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Extremely narrowed and amplified gain spectrum induced by the Doppler effect

Li-Da Zhang[1,2,3], **Yun Jiang**[1,2], **Ren-Gang Wan**[1,2], **Si-Cong Tian**[1,2], **Bing Zhang**[1,2], **Xiao-Jun Zhang**[1,2], **Jin-Yue Gao**[1,2], **Yue-Ping Niu**[3,4] **and Shang-Qing Gong**[4]

[1] College of Physics, Jilin University, Changchun 130023, People's Republic of China
[2] Key Laboratory of Coherent Light and Atomic and Molecular Spectroscopy of Ministry of Education, Jilin University, Changchun, People's Republic of China
[3] State Key Laboratory of High Field Laser Physics, Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Science, Shanghai 201800, People's Republic of China
[4] Department of Physics, East China University of Science and Technology, Shanghai 200237, People's Republic of China

E-mail: zhanglidacn@yahoo.cn and jygao@mail.jlu.edu.cn

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Abstract

The effect of Doppler broadening on the gain spectrum is theoretically investigated in a hot *N*-type active Raman gain atomic system. It is found that the gain peak in the spectrum can be extremely narrowed and amplified by two orders at room temperature as compared with that in the cold atomic system. This remarkable result originates from the shift of the dressed levels and the modification to the transition probability between dressed states when the Doppler effect is considered in the hot atomic system. The enhanced subluminal and superluminal pulse propagation for the probe field in the hot atomic system has also been obtained by numerical simulation; the probe pulse undergoes no absorption and little pulse distortion.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Tradionally, when taking the Doppler effect into account it always results in a broadened and attenuated gain or absorption spectrum, leading to a reduced resolution. Even though some methods such as saturation spectroscopy [1], two-photon absorption spectroscopy [1] and coherent Raman scattering spectroscopy [1], etc, have been developed to reduce the Doppler effect, all those techniques are limited to the natural width. In the last two decades, sub-Doppler linewidth resolution has been proposed and realized with electromagnetically induced transparency (EIT) [2–4] in hot atoms. Recently, there are some reports that a subnatural linewidth transparent window for the probe can also be obtained in a Λ- and four-level *N*-EIT system [5–8] when the Doppler effect is considered.

There are also some published results on the effect of Doppler broadening on the gain spectrum in four-level *N* systems [9–11]. However, the gain spectrum obtained in those papers is due to the evolution from EIT into amplification with or without population inversion; the gain is attained due to constructive interference [12]. In contrast, the gain spectrum based on active Raman gain (ARG) [13–18] results from the stimulated Raman process, which is basically different from those published papers. Due to the essential physical differences between ARG and EIT, many works about ARG have been developed, such as the realization of superluminal and subluminal pulse propagation [13, 14, 18], large and rapidly responding Kerr effect [19], etc. Hence, it is meaningful to study the effect of Doppler broadening on the gain spectrum in the ARG scheme. Here, we theoretically demonstrate an extraordinary case that the gain spectrum can be extremely narrowed and amplified by the Doppler effect in an *N*-type atomic system based on ARG. In our calculation the gain peak in the spectrum is found to be 200 times narrower than the natural width and 157 times stronger in the spectral intensity when the Doppler effect is considered at room temperature as compared with that in cold atoms, resulting in a considerably increased resolution and improved sensitivity. We also theoretically get a Doppler-broadening

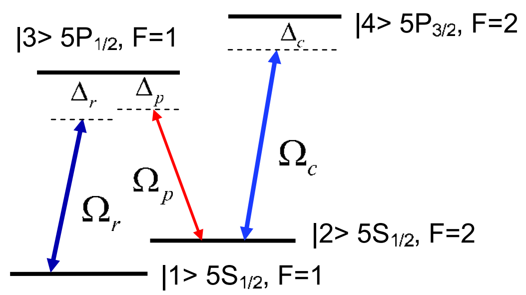

Figure 1. Schematic atomic level diagram. The ground state hyperfine levels $^5S_{1/2}$, F = 1, 2 of ^{87}Rb are chosen to be $|1\rangle$ and $|2\rangle$, $^5P_{1/2}$, F = 1 to be $|3\rangle$ and $^5P_{3/2}$, F = 2 to be $|4\rangle$ respectively in the *N*-level scheme.

enhanced subluminal or superluminal effect on the pulse propagation [14, 20–22] in hot atoms by numerical simulation. The probe pulse undergoes no absorption and little pulse distortion.

