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# Control of turbulence in a two-dimensional coupled map lattice

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**Abstract**

A turbulent state in a model of two-dimensional coupled map lattices (CMLs) is controlled to homogeneous stable states by using uniform phase space compression, and an equation for controlling the CML model to desired stable states is given in a certain region. Any desired stable patterns in a 2D transverse section are obtained by using nonuniform phase space compression under the turbulent state. The method is also shown to be successful for suppressing turbulence in the CML model to spatiotemporally periodic states by compressing a part of the phase space and be robust against noise. 

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*Keywords:* Control of turbulence; Coupled map lattice model; Phase space compression

In recent decades, turbulence has evolved into a very active field of theoretical physics [1,2]. It is being studied from the point of view of dynamical systems, which rely heavily on simplified models of turbulent behavior. Coupled map lattice (CML) models, which are discrete in both space and time, are often taken as the convenient tools to study turbulence. The investigation of turbulence in CMLs provides reasonable understanding of turbulent phenomena. Another important thing is that the study of controlling turbulence in such systems can offer effective methods for potentially practical applications such as in plasma devices, lasers and reaction–diffusion systems and so on. Some methods of controlling turbulence in CMLs were suggested [3–10]. Generally speaking, there are two kinds of control method in CML: feedback control [3–8] and nonfeedback one [9,10]. Among these methods, we have proposed controlling turbulence via phase space compression [10]. We obtained global and local control of turbulence by compressing system evolution orbit at every site in the control region, and found the functional relationship between control results and control parameters. The result in Ref. [10] is only a numerical analysis for controlling turbulence in a one-dimensional coupled logistic map and not applied to other systems. Furthermore, Ref. [10] did not involve whether the method can be successful to control a part of the system sites and whether it is robust against noise which are much more important for possible applications in practice.

In this Letter, we first choose a two-dimensional CML model then control the turbulent state in the

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model to homogeneous stable state by uniform phase space compression. The numerical results give an equation for controlling the CML to the desired stable states. According to the equation, we control the turbulent state to various desired stable patterns in a 2D transverse section by nonuniform phase space compression. We also show that the control method is robust against noise and obtain the same control equation above. Finally, we control turbulence in the model to spatiotemporally periodic states by manipulating parts of the system sites and give the controlled results in which there exists small additive noise. Unlike other feedback algorithms [3–8], the control method does not require any a prior knowledge of the system dynamics or explicit changes in its parameters. Thus this simple method will be easier to implement.

Let us consider a well-known two-dimensional CML model [11]

$$\begin{aligned} I_{n+1}(i,j) &= (1-D) f\big[I_n(i,j)\big] \\ &\quad + \frac{D}{4}\Big\{ f\big[I_n(i-1,j)\big] + f\big[I_n(i+1,j)\big] \\ &\qquad + f\big[I_n(i,j-1)\big] + f\big[I_n(i,j+1)\big]\Big\}, \end{aligned} \tag{1}$$

where $n = 1, 2, \ldots, N$ are the discrete time steps, $i, j$ denotes the two-dimensional lattice point ($i, j = 1, 2, \ldots, L =$ system size), $D$ is the coupling strength to the nearest neighbor sites (generally speaking, it describes diffusive or diffractive coupling in space), and $f(I_n)$ governs the local dynamics. The earlier theoretical work on the bifurcation and chaos of laser oscillation output in a ring cavity with gain medium obtained a first-order iterative equation [12]

$$I_{n+1} = f(I_n) = I_n \exp\left(\frac{\alpha}{1+I_n} - \Gamma\right), \tag{2}$$

where $I_n$ stands for the light intensity, $\alpha$ is the gain coefficient of the medium in the cavity, $\Gamma$ is the losses of the cavity. When $|1 - \Gamma(1 - \Gamma/\alpha)| > 1$, the state of Eq. (2) is unstable. Ref. [12] solved Eq. (2) numerically by increasing $\alpha$ slowly and at $\Gamma = 4$. The solution undergoes a Hopf bifurcations, and finally gets into chaos at $\alpha = 12.7$. $I^* = (\alpha - \Gamma)/\Gamma$ is unstable steady state in the chaotic region. Ref. [13] considered Eq. (1) as the spatially extended system of Eq. (2), it assumed that the incident light of laser is plane wave (homogeneous distribution in its initial transverse section) and the light intensity outside laser beam is zero (boundary value is zero). Instead of random initial conditions and periodic boundary conditions which are often used in CML models, Ref. [13] took homogeneous initial state and rigid boundary condition

