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The electromagnetically induced negative refractive index in the Er\textsuperscript{3+}:YAlO\textsubscript{3} crystal

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Abstract

We carried out the negative refractive index in the solid medium Er\textsuperscript{3+}:YAlO\textsubscript{3} crystal with a $\Lambda$-type four-level scheme proposed for atomic vapour by Thommen and Mandel, and Kästel (\textit{Phys. Rev. Lett.} 2006 \textbf{96} 053601 and 2007 \textbf{98} 069301) based on quantum interference and electromagnetically induced transparency (EIT). The results show that the frequency band with the negative index is much wider ($\sim 1$ MHz) than reported previously. Usually, \text{Im}[n]$ is always positive, corresponding to absorption, and the figure of merit (FOM, the ratio of real to imaginary, namely $|\text{Re}[n]/\text{Im}[n]|$) is only on the order of unity. We achieve $\text{FOM} = |\text{Re}[n]/\text{Im}[n]| = 4.6$. The corresponding negative \text{Im}[n]$ is related to the stimulated emission of $^4I_{13/2} \rightarrow ^4I_{15/2}$ of the rare earth ion Er\textsuperscript{3+} under external electromagnetic fields. It is concluded that the rare earth ion doped material with abundant energy levels and various electric and magnetic transitions is an outstanding and practical candidate for the electromagnetically induced negative refractive index material.

1. Introduction

Owing to fascinating and counterintuitive electromagnetic and optical features as well as potential applications such as perfect lens and optical cloaking, negative-index metamaterials (NIMs), or left-handed metamaterials (LHMs), have attracted considerable attention \cite{1–3}. Most LHMs are artificially assembled effective media, but negative refraction does occur in natural materials \cite{4}. We can realize LHMs by two kinds of approach. The first requires delicate manufacturing of spatially periodic structures based on classical electromagnetic theory, such as artificial composite metamaterials \cite{5, 6}, photonic crystals \cite{7} and chiral materials \cite{8}. In particular, recently the three-dimensional NIMs constructed with a fishnet structure and silver nanowires designed for visible light have been fabricated by the group of Xiang Zhang \cite{9, 10}. This has brought an enormous boost to applications. However, the fabrication of these small periodic structures remains a challenge. In particular, creating a magnetic response at optical frequencies is still difficult. For the second, we can use the quantum interference effect to drive the refractive index negative, e.g. coherent atomic vapours were first simultaneously and independently worked out by Shen and Oktel in 2004 \cite{11, 12}. The physical mechanism of the quantum optics approach is based on the quantum interference and coherence that arise from the transition processes in a multilevel system \cite{11–14}. Thommen and Mandel \cite{13} and Kästel \cite{14} proposed a novel scheme to induced NIMs by quantum interference in an atomic four-level system. In this paper we acquire the requirements for energy level configuration in the most promising scheme and first perform the programs in \cite{13} in the solid medium Er\textsuperscript{3+}:YAlO\textsubscript{3} crystal. The results are spectacular and the gain is simultaneously achieved. The frequency band with a negative index is wider ($\sim 1$ MHz) than what has been reported previously.

2. Descriptions of the model

Let us consider a $\Lambda$-type four-level $|1\rangle$–$|4\rangle$ scheme \cite{13, 14} corresponding to the energy level configuration $^4I_{15/2}$, $^4I_{13/2}$, $^4I_{9/2}$ and $^2H_{11/2}$ of the rare earth ion Er\textsuperscript{3+} doped YAlO\textsubscript{3} crystal, respectively, as depicted in figure 1. The levels $|1\rangle$ and $|2\rangle$ have the same parity and $\hat{\mu}_{12} = \langle 1|\hat{\mu}|2\rangle \neq 0$, where $\hat{\mu}$ is...
A four-level medium can be electromagnetically induced negative refraction, levels |1⟩–|4⟩ corresponding to the energy level of Er³⁺ ions ⁴I_{13/2}, ⁴I_{15/2}, ⁴I_{11/2}, and ⁴H_{11/2}, respectively.

