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Technical Notes

Kinematics analysis of six-bar parallel mechanism and its applications in synchrotron radiation beamline

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A B S T R A C T

Six-bar parallel mechanism is now widely applied in synchrotron radiation beamline, while the six-dimensional adjustment is difficult and inefficient for lack of theoretical direction. This paper introduces a special six-bar parallel mechanism. By means of coordinate transformations, the inverse kinematics of six-bar parallel mechanism is studied, and the precise equations for six bars' lengths are obtained. Based on the inverse kinematics, forward kinematics of six-bar parallel mechanism is obtained with trust region method working for nonlinear optimization. The corresponding MATLAB program is also designed. The results show that trust region method is an effective way to solve forward kinematics, and the program is stable, reliable and rapid. This method has small errors with linear precision of 10^{-12} mm and rotational precision of 10^{-15} deg. Using differential snail adjustment, monochromator chamber's attitude can reach a linear resolution of 5 μm and a rotational resolution of 3″, which entirely satisfies the practical requirements.

1. Introduction

Parallel mechanisms are being preferred in many applications, as they have advantages of higher stiffness, better accuracy, greater stability and more compact structure over serial mechanisms [1–3]. Beamline is an important part of synchrotron radiation facility, which is primarily made up of monochromator, focusing mirror system and so on. The key components, mirrors in these systems, are all adjusted by fine mechanisms. Wherein six-bar parallel mechanism, which has six degrees of freedom, is playing a more and more important role in the field [4–8].

In recent years, study of parallel mechanisms has been a hotspot of mechanism study in the world [9]. Referring to the kinematics analysis of parallel mechanisms, given the position and orientation of the moving platform, to determine the lengths of all bars, the whole process is called inverse kinematics. Contrarily, given the lengths of all bars, to determine the position and orientation of the moving platform, is called forward kinematics [10,11]. It is known that it's relatively easy to work out inverse kinematics of parallel mechanisms, while forward kinematics is much more complicated and is becoming a big problem all over the world [9]. Moreover, forward kinematics is the foundation for applications of parallel mechanisms [3]. To solve the problem, there are two algorithms: analytic solution and numerical method. Analytic solution can obtain all results of the equations, but it is complicated thus limiting its applications. Though numerical method depends on the iterative initial value, its mathematic models can be easily built and real solutions of all kinds of parallel mechanisms will be obtained quickly. Therefore, numerical method is widely used to solve forward kinematics problem [11].

A special six-bar parallel mechanism is being applied in the monochromator of STXM beamline, one of the first-built beamlines in Shanghai Synchrotron Radiation Facility (SSRF). Its physical and mathematic model is set up in the paper. By means of coordinate transformations, the inverse kinematics of six-bar parallel mechanism is studied, and the precise equations for six bars' lengths are obtained. Curves of six bars' lengths are represented after simulation and calculation by MATLAB. Then based on inverse kinematics, trust region method, a nonlinear programming algorithm, is used to solve the forward kinematics problem. MATLAB simulation and calculations show that the results are quite accurate and can be obtained very fast by using the trust region method, which provides important guidance in our practical work.

2. Inverse kinematics of six-bar parallel mechanism

Parallel mechanisms were first introduced by Stewart in the 1960 s [12]. Its original model is shown as Fig. 1 and that's the classic model of six-bar parallel mechanism. It has six degrees of

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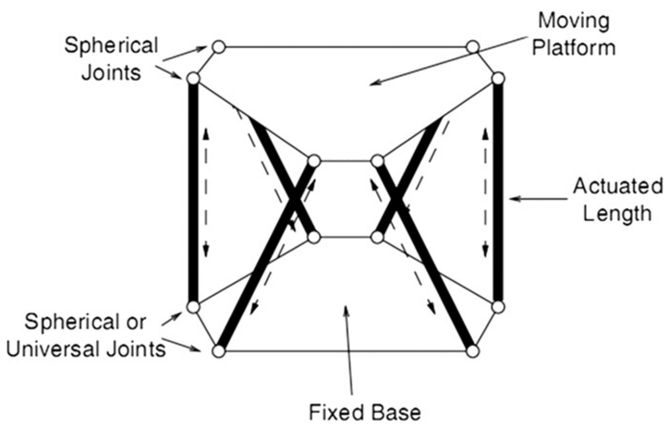

Fig. 1. Classic model of six-bar parallel mechanism.