$$\begin{aligned} &I_n(k,j) = I_n(i,l) = I_n(k,l) = C_1, \\ &I_0(i,j) = C_2, \\ &i, j = 1, 2, \ldots L, \quad k, l = 0, L+1, \end{aligned} \tag{3}$$

where $C_1$ is boundary value and $C_2$ is initial value of system, they may be same or different constants. Ref. [13] solved Eq. (1) numerically by increasing $\alpha$ ($\alpha > 11.0$), other parameters are fixed at $\Gamma = 11.0$, $D = 0.2$, $L = 64$, $C_1 = 0$, $C_2 = 0.5$. The solution is initially in the homogeneous stationary state except the edge regions. As $\alpha$ exceeds the bifurcation point $\alpha = 13\frac{4}{9}$, homogeneous travelling wave solution appears. With further increase of $\alpha$, homogeneous travelling wave solution undergoes a sequence of bifurcations with its period doubling successively in time, and finally gets into turbulent states.

In order to start to control turbulence in Eq. (1) at the $(n+1)$th iterative, we select the motion of Eq. (1) that is turbulent at $\alpha = 15.0$, $\Gamma = 11.0$, $D = 0.2$ [13] in all numerical simulations. Suppose the strange attractor in CML (1) occupies a bounded phase space $V$ after transient iterations. We choose a nonempty subset $W$, $W \subset V$. Let $I_n(i,j)$ $(i, j = 1, 2, \ldots, L)$ be limited to $W$, thus $I_n(i,j)$ is changed to

$$\begin{aligned} &I_n(i,j) = \begin{cases} I_n(i,j), \\ \quad I_n(i,j)_{\min} < I_n(i,j) < I_n(i,j)_{\max}, \\ I_n(i,j)_{\max}, \quad I_n(i,j) \geqslant I_n(i,j)_{\max}, \\ I_n(i,j)_{\min}, \quad I_n(i,j) \leqslant I_n(i,j)_{\min}, \end{cases} \\ &i, j = 1, 2, \ldots, L, \end{aligned} \tag{4}$$

where $I_n(i,j)_{\max}, I_n(i,j)_{\min} \in W$. For global control: if we use uniform phase space compression at all system sites, we can take $I_n(i,j)_{\max} = I_{\max}$, $I_n(i,j)_{\min} = I_{\min}$, $i, j = 1, 2, \ldots, L$, where $I_{\max}$, $I_{\min} \in W$; if we use nonuniform phase space compression at all system sites then $I_n(i,j)_{\max}$ and $I_n(i,j)_{\min}$ will take different values, respectively, in the selected lattice sites or regions. If we control a part of phase space to suppress the turbulence, $I_n(i,j)_{\max}$ and $I_n(i,j)_{\min}$ are turned on at the selected sites and turned off at the rest of the system sites.

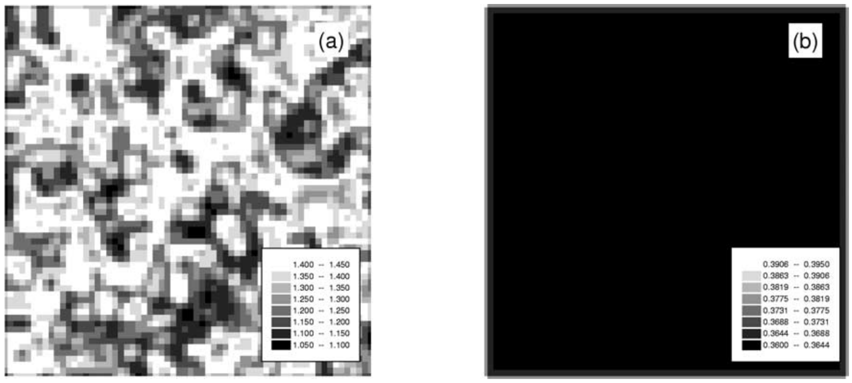


Fig. 1. Control of turbulence by uniform phase space compression. The system parameters are $\alpha = 15.0$, $\Gamma = 11.0$, $D = 0.2$, $C_1 = 0$, $C_2 = 0.5$, $L = 64$, the control parameters $I_{\max} = I^* = 4/11$, $I_{\min} = 0$. (a) Pattern at $n = 1000$ before control. (b) Pattern which result is $I_c = I^*$ after control.