2. Model and analytic solution

Here, we consider an *N*-type level scheme. The relevant atomic levels in ^{87}Rb are shown in figure 1. $^5S_{1/2}$, F = 1, 2 are chosen to be the lower states $|1\rangle$ and $|2\rangle$, $^5P_{1/2}$, F = 1 to be the upper state $|3\rangle$ and $^5P_{3/2}$, F = 2 to be $|4\rangle$. Raman resonance is induced by transitions $|3\rangle\leftrightarrow|1\rangle$ and $|3\rangle\leftrightarrow|2\rangle$, which are driven by the Raman and probe fields with respective Rabi frequencies Ω_r and Ω_p, while the transition $|4\rangle\leftrightarrow|2\rangle$ is driven by a controlled field of Rabi frequency Ω_c ($\Omega_r = \vec{d}_{31}\cdot\vec{E}_r/2\hbar$, $\Omega_p = \vec{d}_{32}\cdot\vec{E}_p/2\hbar$, $\Omega_c = \vec{d}_{42}\cdot\vec{E}_c/2\hbar$, and $\vec{d}_{ij}$ is the dipole moment between $|i\rangle$ and $|j\rangle$). The frequency detunings of the Raman, the probe and the controlled field are defined as $\Delta_r = \omega_r - \omega_{31}$, $\Delta_p = \omega_p - \omega_{32}$, and $\Delta_c = \omega_c - \omega_{42}$ respectively.

In the interaction picture and under the dipole and rotating wave approximations the Hamiltonian is

$$H_I = -\hbar\begin{bmatrix} 0 & 0 & \Omega_r & 0 \\ 0 & \Delta_r-\Delta_p & \Omega_p & \Omega_c \\ \Omega_r & \Omega_p & \Delta_r & 0 \\ 0 & \Omega_c & 0 & \Delta_r-\Delta_p+\Delta_c \end{bmatrix}. \tag{1}$$

By including the spontaneous emission and decoherence rate, the related density-matrix equations can be written down as

$$\begin{aligned}
\dot\rho_{12} &= \mathrm{i}(\Omega_r\rho_{32} - \Omega_p\rho_{13} - \Omega_c\rho_{14}) + \mathrm{i}\Delta_{12}\rho_{12} \\
\dot\rho_{13} &= \mathrm{i}(\Omega_r\rho_{33} - \Omega_r\rho_{11} - \Omega_p\rho_{12}) + \mathrm{i}\Delta_{13}\rho_{13} \\
\dot\rho_{32} &= \mathrm{i}(\Omega_r\rho_{12} + \Omega_p\rho_{22} - \Omega_p\rho_{33} - \Omega_c\rho_{34}) + \mathrm{i}\Delta_{32}\rho_{32} \\
\dot\rho_{14} &= \mathrm{i}(\Omega_r\rho_{34} - \Omega_c\rho_{12}) + \mathrm{i}\Delta_{14}\rho_{14} \\
\dot\rho_{24} &= \mathrm{i}(\Omega_p\rho_{34} - \Omega_c\rho_{22} + \Omega_c\rho_{44}) + \mathrm{i}\Delta_{24}\rho_{24} \\
\dot\rho_{34} &= \mathrm{i}(\Omega_r\rho_{14} + \Omega_p\rho_{24} - \Omega_c\rho_{32}) + \mathrm{i}\Delta_{34}\rho_{34},
\end{aligned} \tag{2}$$