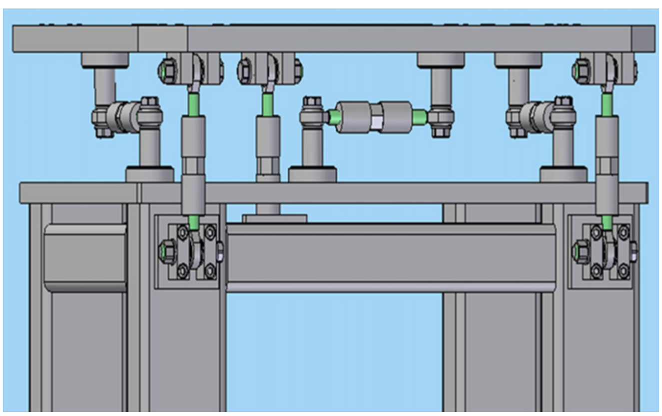

Fig. 2. Physical model of six-bar parallel mechanism.

freedom and its six flexible bars are connected to mobile joints at the fixed base and moving platform.

A special six-bar parallel mechanism, as shown in Fig. 2, is always applied in synchrotron radiation beamline facilities. Unlike the classic model, it has three vertical and three horizontal bars, whose lengths are fine adjusted by differential screws in particular. Upper and lower joints are not at the same plane. This mechanisms' configuration is space-saving and easy to operate.

2.1. Establishing coordinate systems

From the model shown in Fig. 2, the fixed base coordinate frame *O-xyz* is established firstly. Set the fixed plane *Oxy* by the lower joints of three vertical bars. Point *O*, origin of the frame, is located in the center of fixed base. Directions of coordinate axes *x*, *y*, *z*, are determined by the right-hand rule, as shown in Fig. 3. Therefore, coordinates of six lower joints in *O-xyz* are $A_i=\{A_{ix}, A_{iy}, A_{iz}\}^T$ ($i=1, 2, \ldots, 6$).

Establish the moving platform frame O'-$x'y'z'$. Set the moving plane $O'x'y'$ by the upper joints of three vertical bars. Actually, mirrors in synchrotron radiation beamline facilities are always higher than the moving plane, thus O' is located right above plane $O'x'y'$ with the height of *h* in the paper. Three axes of two frames are parallel to each other at the initial time. Namely, coordinates of six upper joints in O'-$x'y'z'$ are $B_i'=\{B_{ix}', B_{iy}', B_{iz}'\}^T$ ($i=1, 2, \ldots, 6$).

2.2. Inverse kinematics

Set (α, β, γ) are the RPY (Roll-Pitch-Yaw) angles and $P=\{X_P, Y_P, Z_P\}^T$ is the coordinate of O' in *O-xyz*. Coordinates of six upper joints

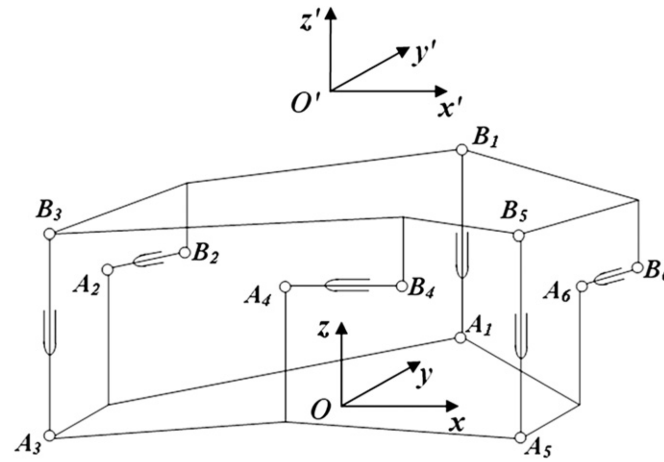

Fig. 3. Kinematic diagram of six-bar parallel mechanism.

in *O-xyz* can be written in the following way by means of coordinate transformations

$$B_i = RB_i' + P (i=1,2,\ldots,6) \quad (1)$$

whereas *R* is the attitude matrix from O'-$x'y'z'$ to *O-xyz* [11,15]

$$R = Yaw(z,\gamma)Pitch(y,\beta)Roll(x,\alpha) = \begin{bmatrix} c\gamma & -s\gamma & 0 \\ s\gamma & c\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\beta & 0 & s\beta \\ 0 & 1 & 0 \\ -s\beta & 0 & c\beta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\alpha & -s\alpha \\ 0 & s\alpha & c\alpha \end{bmatrix} \quad (2)$$

where *c* is on behalf of cosine rules and *s* is sine rules.

Bars' length vectors l_i in *O-xyz* are

$$l_i = B_i - A_i = (B_{ix}-A_{ix})x + (B_{iy}-A_{iy})y + (B_{iz}-A_{iz})z = l_{ix}x + l_{iy}y + l_{iz}z (i=1,2,\ldots,6) \quad (3)$$

Given the position and orientation of the moving platform, namely, $(\alpha, \beta, \gamma, X_P, Y_P, Z_P)$ are assumed to be known, lengths of six bars are as follows

$$l_i = \sqrt{(B_{ix}-A_{ix})^2 + (B_{iy}-A_{iy})^2 + (B_{iz}-A_{iz})^2} (i=1,2,\ldots,6) \quad (4)$$

Eq. (4) are exactly the inverse kinematic equations of six-bar parallel mechanism.