## Controlling turbulence by uniform phase space compression

To illustrate the effects of uniform phase space compression on the two-dimensional CML model, we choose eight shades of color to represent the system states. The results presented in Figs. 1(a) and (b) show the transverse patterns of Eq. (1) before and after control with $\alpha = 15.0$, $\Gamma = 11.0$, $D = 0.2$, $L = 64$, $C_1 = 0$, $C_2 = 0.5$. We take $n = 1000$ in Fig. 1(a), and in Fig. 1(b) we fix the lower boundary of $V$ unchanged $I_{\min} = 0$ and let $I_{\max} = I^* = 4/11$. One can see, the motion of the system is turbulent before control and the turbulent motion can be successfully suppressed to the homogeneous steady state $I^*$ (except edge regions) after the uniform phase space compression is input. By selecting different compression parameters $I_{\max}$ and $I_{\min}$ to control turbulence in Eq. (1), one can obtain other homogeneous stable states. In numerical simulations, we find that when $0 \leqslant I_{\max} \leqslant I^*$ and $I_{\min} = 0$, turbulence in Eq. (1) can be controlled to homogeneous stable state $I_c$ except edge regions. The curve of stabilized state $I_c$ versus $I_{\max}$ is shown in Fig. 2 where the squares represent the results of numerical simulations, their functional relationship may be written as

$$I_c = I_{\max} \exp\left(\frac{\alpha}{1 + I_{\max}} - \Gamma\right), \quad \text{for } I_{\min} = 0. \tag{5}$$

Obviously at $0 \leqslant I_{\max} \leqslant I^*$, Eq. (5) has the same form as the local dynamics of Eq. (1). $I_c$ is only the function of system parameters $\alpha$ and $\Gamma$, control parameters $I_{\max}$ and $I_{\min}$, which bears no relation to coupling strength $D$. Fig. 2 shows that $I_c$ has a maximum value which is close to the upper boundary of phase space $V$, so Eq. (5) is very useful for controlling turbulence in CML (1). We can control the turbulent state to any desired homogeneous stable state $I_c$ by appropriately choosing $I_{\max}$ from Eq. (5). Furthermore, it is worthwhile to point out that all results of the numerical simulations in Fig. 2 are still obtained under random initial conditions and periodic boundary conditions.

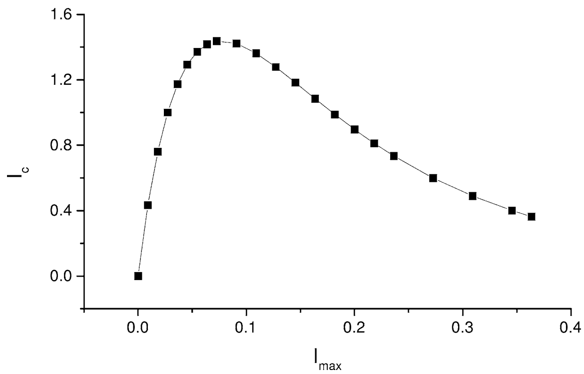


Fig. 2. Stabilized state $I_c$ versus $I_{\max}$ with $I_{\min} = 0$. The data points represent the numerical results, which are controlled to homogeneous stable states except edge regions. Each control begins at $n = 1001$, the system parameters are the same as in Fig. 1.

Adding random noise to dynamical equation (1) in numerical simulations, we control the turbulent state to homogeneous stable state $I_c$ and obtain the same results in Fig. 2. Only in the edge regions, the stabilized states are different from ones of no noise. So the method possesses robustness while it works.

