# Focus retrocollimated interferometry for focal-length measurements 

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#### Abstract

Focus retrocollimated interferometry is developed for the measurement of focal lengths of optical lenses and systems, and achievable accuracy is discussed. It is shown that this method can be used to measure both short and long focal lengths simply and with high accuracy. © 2002 Optical Society of America

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## 1. Introduction

The focal length is a fundamental and important parameter of an optical lens or system for which a number of measurement methods have been reported. ${ }^{1-8}$ Methods that use nodal slides, image magnification, etc. are valuable for lenses with a short focal length ( $<1 \mathrm{~m}$ ) and provide measurement accuracy of between $\pm 0.04 \%$ and $\pm 0.3 \%$. Talbot interferometry has been proposed ${ }^{1,2}$ for the measurement of long focal lengths but has proved to yield inaccurate results. By technical modification, one can use Talbot methods to measure not only long focal lengths but also short focal lengths to a relative accuracy of between $\pm 0.2 \%$ and $\pm 0.03 .^{3-8}$
Accurate and simple focal-length measurements are necessary for the measurement and manufacture of radii of curvature. ${ }^{4,5}$ Here I present details of focus retrocollimated interferometry (FRCI) for focallength measurement that provides a twofold increase in simplicity and accuracy.

## 2. Measurement Principle

The optical principle of FRCI is shown in Fig. 1, where L is the lens under test, F and $\mathrm{F}^{\prime}$ are its front and back foci, respectively, S is the spherical reference positioned at $\mathrm{F}^{\prime}$ to realize two-beam interference, $\mathrm{O}^{\prime}$ is the center of curvature of S , and O is the image of $O^{\prime}$ through $L$. With this arrangement, the beam from O is normally incident upon S after it

[^0]passes through L, and the retroreflected beam converges to O after it passes twice through L . Let $f$ and $f^{\prime}$ be the front and the back focal lengths of $L$, respectively, $R$ is the radius of curvature of S , and $x$ is the imaging distance of $\mathrm{O}^{\prime}$ through L . According to Newton's formula there is
\[

$$
\begin{equation*}
f f^{\prime}=R x . \tag{1}
\end{equation*}
$$

\]

Since

$$
\begin{equation*}
f^{\prime}=f, \tag{2}
\end{equation*}
$$

the focal length of L is given by

$$
\begin{equation*}
f=\sqrt{x \times R}, \tag{3}
\end{equation*}
$$

where $R$ is known so only $x$ has to be measured.

## 3. Measurement Method and Procedure

To achieve high accuracy we used the interferometer shown in Fig. 2 to measure $x$. The interferometer consists of a laser, a beam expander, a beam splitter (BS), a half-mirror $\left(\mathrm{P}_{1}\right)$, a transmission sphere $\left(L_{1}\right)$, a plane mirror $(\mathrm{P})$, a spherical reference (S), and a slide mechanism (not shown). The laser beam was used to expand the beam in the parallel direction and was then used to divide the beam into a reference beam and a test beam by $\mathrm{P}_{1}$.
Three steps are required for the measurement of $x$ as shown in Figs. 2(b)-2(d).
First, as shown in Fig. 2(b), L is inserted into the test arm and moved until $\mathrm{F}^{\prime}$ is positioned at S . This can be accomplished by observation of the interference pattern until a null fringe pattern appears. The distance between $L$ and $S$ must remain unchanged during the following two adjustment steps.
Second, as shown in Fig. 2(c), $\mathrm{L}_{1}$ is inserted between $\mathrm{P}_{1}$ and $\mathrm{L}, \mathrm{P}$ between L and $\mathrm{S}, \mathrm{L}_{1}$ is moved until


Fig. 1. Principle of the focus retrocollimated focal-length measurements.


Fig. 2. Focal-length measurement configuration and procedures.
its back focus coincides with F. Again, the interference pattern should be observed until a null fringe pattern is obtained.
Third, as shown in Fig. 2(d), P is removed and $\mathrm{L}_{1}$ is moved until its back focus coincides with O. Again, the interference pattern should be observed until a null fringe pattern is obtained.
$\mathrm{L}_{1}$ moves a distance equivalent to $x$. If $x$ is substituted into Eq. (3), one can obtain the value of $f$.

## 4. Error Analysis

According to Eq. (3), the possible measurement uncertainty in the value of $f$ is mainly the result of measurement errors $\Delta R / R$ in $R$ and $\Delta x$ in $x$. Differentiating Eq. (3) we have

$$
\begin{equation*}
\frac{\delta f}{f}=\frac{1}{2}\left(\frac{\Delta x}{x}+\frac{\Delta R}{R}\right) . \tag{4}
\end{equation*}
$$

From Eq. (4) the relative error $\Delta f / f$ in $f$ is given by

$$
\begin{align*}
\frac{\Delta f}{f} & = \pm \frac{1}{2}\left[\left(\frac{\Delta x}{x}\right)^{2}+\left(\frac{\Delta R}{R}\right)^{2}\right]^{1 / 2} \\
& = \pm \frac{1}{2}\left[\left(\frac{R}{f^{2}}\right)^{2}(\Delta x)^{2}+\left(\frac{\Delta R}{R}\right)^{2}\right]^{1 / 2} . \tag{5}
\end{align*}
$$