The magnetic dipole operator; the fields E_p and E_d with the Rabi frequencies Ω_p and Ω_d are introduced to couple and drive |2⟩ → |4⟩ and |1⟩ → |3⟩ electric dipole transitions, respectively. A weak probe field E_p with a Rabi frequency Ω_p interacts with |3⟩ → |4⟩ electric dipole transitions. The scheme has three features: (1) the electric dipole transition [3] → [4] and the magnetic dipole transition |1⟩ → |2⟩ do not have to involve common states. This is a very important feature, which greatly enhances the freedom of choice of levels and makes the scheme much more applicable than previous proposals. (2) The energy separations are hω₁,₂ ≈ 6600 cm⁻¹, hω₃₄ ≈ 6740 cm⁻¹, hω₄₂ ≈ 12 650 cm⁻¹, and hω₃₁ ≈ 12 250 cm⁻¹, respectively (see table 1) [15, 16]. Consequently, there are ω₁₂ ≈ ω₃₄, ω₁₃ ≈ ω₄₂ or degeneration conditions. Left-handed properties of a four-level medium can be electromagnetically induced provided the medium verified the condition [13]. Using μ₁₂ = (1/|e|²) ≠ 0, coherence ρ₁₂ drives a magnetic dipole oscillating at the frequency ω₁₂ ≈ ω₄₂, which ensures tuning simultaneously the dielectric permittivity ε, and the magnetic permeability μ. (3) A coherent cross-coupling of electric and magnetic dipole transitions, which couple the corresponding component of the probe field.

In the framework of the semi-classical theory, by the standard density matrix formalism under the dipole approximation and the rotating wave approximation, the interaction Hamiltonian H_I can be represented in the interaction picture by

\[ H_I = -\hbar \left[ \Omega_p |4⟩⟨3| + \Omega_d |2⟩⟨3| + \hbar c.\right] \]  

The master equation of motion for the density operator in the system can be written as

\[ \dot{\rho} = -\frac{i}{\hbar} [H_I, \rho] - \frac{1}{2} \{\Gamma, \rho\}, \]

in which [\( \Gamma, \rho \)] = \( \Gamma \rho + \rho \Gamma \), \( \Gamma_{ij} = \Gamma_m \delta_{ij} \), \( \Gamma_m \) is the relaxation rate of the level |m⟩. By expanding equation (2), we can easily arrive at their equations of motion:

\[ \rho_{44} = -(\Gamma_{43} + \Gamma_{42} + \Gamma_{41}) \rho_{44} + i\Omega_e (\rho_{34} - \rho_{43}) + i\Omega_p (\rho_{34} - \rho_{43}), \]

\[ \rho_{33} = \Gamma_{34} \rho_{44} - \Gamma_{31} \rho_{33} - \Gamma_{32} \rho_{33} + \Gamma_{34} \rho_{13} - \rho_{31}, \]

\[ \rho_{22} = -\Gamma_{24} \rho_{44} + \Gamma_{23} \rho_{32} - \Gamma_{21} \rho_{22} + i\Omega_e (\rho_{24} - \rho_{23}), \]

\[ \rho_{43} = -(\Gamma_{43} + \Gamma_{34}) \rho_{34} + i\Omega_e (\rho_{34} - \rho_{44}) + i\Omega_p (\rho_{34} - \rho_{43}), \]

\[ \rho_{11} + \rho_{22} + \rho_{33} + \rho_{44} = 1, \]

where \( \Gamma_j \) denotes the population spontaneous decay from |j⟩ to |j⟩, while \( \gamma_{ij} \) are total coherence relaxation rates between |i⟩ and |j⟩, given by

\[ \gamma_{34} = \gamma_{34} = (\Gamma_{41} + \Gamma_{42} + \Gamma_{43} + \Gamma_{32} + \gamma_{34}^{dph})/2, \]

\[ \gamma_{42} = \gamma_{42} = (\Gamma_{41} + \Gamma_{42} + \Gamma_{43} + \Gamma_{21} + \gamma_{42}^{dph})/2, \]

\[ \gamma_{41} = \gamma_{41} = (\Gamma_{41} + \Gamma_{42} + \Gamma_{43} + \gamma_{41}^{dph})/2, \]

\[ \gamma_{32} = \gamma_{32} = (\Gamma_{32} + \Gamma_{31} + \Gamma_{21} + \gamma_{32}^{dph})/2, \]

\[ \gamma_{31} = \gamma_{31} = (\Gamma_{32} + \Gamma_{31} + \gamma_{31}^{dph})/2, \]

\[ \gamma_{21} = \gamma_{21} = (\Gamma_{21} + \gamma_{21}^{dph})/2. \]