Furthermore, varied lengths of six bars are defined as

$$\Delta l_i = l_i - l (i=1,2,\ldots,6) \quad (5)$$

where *l* is the initial length of six bars.

3. Forward kinematics of six-bar parallel mechanism

Forward kinematics is the process of determining the position and orientation of the moving platform, $(\alpha, \beta, \gamma, X_P, Y_P, Z_P)$, by given the lengths of six bars l_i ($i=1, 2, \ldots, 6$). From Eq. (4), constraint equations between the position and orientation of the moving platform and the bars' lengths are as follows

$$f_i(\alpha,\beta,\gamma,X_P,Y_P,Z_P,l_i) = 0 \ (i=1,2,\ldots,6) \quad (6)$$

Eq. (6) are nonlinear transcendental equations with six unknowns with trigonometric functions.

The key of forward kinematics is exactly to solve the nonlinear transcendental equations above. Since the emergence of parallel mechanism in 1960 s, lots of mechanists have obtained the closed-form solutions of some kinds of parallel mechanisms using analytic solutions. But due to the non-linearity of these equations, there is no complete method that would get the closed-form solution [3]. While numerical method is a sufficient way that can solve the forward kinematics problem of all kinds of parallel mechanisms quickly and conveniently [11].

Trust region method and line search method are two generally used nonlinear programming. While trust region method is a novel algorithm developed in latest 30 years. It is reliable and has the advantage of global convergence which makes the initial values easier to be found. Hence, study of trust region method has recently become a very important aspect in the field of nonlinear programming.

The basic idea of trust region method is to solve a relatively easy subproblem in the neighborhood of current iterate, and a trust region radius is obtained sequentially. Then the trust region radius will be expanded or contracted by judging a ratio of the objective function. The steps are [13]:

Step 1: Suppose an initial value x_0, and an initial trust region radius $\delta_0 \in (0, 1)$. Let δ, η and ε be some constants and satisfy $\delta > 0$, $\eta \in (0, 0.25)$, $\varepsilon \geq 0$. Let $k=0$.

Step 2: If $\|\nabla f(x_k)\| \leq \varepsilon$, then stop; otherwise, go to Step3. $\nabla f(x_k)$ is the Jacobian matrix of objective function $f(x)$ by current iterate x_k.

Step 3: Construct a trust region subproblem using quadratic approximation, as Eq. (7). Then d_k will be obtained.

$$\min \phi_k(x) = \frac{1}{2} d^T B_k d + \nabla f(x_k)^T d \qquad st \ \ \|d\| \leq \delta_k \tag{7}$$

where B_k is a $n \times n$ real symmetric matrix.

Step 4: Calculate the ratio r_k of the true reduction to the certain reduction.

$$r_k = \frac{\Delta f_k}{\Delta \phi_k} = \frac{f(x_k) - f(x_k + d_k)}{\phi_k(0) - \phi_k(d_k)} \tag{8}$$

Step 5: If $B_k < 0.25$, then set $\delta_{k+1} = 0.25\delta_k$, update $x_{k+1} = x_k$ and $k = k+1$, jump to Step2; If $r_k > 0.75$, then set $\delta_{k+1} = \min(2\delta_k, \delta)$, update $x_{k+1} = x_k + d_k$ and $k = k+1$, jump to Step2; When $0.25 \leq r_k \leq 0.75$ and $r_k \geq \eta$, then set $x_{k+1} = x_k + d_k$; otherwise set $x_{k+1} = x_k$. Update $\delta_{k+1} = \delta_k$ and $k = k+1$, jump to Step 2.

Note: The constants 0.25 and 0.75 above are experienced values and can be replaced by other values between 0 and 1.

As we know, numerical methods always go with a huge workload. And MATLAB is just such a powerful tool in numerical calculation, where nonlinear equations can be solved quickly with its optimization toolbox of trust region method [14]. Therefore, MATLAB is perfectly applied to solve the forward kinematics problem of six-bar parallel mechanism in this paper.

4. Application

SSRF is a third-generation of synchrotron radiation light source in China. Grating monochromator is the key part of STXM beamline at SSRF, and its chamber is adjusted by six-bar parallel mechanism. Table 1 shows the requirements of adjust range and resolution.

Referring to the structure in Fig. 3, joints' positions of monochromator chamber are shown in Fig. 4, where unit is mm. Initial length of each bar is considered as $l = 211$ mm, $h = 30$ mm, and

$$A_{iz} = \begin{cases} 0; & (i = 1,3,5) \\ 157.5\,\text{mm}; & (i = 2,4,6) \end{cases},$$

$$B'_{iz} = \begin{cases} -30\,\text{mm}; & (i = 1,3,5) \\ -83.5\,\text{mm}; & (i = 2,4,6) \end{cases}$$

Table 1
Technical requirements of chamber's attitude adjustment.