Table 1. Relations between $\Delta W$ and Wave Aberration

| Observation Method | $\|\Delta W\|$ | Wave Aberration |
| :--- | :--- | :---: |
| Visual | $\leq \lambda / 2$ | $>3 \lambda$ |
|  | $\lambda / 10$ to $\lambda / 20$ | $<3 \lambda$ |
| Digital analysis | $\lambda / 20$ to $\lambda / 50$ |  |

Table 2. Effect of Interferometry Sensitivity on Null Position Determination

| $\|\Delta x\|(\mu \mathrm{m})$ |  | $\|\Delta W\|(\lambda=0.6328 \mu \mathrm{~m})$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\lambda / 2$ | $\lambda / 5$ | $\lambda / 10$ | $\lambda / 20$ | $\lambda / 50$ |
| $f / D$ | 2 | 10.1 | 4.1 | 2.0 | 1.0 | 0.4 |
|  | 3 | 22.8 | 9.1 | 4.6 | 2.3 | 0.9 |
|  | 4 | 40.5 | 16.2 | 8.1 | 4.1 | 1.6 |

Equation (5) shows that the smaller $|\Delta x|$ and $|\Delta R / R|$ the smaller $|\Delta f / f|$. $|\Delta f / f|$ decreases when $f$ increases. When $\Delta x, R$, and $\Delta R / R$ are given, however, there is a limit to the amount of decrease because $\Delta R / R$ is independent of $f$.
The high accuracy of $\Delta R / R$ can be determined by interferometry. The achievable accuracy is generally at a level between $0.1 \%$ and $0.01 \%$ and can reach $0.001 \%$ for $100 \mathrm{~mm}<R<1000 \mathrm{~mm}$ by careful alignment. 9 Under ideal alignment conditions, $\Delta x$ can be determined mainly based on the accuracy of the slide and the sensitivity of the interferometer. According to wave theory aberration, ${ }^{10}$ we have

$$
\begin{equation*}
\Delta x=8\left(\frac{f}{D}\right)^{2} \Delta W \tag{6}
\end{equation*}
$$

where $\Delta W$ is the interferometry sensitivity as determined by both observation and wave aberration of $L$ and $f / D$ is the relative aperture of $L$. The relation between $\Delta W$ and wave aberration is listed in Table 1.


Fig. 3. Variation of $|\Delta f / f|$ with $f$ for $R=20 \mathrm{~mm} . \quad|\Delta R / R|$ equals $0.1 \%$ in (a) and $0.01 \%$ in (b). For $|\Delta x|$, the filled squares represent $5 \mu \mathrm{~m}$, the filled circles represent $10 \mu \mathrm{~m}$, the filled triangles represent $20 \mu \mathrm{~m}$.


Fig. 4. Variation of $|\Delta f / f|$ with $f$ for $R=200 \mathrm{~mm} . \quad|\Delta R / R|$ equals $0.1 \%$ in (a), $0.01 \%$ in (b), $0.001 \%$ in (c). The symbols have the same meaning as in Fig. 3.

The values of $|\Delta x|$ calculated with Eq. (6) and Table 1 for some $\Delta W$ and $f / D$ are listed in Table 2.

A quantitative analysis was carried out for $\Delta f / f$ to obtain a clear knowledge of the variation of $\Delta f / f$ with $f$ to achieve $\Delta R / R$ and $\Delta x$. Figures 3-6 show curves of $|\Delta f / f|$ versus $f$ for different $R,|\Delta R / R|$, and $|\Delta x|$.
From Fig. 3-6 one can conclude that
(1) It is feasible and easy to measure a focal length at a level of $0.05 \%$ accuracy. With an accurate spherical reference, this accuracy can be lowered to the range of approximately $0.005 \%$ and below this level requires careful consideration of $\Delta x / x$.
(2) For shorter $x\left(|\Delta x / x|>10^{-3}-10^{-4}\right), \Delta x$ is severe. A smaller $\Delta x$ has a larger deviation in $f$, hence, its value should be known accurately. In other words, extreme care of the null position is essential for shorter imaging distance to achieve high accuracy.
(3) High accuracy can also be achieved by choosing an appropriate spherical reference. If $R$ is so small that $|\Delta x / x|$ is negligible compared with $|\Delta R / R|$ and $|\Delta R / R|$ can be known as accurately as possible, this method can give a much smaller value of $|\Delta f / f|$.




Fig. 5. Variation of $|\Delta f / f|$ with $f$ for $R=800 \mathrm{~mm}$. The values of $|\Delta R / R|$ are the same as in Fig. 4, and the symbols have the same meaning as in Fig. 3.



Fig. 6. Variation of $|\Delta f / f|$ with $f$ for $R=5000 \mathrm{~mm} . \quad|\Delta R / R|$ equals $0.1 \%$ in (a) and $0.01 \%$ in (b). The symbols have the same meaning as in Fig. 3.

## 5. Conclusion

The principle of FRCI is entirely different from earlier interferometry methods in two respects:
(1) The entire measurement is performed by twobeam interference.
(2) The spherical reference is at the back focus of the test lens.

These two characteristics provide accurate focallength measurements. The results show that FRCI can be used to measure focal lengths within any range and a higher degree of accuracy can be achieved if the appropriate slide mechanism and radius of curvature are chosen. Because the alignments are routine, the FRCI is straightforward and easy to implement. This method should provide new possibilities for the use of focal-length measurements.

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