Attractor of spatiotemporal chaos arises from the competitive effect of repeated stretching and folding in phase space from a theoretical viewpoint. Such a process produces very complex and irregular structures. Because phase space compression limits free contraction and expansion of the chaotic attractor, by appropriately choosing phase space compression parameters, one can control turbulent state to different stable states. And phase space compression can effectively suppress additive noise.

## Controlling turbulence by nonuniform phase space compression

We choose different $I_n(i, j)_{\max} = I_{\max}$ in different lattice regions and $I_n(i, j)_{\min} = 0$ in the entire lattice region. Taking desired values of $I_{\max}$ at different selected lattice regions according to Eq. (5) or Fig. 2 and controlling turbulence in Eq. (1), we obtain various desired stable patterns shown in Fig. 3. In Fig. 3, the system parameters are the same as in Fig. 1, phase space compression starts at $n = 1001$. Here we give four globally stable patterns, they are (a) strip pattern, (b) circle pattern, (c) square pattern, and (d) starlike pattern. Certainly, by choosing different phase space compression parameters according to Eq. (5) and/or selecting different lattice regions, one can obtain other desired stable patterns. Any desired stable patterns in a 2D transverse section are difficultly obtained by most of the exiting control strategy since parametric changes are needed to achieve the control. The present method can effectively obtain them because a controlled site in this method exerts its influence only to its neighboring sites under the control situations and according to control equation (5) we can obtain any desired stabilized values in the phase space of chaotic attractor. It should be remarked that the state at the edge of every control region is different from the one in the control region. This phenomenon is called edge effect in Ref. [10].

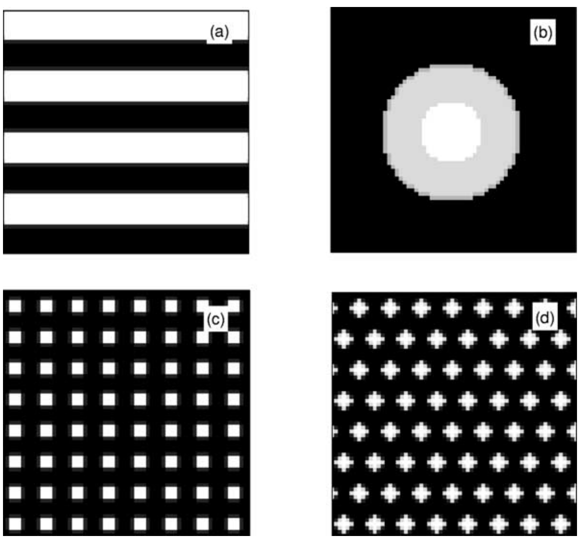


Fig. 3. Various desired stable patterns obtained by nonuniform phase space compression. Control parameter $I_{\max}$ in different control regions is selected according to Eq. (5), $I_{\min} = 0$. Other parameter values are the same as in Fig. 1 before and after control.

## Controlling turbulence by compressing a part of phase space

Controlling the system dynamics over the entire spatial domain is neither very efficient nor always practically feasible. This method can also be used to suppress turbulence in the CML model to spatiotemporally periodic states by compressing a part of phase space. We take the same turbulent state as shown in Fig. 1(a) and use uniform phase space compression to control turbulence of Eq. (1) but select the control sites in the following way:

$$I_n(i, j)_{\max} = I_{\max},$$
$$I_n(i, j)_{\min} = 0,$$
$$i = 1, 2, \ldots, L, \quad j = 1, 1 + d, 1 + 2d, \ldots, \tag{6}$$

where $d$ is the distance between two neighboring control sites. We leave the rest of the system sites undisturbed, and let $I_{\max} = I^* = 4/11$. Fig. 4 shows that the turbulent state is successfully controlled to the spatially periodic state, respectively, for $d = 2, 3, 4$, in which the patterns change in (a) time period-2, (b) time period-4, (c) time period-4. When $d \geqslant 5$, the turbulent state can be suppressed but not controlled to spatially periodic state. Thus, this method can be used to suppress turbulence in systems where controlling all system sites may not be feasible.