Parameters	Adjust range	Resolution
X_P	± 5 mm	0.02 mm
Y_P	± 5 mm	0.02 mm
Z_P	± 5 mm	0.01 mm
α	$\pm 1.0°$	0.1°
β	$\pm 1.5°$	5″
γ	$\pm 1.0°$	5″

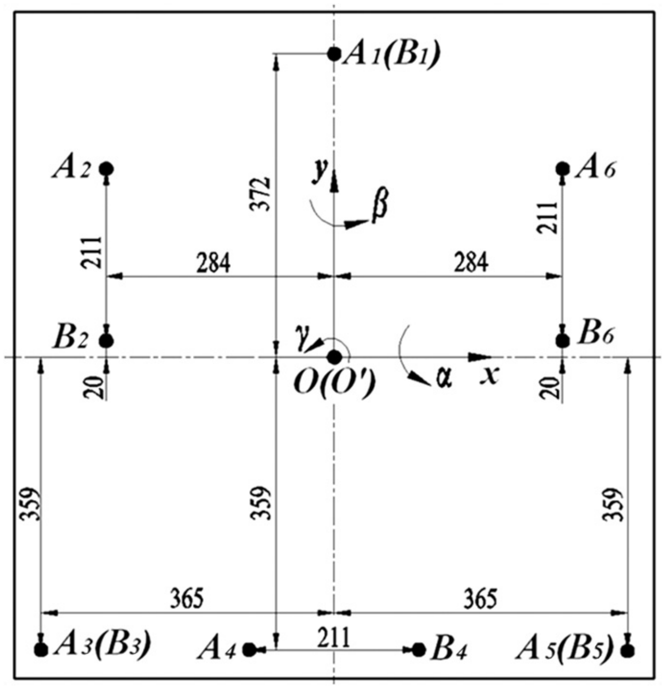

Fig. 4. Joints distribution map of chamber's attitude adjustment.

4.1. Inverse kinematics

As Table 1 required, inverse kinematic equations of six-bar parallel mechanism are programmed in MATLAB. MATLAB simulation shows the curves of six bars' lengths.

(1) Set $-1.5° \leq \alpha \leq 1.5°$, $\beta = \gamma = 0$, and $X_P = Y_P = Z_P = 0$. After MATLAB calculation and simulation, curves of α vs. six bars' varied lengths Δl_i are shown in Fig. 5(a). Similarly, set $-1° \leq \beta \leq 1°$, $\alpha = \gamma = 0$ and $X_P = Y_P = Z_P = 0$, curves of β vs. Δl_i are as Fig. 5(b). Set $-1° \leq \gamma \leq 1°$, $\alpha = \beta = 0$ and $X_P = Y_P = Z_P = 0$, curves of γ vs. Δl_i are as Fig. 5(c).

(2) Set -5 mm $\leq X_P \leq$ 5 mm, $\alpha = \beta = \gamma = 0$ and $Y_P = Z_P = 0$, we get the curves of X_P vs. six bars' varied lengths Δl_i. In view of values of Δl_i for six bars differ greatly, 2-D lines with different y-value on left and right side are plotted using plotyy function in MATLAB. As shown in Fig. 5(d), left side Δl_1 is corresponding to $\Delta l_i = f(X_P)$ (i = 1, 2, 3, 5, 6), and right side Δl_4 is corresponding to $\Delta l_4 = f(X_P)$. Similarly, set -5 mm $\leq Y_P \leq$ 5 mm, $\alpha = \beta = \gamma = 0$ and $X_P = Z_P = 0$, curves of Y_P vs. Δl_i are as Fig. 5(e). Initial bars' length $l = 211$ mm and $Z_P = 240$ mm, consequently set 235 mm $\leq Z_P \leq$ 245 mm, $\alpha = \beta = \gamma = 0$ and $X_P = Y_P = 0$, curves of Z_P vs. Δl_i are as Fig. 5(f).

From Fig. 5, the relationship between (α, β, γ, X_P, Y_P, Z_P) and Δl_i (i = 1, 2, …, 6) is very obvious. Take Fig. 5(d) for example, in the range of -5 mm $\leq X_P \leq$ 5 mm, Δl_4 is much larger than others and it can be considered as the main factor of X_P, which provides a big hand for our practical fine adjustment in synchrotron radiation beamline.