The \( \gamma_{ij}^{dph} \) are the dephasing decay of the quantum coherence of the |j⟩ → |j⟩ transition. The \( \Delta_{ij} \) are given by \( \Delta_{41} = \Delta_c + \Delta_{02} + \Delta_{021}, \Delta_{42} = \Delta_c + \Delta_{012}, \Delta_{43} = \Delta_p - \Delta_d + \Delta_{02} - \Delta_{02}, \Delta_{31} = \Delta_c - \Delta_p + \Delta_d + \Delta_{02}, \Delta_{32} = \Delta_c - \Delta_p + \Delta_d + \Delta_{02} \) and \( \Delta_{21} = \Delta_{021} \). Here \( \Delta_{0ij} \) (i, j = 1, 2, 3, 4) are the deviations of transition frequencies from the corresponding inhomogeneous line centres, and \( \Delta_i \) (i = c, d, p) are the detuning of the couple, drive and probe fields, respectively.

In the limit of a weak probe, under the steady state condition, the solutions of equation (3) for \( \rho_{43} \) to the first order of the probe field and for other \( \rho_{ij} \) to the zero order of the probe field are as follows:

\[ \rho_{43} = \frac{i\Omega_p}{(\gamma_{34} + i\Delta_{34})} 2\gamma_{34} \gamma_{31} \left[ 2\gamma_{42} \gamma_{41} (\Gamma_{43} + \Gamma_{41} + \Gamma_{2} - \Gamma_{32}) + \Gamma_{21} \Gamma_{41} (\gamma_{21}^{dph} + \Delta_{21}^{dph}) \right], \]
\rho_{21} = \frac{\Omega_p^{(3)}}{\Omega_p^{(3)}} - \frac{\Omega_d^{(4)}}{\Omega_e^{(4)}} \rho_{43} \\
= \frac{\Omega_p^{(4)}}{\Omega_e^{(4)}} \times \langle \gamma_1 + \Delta_3 | \hat{\mathcal{O}}_d \rangle \left[ 2 \Omega_e^{(2)} \langle \gamma_2 | (\Gamma_3 + \gamma_1) \rangle \times (-\Gamma_3 - \Gamma_4 - \Gamma_2 + \Gamma_2 \Gamma_3 \Gamma_4) \right. \\
\left. - \Gamma_2 \Gamma_4 (\Gamma_3 + \gamma_1) \langle \gamma_2 | + \Delta_2 \rangle \right] \times \frac{\Omega_d^{(4)}}{\Omega_e^{(4)}} \rho_{43}, \quad (6)

where

\[ D = \left[ \gamma_1 \gamma_2 \hat{\mathcal{O}}_d^{(2)} \hat{\mathcal{O}}_d^{(3)} (\Gamma_3 + \gamma_4 + \Gamma_3 + \gamma_4) \right. \\
\left. - 2 \Omega_e^{(2)} (\gamma_2^2 + \Delta_2^2) \gamma_2 \Gamma_1 (\gamma_3 + \gamma_4 + \Gamma_3 + \gamma_4) \right. \\
\left. + \Gamma_4 (\gamma_3^2 + \Delta_3^2) \left[ 2 \gamma_3 \gamma_2 \hat{\mathcal{O}}_d^{(2)} (\Gamma_2 + \gamma_2) + \Gamma_2 \Gamma_3 \gamma_2 \gamma_3 \right] \right]. \quad (7)

Under the electromagnetically induced transparency (EIT) scheme, the ions are initially populated in state \( |1\rangle \) and with strong pump approximation \( \Omega_{e} \gg \Omega_{d} > \Omega_{p} \). If we choose the probe field \( \hat{E}_p \) parallel to electric dipole momentum and its polarizability, the complex polarizability tensor \( \alpha_e \) which is related to the induced electric dipole moment \( \hat{p}(\omega_p) = \varepsilon_3 \alpha_e(\omega_p) \hat{\mathcal{E}}_p(\omega_p) \) is a scalar and given as

\[ \alpha_e = \frac{\gamma_3}{\varepsilon_3} \hat{\mathcal{E}}_p \] \[ \text{(9)} \]

Taking into account the local-field effects that lead to the Clausius–Mossotti relation [17] between the polarizability and the susceptibility, the complex electric susceptibility for the system with doped density \( N \) is

\[ \chi_e = N \alpha_e \left( 1 - \frac{N \alpha_e}{3} \right)^{-1}. \quad (9) \]