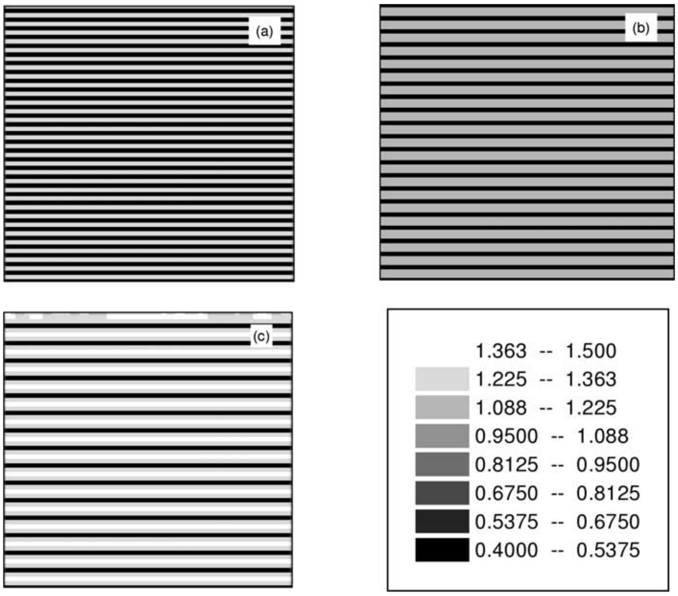


Fig. 4. Spatial patterns obtained by compressing a part of phase space. Control parameters are $I_{\max} = I^*$, $I_{\min} = 0$. Control intervals are (a) $d = 2$, (b) $d = 3$, (c) $d = 4$. Other parameter values are the same as in Fig. 1 before and after control.

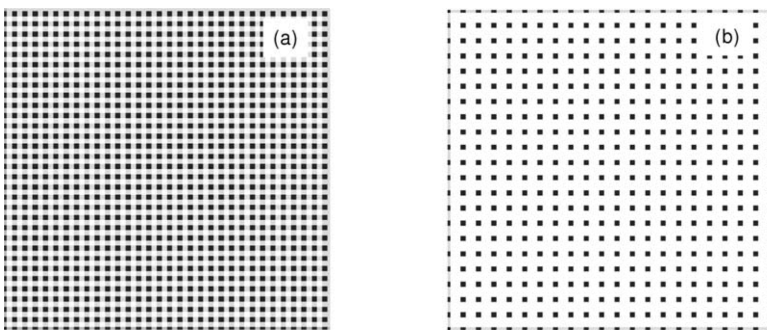


Fig. 5. Spatial patterns obtained by compressing a part of phase space. Control parameters are $I_{\max} = I^*$, $I_{\min} = 0$. Control intervals are (a) $d = 2$, (b) $d = 3$. Other parameter values are the same as in Fig. 1 before and after control.

If we select control sites in Eqs. (6) as follows:

$$i, j = 1, 1 + d, 1 + 2d, \ldots,$$

the turbulence of the CML can be control to square structure in a 2D transverse section as shown in Fig. 5 with $d = 2$ or 3. Although the transverse patterns in Fig. 5 are similar to the one in Fig. 3(c), they are different. The spatial patterns in Fig. 3 are stable, but the spatial patterns in Fig. 5 are time period-4. To illustrate the spatiotemporal patterns of CML (1) under this situation, we plot the time evolution of some individual site before and after control in Fig. 6. The control is turned on at iteration step 1001. It is shown in Fig. 6 that the turbulent state is controlled to different time period-4 states with different parameter $d$. Compared with the patterns in Ref. [7]: (1) Our simulation results may or may not be the solutions of the original system, since the nonfeedback does not vanish when the system is under control; but their stabilized spatiotemporal pattern states are surely the solutions of the original system, since the feedback vanishes when the control is achieved. (2) More than two states in our patterns can be obtained by using appropriate parameter $d$. In other words, our patterns have richer struc-