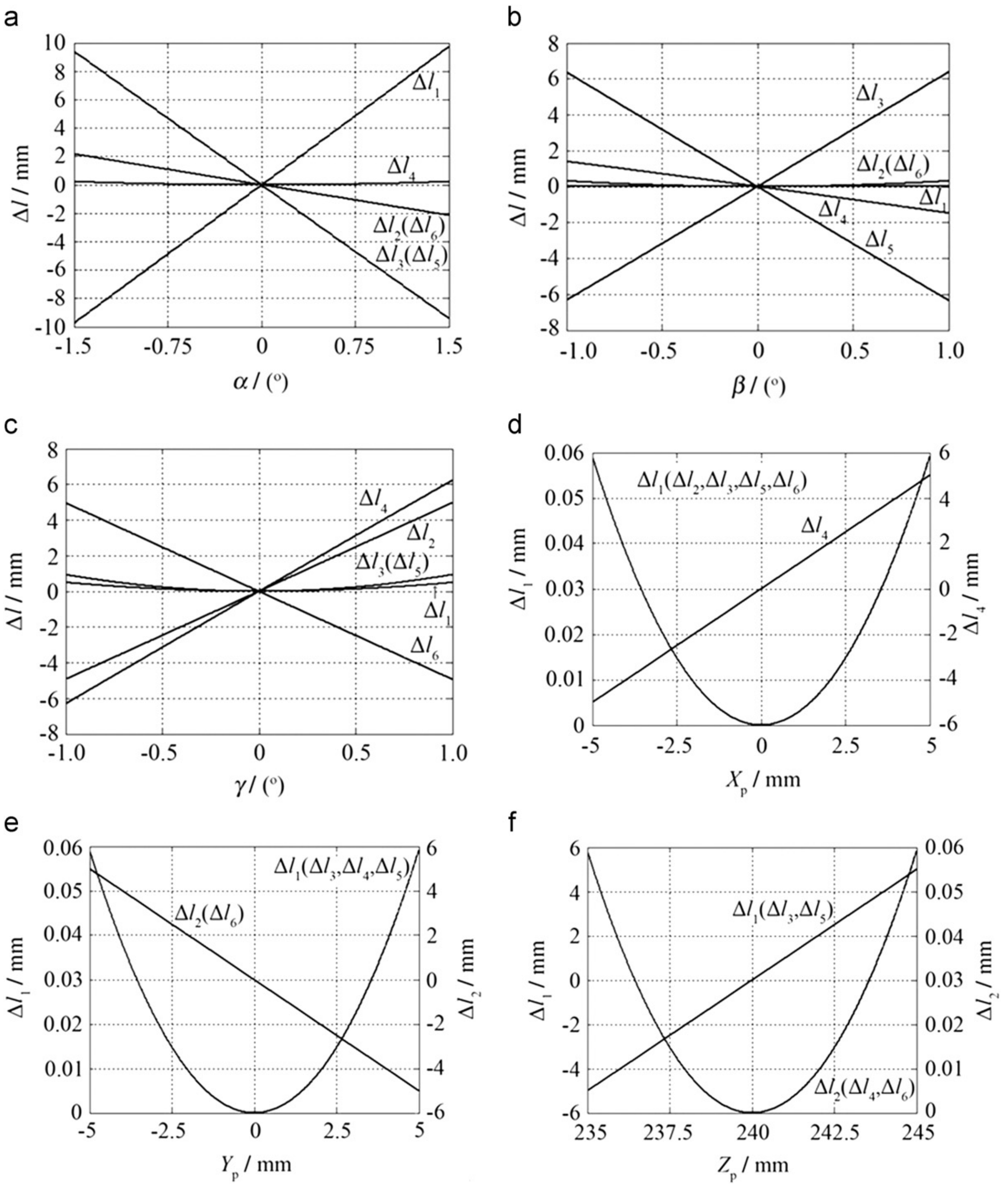

Fig. 5. Curves of chamber's attitude parameters vs. bars' lengths. (a) α vs. Δl_i. (b) β vs. Δl_i. (c) γ vs. Δl_i. (d) X_P vs. Δl_i. (e) Y_P vs. Δl_i. (f) Z_P vs. Δl_i

4.2. Forward kinematics

Forward kinematics of parallel mechanism, as analyzed before, is based on its inverse kinematics. A block diagram of forward kinematics using trust region method is programmed in MATLAB according to the technical requirements of Table 1, shown in Fig. 6. Where IK is on behalf of inverse kinematics and FK is forward kinematics.

When programs are running, MATLAB is able to choose a best trust region radius automatically by using fsolve function, and the highest convergence rates are obtained. As seen in the process of MATLAB compute, an optimal solution can be worked out by forward kinematics after only 3 to 5 iterate times within 1 second.

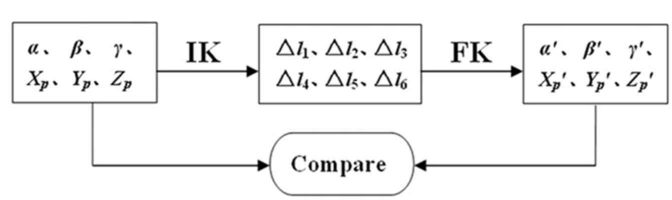

Fig. 6. Block diagram of forward kinematics.