The relative permittivity can be written as

\[ \varepsilon_r = 1 + \chi_e. \quad (10) \]

After considering the local field effects and using Maxwell’s relation and \( \hat{B} = \mu_0 (\hat{H} + \hat{M}) \) as well as \( \hat{B} = \mu_0 \mu_s \hat{H} \), the relative permeability \( \mu_r = \mu / \mu_0 \) is obtained:

\[ \mu_r = \frac{1}{1 + N \mu_0 \eta(\omega_p) (1 + \chi_e(\omega_p))/3}, \quad (11) \]

where the coefficient \( \eta(\omega_p) \) is a unitary complex number depending on the polarization of the field \( \hat{E}_p \) relative to the magnetic dipole \( \mu_1 \) and electric dipole \( d_{34} \). Based on Judd–Ofelt theory [18, 19], we work out the coefficient \( \eta(\omega_p) \). If the contributions from nuclear spin and electronic spin can be ignored, the response of rare earth ions to the magnetic component of the probe field would reach maximum as the induced magnetic dipole of ions oscillate in phase with the probe beam [11]. The magnetic dipole between levels \( |1\rangle \) and \( |2\rangle \) becomes

\[ \mu_{12} = \frac{|e h|}{2 \mu_r c} \langle \hat{L} + 2 \hat{S} \rangle \langle \hat{L} + 2 \hat{S} \rangle \left[ 4 f^N \gamma S L M |(\hat{L} + 2 \hat{S}) \langle 1 | | (2 f^N \gamma S L M - M) \right] \]

\[ = (-1)^{J + M} \left( \begin{array}{c} J \ 1 \ J' \\
- M \ - \rho \ M' \end{array} \right) \]

\[ \times \langle 4 f^N \gamma S L |(\hat{L} + 2 \hat{S}) \langle 1 | | (4 f^N \gamma S L) \rangle \rangle \] \[ \text{where} \quad \gamma \text{is an additional quantum number to} \quad \text{discern between levels with the same} \quad S \text{and} \quad L \text{quantum numbers,} \quad S \text{is the spin angular momentum,} \quad L \text{is the orbital angular momentum and} \quad M \text{is the magnetic number.} \quad \text{The matrix element in equation (12) is calculated by the application of the Wigner–Eckart theorem to remove the} \quad \rho \text{dependence. Here,} \quad (J \ - \ M \ - \ \rho \ M') \quad \text{is a 3-j-symbol} [20] \quad \text{and} \quad (4 f^N \gamma S L |\langle \hat{L} + 2 \hat{S} \rangle | | (4 f^N \gamma S L') \rangle \rangle \quad \text{is a reduced matrix element. From the selection rule for the magnetic dipole transition} \quad |1\rangle - |2\rangle, \quad S = S' = 3/2, \quad J = J - 1 = 13/2, \quad \text{is reduced to} \quad |S + L = J + 1 (S + J = L)(J + L = S) |^{1/2}. \quad (13) \]

According to the selection rule, electric dipole transitions are allowed between two states of opposite parity. In a first approximation, no intra-configurational 4f–4f transitions can occur by an electric dipole mechanism, because the matrix element \( \langle f | d_{34} | l \rangle \) is equal to zero. Nevertheless, most observed transitions in the absorption and luminescence spectra of trivalent lanthanide ions are electric dipole transitions. A non-zero electric dipole matrix element can result from an admixing into \( |i\rangle \) and \( |f\rangle \) of states built from configurations of opposite parity to the \( 4f^p \) electric states. The electric dipole matrix element of the f–f transition is given by

\[ d_{34} = \langle 4 f^N | \hat{\mathcal{E}}_p | 3 \rangle \]

\[ = \sum_{k,q} \sum_{\lambda, \gamma} \langle 2\lambda + 1 | (\pm)^{\pi, q} A_{\lambda q} \left( \begin{array}{c} \lambda \\
-p \ q \ \gamma \end{array} \right) \ X (k, \lambda) \] \[ \langle A' | U^{(l)}_{\beta, \gamma} | A \rangle = \delta_{S S'} \sum_{M, M', \gamma} d_{M, M' - 1} \left( -1 \right)^{J - M + S + \kappa + J - \gamma} \times \left( \begin{array}{c} J' \ \lambda \\
-M' \ -p \ q \ \gamma \end{array} \right) \left( \begin{array}{c} L' \ J' \ S \\
L \ J \ \lambda \end{array} \right) \times \delta_{\kappa, \gamma} \sum_{m, m', \lambda} d_{M, M'} \left( -1 \right)^{M + S + \kappa + J - \gamma} \times \sqrt{(2 f + 1) (2 f' + 1)} \langle 4 f^N \gamma | S' L' | | U^{(l)} | 4 f^N \gamma S L \rangle. \quad (15) \]

Here \( | i \ \beta \ \lambda \rangle \) is a 6-j-symbol.