where $\Delta_{12} = \Delta_p - \Delta_r + \mathrm{i}\gamma_{12}$, $\Delta_{13} = -\Delta_r + \mathrm{i}\gamma_{13}$, $\Delta_{32} = \Delta_p + \mathrm{i}\gamma_{32}$, $\Delta_{14} = \Delta_p - \Delta_r - \Delta_c + \mathrm{i}\gamma_{14}$, $\Delta_{24} = -\Delta_c + \mathrm{i}\gamma_{24}$, $\Delta_{34} = \Delta_p - \Delta_c + \mathrm{i}\gamma_{34}$, and $\gamma_{ij} = \frac{1}{2}(\Gamma_i + \Gamma_j)$, $\Gamma_i = \sum_k \Gamma_{ik}$; Γ_{ik} is the spontaneous relaxation rate from level i to k, and Raman detunings $\Delta_r, \Delta_p \gg \Delta_c, \Omega_r, \Omega_c, \gamma_{12}, \gamma_{32}, \gamma_{14}, \gamma_{34}$. Assuming that all atoms are in $|1\rangle$ at the very beginning, there could be some populations in $|2\rangle$ because of the two-photon Raman resonance process in the Lambda system. However, the atoms in $|2\rangle$ will be driven to $|4\rangle$ by the pump effect of Ω_c and then back to the state $|1\rangle$ by spontaneous emission. Therefore, almost all of the populations are in $|1\rangle$ for the duration of the interaction process (by solving the zeroth-order density matrix equations under steady-state conditions, the calculation approximately shows that the populations are almost in level 1 when the conditions $\Omega_r^2/\Omega_c^2 \ll \Delta_r^2/\Gamma_3^2$ for cold atoms and $\Omega_r^2/\Omega_c^2 \ll (\Delta_r - \Delta\omega_D)^2/\Delta\omega_D^2$ for hot atoms are satisfied, $\Delta\omega_D = (\ln 2)kv_p$ is the half Doppler width. It has also been proved by the numerical simulation). Then, the zeroth-order density matrix elements for the probe are $\rho_{11}^{(0)} \simeq 1$, $\rho_{12}^{(0)} \simeq 0$, $\rho_{14}^{(0)} \simeq 0$, $\rho_{ij}^{(0)} \simeq 0 (i, j = 2, 3, 4)$ and $\rho_{13}^{(0)} = \Omega_r(\rho_{11}^{(0)} - \rho_{33}^{(0)})/\Delta_{13}$. Note that we have neglected the four-wave mixing process in our scheme for the weak wave-mixing strength, and we also ignore the hyperfine splitting of the excited level in ^{87}Rb. By solving the above equations under steady-state conditions, the first-order element $\rho_{32}^{(1)}(\omega_p)$ can be obtained:

$$\rho_{32}^{(1)}(\omega_p) = -\frac{\rho_{13}^{(0)}(\Omega_c^2 - \Omega_r^2 + \Delta_{14}\Delta_{34})\Omega_r\Omega_p}{\Delta_{12}\Delta_{32}\Delta_{14}\Delta_{34} - (\Delta_{12}\Delta_{14} + \Delta_{32}\Delta_{34})\Omega_c^2 - (\Delta_{12}\Delta_{32} + \Delta_{14}\Delta_{34})\Omega_r^2 + (\Omega_c^2 - \Omega_r^2)^2}. \tag{3}$$

And $\chi^{(1)}(\omega_p) = N_0|d_{32}|^2\rho_{32}^{(1)}(\omega_p)/2\varepsilon_0\hbar\Omega_p$, where N_0 is the atomic density. Equation (3) does not yet include the Doppler effect which will be taken into account later. For simplicity, we are interested in the case of resonant excitation, i.e. $\Delta_c = 0$. With this assumption, the real and imaginary parts of $\chi(\omega_p)$ varied with Δ_p can be plotted in figure 2.

The dashed curves in figure 2 plot the absorptive and dispersive response of the weak probe field in the cold atomic system (i.e. without the Doppler effect) for different values of Ω_c and Ω_r. The calculations show that in the vicinity of $\Delta_p = \Delta_r$ it is always negative for the absorption of the probe field, which means that the probe field is amplified at all times. For little Ω_c with an appropriate Ω_r, there is only one gain peak in the spectrum, followed by a corresponding normal dispersion curve with a normal positive slope near $\Delta_p = \Delta_r$. The amplitude of the gain peak can be modulated by the strength of the Raman field. As Ω_c increases gradually, the gain peak is divided with a dip in the middle of the two peaks. Then the corresponding dispersion profile is shifted to anomalous dispersion with a negative slope near the range of $\Delta_p = \Delta_r$. Moreover, the gap between the two peaks is broadened and deepened as Ω_c increases. In brief, the gain spectrum evolves from a single to two peaks as Ω_c increases; meanwhile, the corresponding dispersion curve changes from normal dispersion to anomalous dispersion. In other words, the switch between subluminal and superluminal light can be realized by altering the intensity of the controlled field only.