Suppose the moving platform running in sinusoidal swing on α, orbit equation α of its center is as follow

$$\alpha = 2\sin(\pi t/5), t = 0 \sim 10\,\mathrm{s} \tag{9}$$

Substituting the orbit equation $\boldsymbol{\alpha}$ to MATLAB program, we get the varied lengths Δl_i of six bars by inverse kinematics, as in Fig. 7. Then substituting Δl_i to the forward kinematics equations, we get another orbit equation $\boldsymbol{\alpha}'$ of its center by MATLAB. Fig. 8 shows the comparison of two orbits $\boldsymbol{\alpha}'$ and $\boldsymbol{\alpha}$. Meanwhile, deviation curve of $\boldsymbol{\alpha}'$ and $\boldsymbol{\alpha}$ is indicated in Fig. 9.

Suppose the moving platform runs in screw rotation, orbit equation t of its center is

$$\begin{cases} X_P = \sin(t) \\ Y_P = \cos(t), \quad t = 0 \sim 10\,\mathrm{s} \\ Z_P = t \end{cases} \tag{10}$$

Similarly, substituting $\boldsymbol{t}$ to MATLAB program, varied lengths Δl_i of six bars are shown as Fig. 10. Fig. 11 shows the comparison of $\boldsymbol{t}'$ and $\boldsymbol{t}$. Deviation curves in three coordinate axes directions are indicated in Figs. 12–14.

It is apparent that the solved curves by trust region method coincide with the given curves and errors between them are extremely small. When moving in swing equations, the largest angular error is only 4×10^{-15} deg. When moving in screw rotation, the largest axial errors in X, Y, Z direction are 6.4×10^{-12}, 5.2×10^{-12}, 5.2×10^{-12} mm, the largest comprehensive axial error is 7.8×10^{-12} mm. These errors are much smaller than technical requirements in Table 1 and could be ignored in our practical work.

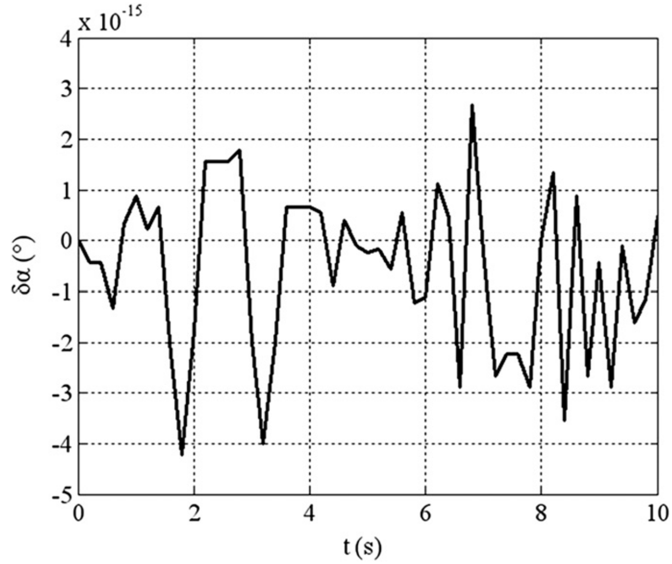

Fig. 9. Deviation $\delta\alpha$ of $\boldsymbol{\alpha}'$ and $\boldsymbol{\alpha}$.

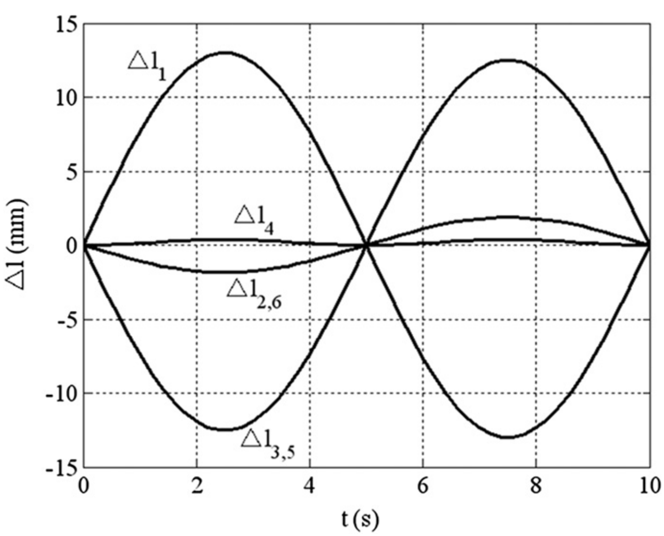

Fig. 7. Variation of bars length Δl.

