculation obtained by the following equation:

$$\lambda_s = \lambda_0(1-\beta_{\|})/(1+\beta_{\|}), \tag{16}$$

where λ_s and λ_0 are the wavelength of the optical radiation and that of the laser undulator, respectively. $\beta_{\|}$ is defined by $v_{\|}/c$, where $v_{\|}$ is the parallel component of the velocity of the electron and c is the velocity of light. The experimental results were in good agreement with the theoretical curve calculated by Eq. (16).

Fig. 8 shows the spectrum of the laser undulator radiation, which was measured by a spectrometer having a resolution of about 11 nm. Theoretically, the spectrum width of the undulator radiation is determined by $\Delta\lambda_s/\lambda_s = 1/N$. The spectrum obtained in this experiment was noisy, and its width was almost equal to the resolution of the spectrometer. Although it was difficult to compare the spectrum width of the radiation obtained in this experiment with theoretical calculation exactly, it is much smaller than the resolution limit of the spectrometer.

5. Conclusions

In this paper various characteristics such as the number of photons, the wavelength, and the radiation solid angle of the laser undulator have been measured quantitatively using a high-power pulsed CO_2 laser and a high-quality electron beam having an energy of 0.65–0.85 MeV. The experimental results were in good agreement with the theoretical calculations using the effective K value of the laser undulator introduced in this paper. The criterion to determine the limitation to the number of periods of the undulator, which was determined by the electron energy and the wavelength of the pump laser, was proposed. The K value obtained in this paper will be useful not only for the undulator radiation, but also for the estimation of the gain of free electron lasers using laser undulator [13].

In this experiment, the relation between the spectrum width of the undulator radiation and the emittance of the electron beam could not be discussed due to the lack of the resolution of the spectrometer and the stability of the accelerating voltage of the electron beam.

References

[1] H. Motz, J. Appl. Phys. 22 (1951) 527.

[2] T. Shintake, K. Huke, J. Tanaka, I. Sato, I. Kumabe, Jpn. J. Appl. Phys. 21 (1982) L601.

[3] T. Shintake, K. Huke, J. Tanaka, I. Sato, I. Kumabe, Jpn. J. Appl. Phys. 22 (1983) 844.

[4] S.C. Chen, T.C. Marshall, Phys. Rev. Lett. 53 (1984) 425.

[5] Y. Kawamura, M. Kawai, K. Toyoda, Jpn J Appl. Phys. 27 (1988) 2013.

[6] J.D. Jackson, Classical Electrodynamics, 2nd Edition, Wiley, New York, 1975, p. 681 (Chapter 14).

[7] R.Z. Olshan, A. Gover, S. Ruschin, H. Kleinman, Phys. Rev. Lett. 58 (1987) 483.

[8] T.Yamazaki, T. Noguchi, S. Sugiyama. T. Mikado, M. Tomimasu, IEEE Trans. Nucl. Sci. NS-32.

[9] T. Tanabe, Y. Kawamura, D. Li, K. Toyoda, Rev. Sci. Instrum. 1010 (1995).

[10] Y. Miyake, K. Etoh, Proceedings of the 12th Symposium. on Ion Source and Ion-Assisted Technology, 1989, p. 195.

[11] M. Kawai, Y. Kawamura, K. Toyoda, Appl. Phys. Lett. 45 (1984) 1287.

[12] C. Pellegrini, Physics of Quantum Electronics, Vol. 17, Free Electron Generation of Coherent Radiation, Addision-Wesley, New York, 1980.

[13] D.A. Deacon, L.R. Elias, J.M.J. Madey, G.J. Ramian, H.A. Schwettman, T.I. Smith, Phys. Rev. Lett. 18 (1977) 892.