Then, we concentrate on the effect of Doppler broadening on the *N*-type atomic system. If the three laser fields are co-propagating in the sample, the Doppler effect can be taken into account by replacing Δ_r, Δ_p, Δ_c by $\Delta_r - kv$, $\Delta_p - kv$, $\Delta_c - kv$ ($k_r \simeq k_c \simeq k_p = k = \omega_p/c$) in equation (3); here, k_i is the wave vector of the corresponding field, and v is the velocity of atoms moving along the propagation direction,

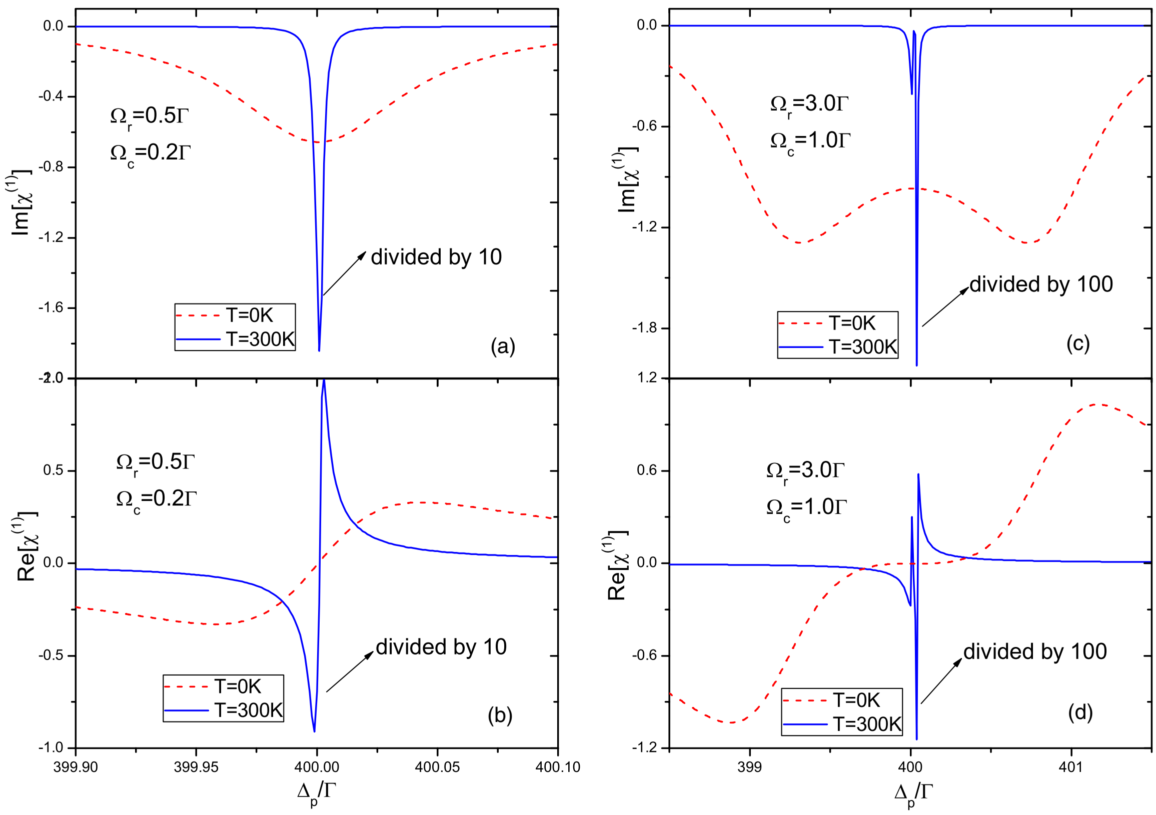

Figure 2. Calculated gain spectra and the corresponding dispersive profiles for the probe versus the probe detuning Δ_p in the N-level system under different conditions (dashed lines (red) without the Doppler effect, and solid lines (blue) with the Doppler effect when the three fields are co-propagating). The data in (a) and (b) for the gain spectra and the related dispersion curves with the Doppler effect are divided by a factor of 10, and divided by a factor of 100 in (c) and (d) for comparison. The relevant parameters are $\gamma_{13} = \gamma_{32} = \gamma_{41} = \gamma_{42} = \Gamma/2$, $\gamma_{12} = 0.001\Gamma$, $\gamma_{34} = \Gamma$, $\Delta_c = 0$, $\Delta_r = 400\Gamma$, $\Gamma = \Gamma_3$ is the natural linewidth of $|3\rangle$.

after integrating different v from $-\infty$ to ∞ in equation (3), i.e.