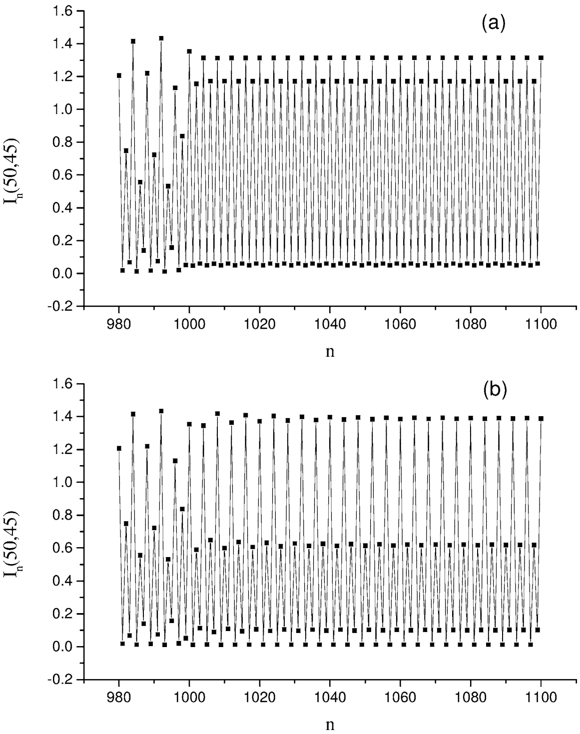


Fig. 6. Time evolution of some individual site before and after control by compressing a part of phase space. The data points represent the numerical results. Compression begins at iteration step 1001. Control intervals are (a) $d = 2$, (b) $d = 3$. Other parameter values are the same as in Fig. 5.

tures than their patterns which only have two states of the original solutions of the system. (3) We cannot give mathematical expression of the controlled results by compression a part of phase space. In numerical simulations, we find that the control easily achieves by controlling a part of phase space under weakly turbulent state. With the increase of the system parameter $\alpha$, success rate decreases, and at strongly turbulent state (for example, $\alpha = 15.6$), $I_n(i, j)$ at all the sites must to be compressed into $W$. Adding small noise, random numbers in the interval $[0, 0.005]$, to Eq. (1), we obtain the patterns in Fig. 7. These patterns show that the method can effectively suppress the turbulent state by compressing a part of phase space under small levels of additive noise.

To summarize, we have shown that turbulence in a two-dimensional coupled map lattice is controlled to homogeneous stable states by using uniform phase space compression. Under this condition, a control equation is given by numerical simulations in a certain region. According to the control equation, we obtain any desired stable patterns in a 2D transverse section by using nonuniform phase space compression. We further investigate the suppression of turbulence in CML (1) by compressing a part of phase space and achieve spatiotemporally periodic patterns. Obtaining global control of turbulence by selecting parts of all system sites provides considerable convenience in experiment. Because this simple method is powerful, flexible, and robust against noise and very easy to implement by using spatial amplitude limiter, it provides a hope of applications in many physical systems. Certainly, the study of control of turbulence in practice is still in its infancy, and many challenging problems await scientists and technologists to solve.

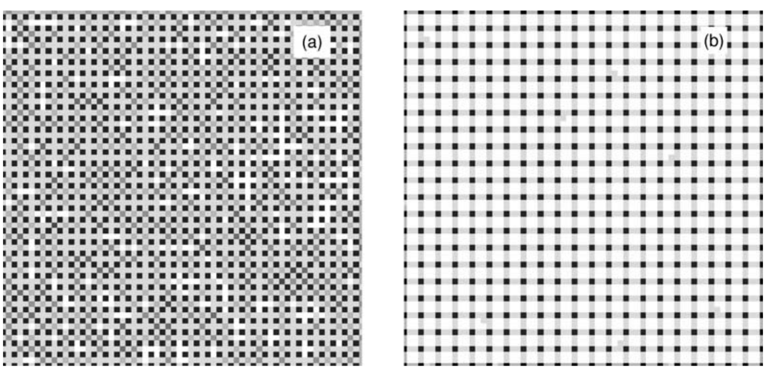


Fig. 7. Spatial patterns obtained by compressing a part of phase space when small noise is added to Eq. (1). Control intervals are (a) $d = 2$, (b) $d = 3$. Control parameters and system parameters are the same as in Fig. 5.

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