In the following numerical calculation, the used parameters of Er\(^{3+}\):YAlO\(_3\) crystals are tabulated in table 1 [15, 16].

### 3. Results and discussion

The real part \( \text{Re}[\varepsilon] \) and imaginary part \( \text{Im}[\varepsilon] \) of relative dielectric permittivity depending on the detuning \( \Delta_p \) is shown
in figure 2, with $\omega_p$ being the probe-field frequency, and let $\Omega_c = 200$ MHz, $\Omega_d = 20$ MHz, $\Omega_p = 8.5$ MHz and $\Delta_c = \Delta_d = 0$. It can be observed in figure 2 that Re[$\varepsilon_r$] is negative in the region $(-321$ kHz, $157$ kHz). Re[$\varepsilon_r$] has the maximum 5.1 and minimum $-3$. Similarly, the dependence of the real part Re[$\mu_r$] and imaginary part Im[$\mu_r$] of relative magnetic permeability on the detuning $\Delta_p$ is presented in figure 3, using the same Rabi frequency and detuning parameters as shown in figure 2. It can be observed in figure 3 that Re[$\mu_r$] is negative in the region $(-869$ kHz, $495$ kHz). Im[$\mu_r$] is positive in the region $(-2000$ kHz, $-212$ kHz). In figure 4, we have obtained the region interval $(\sim 1$ MHz) with $\varepsilon_r(\omega_p) < 0$ and $\mu_r(\omega_p) < 0$ simultaneously. Since the high dissipations and absorptions are the biggest obstacle to potential application of NIMs, obtaining low-loss NIMs in a wide frequency band by EIT is very important. It is indicated that the rare earth ion doped material with abundant energy levels and various electric and magnetic transitions is an outstanding and practical candidate for the electromagnetically induced negative refractive index material. The optical losses are reduced by EIT mainly. There are, however, two points that must be emphasized: (i) the weak probe field $E_p$ with a Rabi frequency $\Omega_p$ is applied to

4. Conclusions

We have worked out that left-handed properties can be electromagnetically induced in a $\Lambda$-type four-level scheme on the Er$^{3+}$ rare earth ion doped YAlO$_3$ crystal. The Er$^{3+}$ ion in the Er$^{3+}$:YAlO$_3$ crystal has acquired the requirements for energy level configuration to achieve induced NIMs by the quantum interference in a four-level system. We have realized a wider region $(\sim 1$ MHz) with $\varepsilon_r(\omega_p) < 0$ and $\mu_r(\omega_p) < 0$ simultaneously. Since the high dissipations and absorptions are the biggest obstacle to potential application of NIMs, obtaining low-loss NIMs in a wide frequency band by EIT is very important. It is indicated that the rare earth ion doped material with abundant energy levels and various electric and magnetic transitions is an outstanding and practical candidate for the electromagnetically induced negative refractive index material. The optical losses are reduced by EIT mainly. There are, however, two points that must be emphasized: (i) the weak probe field $E_p$ with a Rabi frequency $\Omega_p$ is applied to
the electric dipole transition $|3\rangle \rightarrow |4\rangle$ ($^4I_{9/2} \rightarrow ^2H_{11/2}$ of Er$^{3+}$), energy separation $\hbar \omega_{34} = 6740$ cm$^{-1}$ 1481 nm far away from the near-infrared and visible regions where the optical losses are significant [22]. (ii) The figure of merit $FOM = |Re[n]/Im[n]| = 1.233/0.267 = 4.6$ is obtained just for the maximum negative refractive index of $-1.233 - 0.267i$. The above-mentioned results obey the fundamental principle of causality for negative refraction with low optical losses [22–24].

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References

[12] Shen J Q, Ruan Z C and He S L 2004 J. Zhejiang Univ. Sci. (China) 5 1322