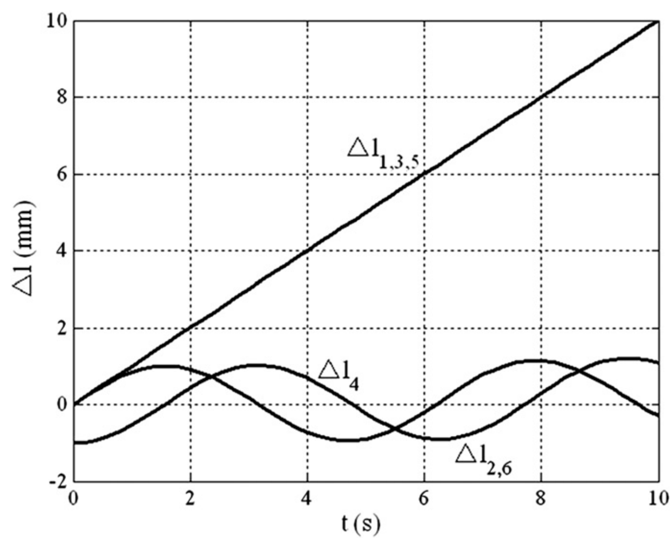

Fig. 10. Variation of bars' length Δl.

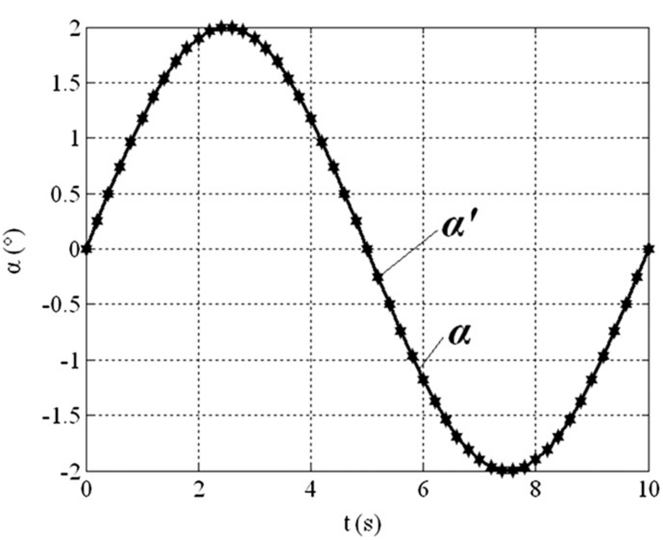

Fig. 8. Given curve $\boldsymbol{\alpha}$ vs. solved curve $\boldsymbol{\alpha}'$.

When adjusting the monochromator chamber's attitude, bars' lengths change can easily get a resolution of 5μm by using differential snail adjustment. Set $\Delta l_i = \pm 5$ μm($i = 1, 2, \ldots, 6$), and there are 2^6 permutations. Substituting each permutation to the forward kinematics program, maximum errors of the position and orientation parameters are obtained and listed in Table 2. The results are smaller than the project's resolution requirements and entirely satisfy the technical requirements of monochromator chamber's attitude.

5. Conclusion

The six-bar parallel mechanism proposed in the paper is a special configuration being applied in several mirror systems of Shanghai Synchrotron Radiation Facility beamlines. After setting up the physical and mathematic model, inverse kinematics of six-bar parallel mechanism is studied in detail by means of coordinate transformations, the precise equations for six bars'

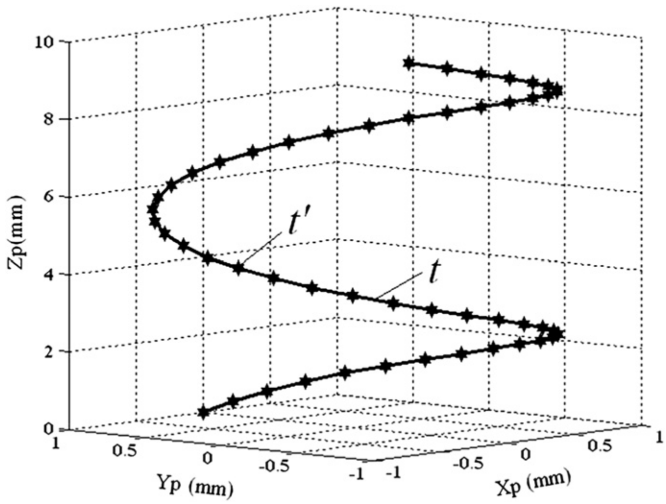

Fig. 11. Given curve ***t*** vs. solved curve ***t*'**.

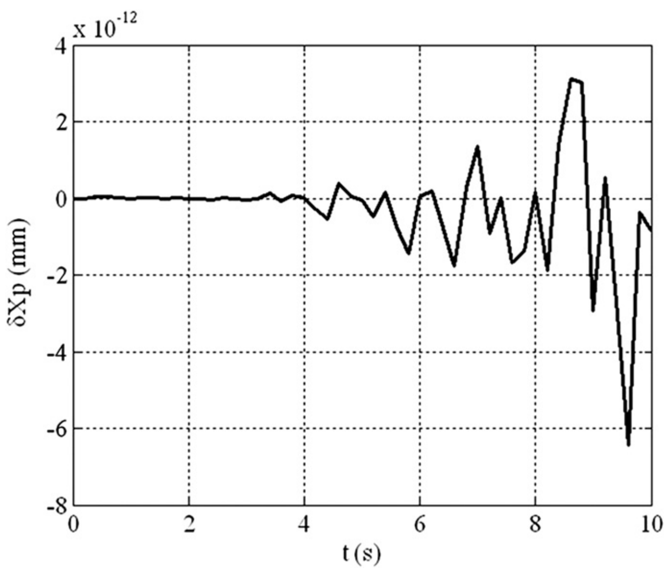

Fig. 12. X-deviation δX_P of ***t*'** vs. ***t***.

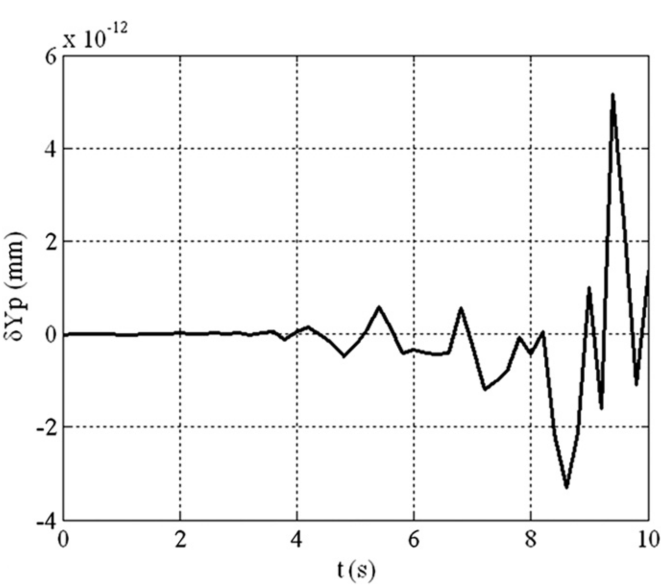

Fig. 13. Y-deviation δY_P of ***t*'** vs. ***t***.

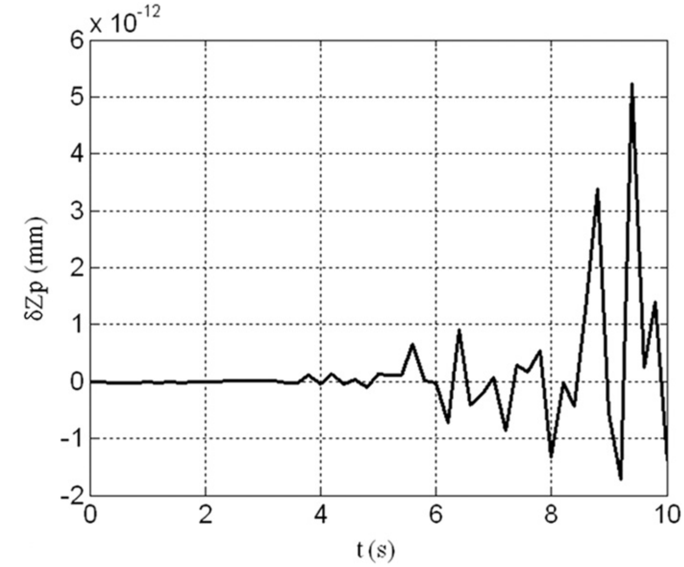

Fig. 14. Z-deviation δZ_P of ***t*'** vs. ***t***.

Table 2
Maximum errors of chamber's attitude adjustment.

Parameters	Maximum errors	Resolution requirements
δX_P	4.9μm	0.02 mm
δY_P	5.1μm	0.02 mm
δZ_P	4.9μm	0.01 mm
$\delta\alpha$	2.9″	0.1°
$\delta\beta$	2.9″	5″
$\delta\gamma$	3.7″	5″

lengths are obtained. Curves of six bars' lengths are represented after simulating and calculating by MATLAB, which make our practical fine adjustment easier, faster and more convenient.

Forward kinematics of six-bar parallel mechanism is exactly to solve a series of nonlinear transcendental equations with six unknowns. As an important nonlinear numerical algorithm, trust region method has advantages of global convergence and stability. Based on the inverse kinematics, the corresponding program is designed with trust region method in MATLAB, forward kinematics is solved finally in this paper. As seen in the process of MATLAB compute, an optimal solution can be worked out by forward kinematics after only 3 to 5 iterate times within 1 second. Given the orbit equation of the moving platform, solved curves will be obtained by MATLAB automatically. Simulation results show that errors between given orbits and solved orbits are extremely small, with linear precision of 10^{-12} mm and rotational precision of 10^{-15} deg. Guided by the analysis above, using differential snail adjustment, monochromator chamber's attitude can reach a linear resolution of 5 μm and a rotational resolution of 3″, which entirely satisfies the practical requirements.

Acknowledgments

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