$$\chi^{(1)}(\omega_p) = \frac{|d_{32}|^2}{2\varepsilon_0\hbar} \int_{-\infty}^{\infty} \frac{N(v)\rho_{32}^{(1)}(\omega_p, v)}{\Omega_p}\, \mathrm{d}v \qquad (4)$$

where $N(v) = N_0\, \mathrm{e}^{-v^2/v_p^2}/(\sqrt{\pi}\, v_p)$ is the Maxwell velocity distribution, and $v_p = \sqrt{2k_B T/m}$ for the most probable velocity [21]. At room temperature ($T = 300$ K), the gain spectrum and the corresponding dispersive response for the probe field are calculated and presented in figure 2 by the solid blue curve. It can soon be found that the peaks in the gain spectrum are extremely narrowed and amplified when the Doppler effect is considered under the same conditions. In figure 2(c), the higher peak is amplified by 157 times and narrowed down to 0.005Γ when $\Omega_c = 1\Gamma$ and $\Omega_r = 3\Gamma$. The dispersion curve also shows a much steeper slope near $\Delta_p = \Delta_r$ than that in the cold atomic system. It means that the superluminal or subluminal effect on the light propagation can be enhanced by the Doppler effect. In fact, this remarkable result originates from the shift of the dressed levels and the modification to the probability amplitude of transitions from $|3+\rangle$ to $|2\pm\rangle$ caused by different detunings of atoms moving with various velocities, which will be discussed in the next section.

3. Theoretical explanation

The results described above can be clearly explained in the dressed state representation. Figure 3 shows the related dressed states for the N-type level scheme. The eigenvalues and eigenvectors of the dressed states $|3\pm\rangle$ and $|2\pm\rangle$ for atoms at certain v can be quickly obtained in a simple expression (for simplicity we have considered the resonance case that $\Delta_c = 0$):

$$\begin{aligned}
E_{3+} &= \hbar\lambda_{3+},\ |3+\rangle = \frac{\Omega_r}{\sqrt{\lambda_{3+}^2 + \Omega_r^2}}|1\rangle - \frac{\lambda_{3+}}{\sqrt{\lambda_{3+}^2 + \Omega_r^2}}|3\rangle\,;\\
E_{3-} &= \hbar\lambda_{3-},\ |3-\rangle = \frac{\Omega_r}{\sqrt{\lambda_{3-}^2 + \Omega_r^2}}|1\rangle - \frac{\lambda_{3-}}{\sqrt{\lambda_{3-}^2 + \Omega_r^2}}|3\rangle\,.\\
E_{2+} &= \hbar\lambda_{2+},\ |2+\rangle = \frac{\Omega_c}{\sqrt{\lambda_{2+}^2 + \Omega_c^2}}|2\rangle - \frac{\lambda_{2+}}{\sqrt{\lambda_{2+}^2 + \Omega_c^2}}|4\rangle\,;\\
E_{2-} &= \hbar\lambda_{2-},\ |2-\rangle = \frac{\Omega_c}{\sqrt{\lambda_{2-}^2 + \Omega_c^2}}|2\rangle - \frac{\lambda_{2-}}{\sqrt{\lambda_{2-}^2 + \Omega_c^2}}|4\rangle\,.
\end{aligned} \qquad (5)$$

Here, $\lambda_{3\pm} = \left(-\Delta_r + kv \pm \sqrt{(\Delta_r - kv)^2 + 4\Omega_r^2}\right)/2$ and $\lambda_{2\pm} = \left(kv \pm \sqrt{k^2v^2 + 4\Omega_c^2}\right)/2$. First, we consider the simple case of a cold atomic system in which the Doppler effect does not need to be considered. Then we set v = 0. On account of $\Delta_r \gg \Omega_r$, the first two equations in equation (5) could be

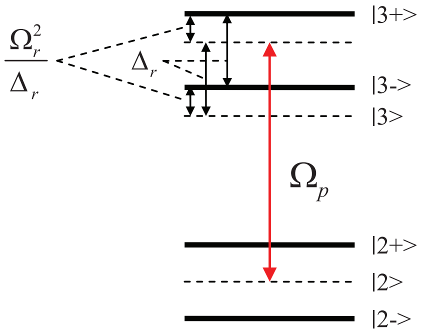

Figure 3. The dressed state diagram for the *N*-type level scheme. The controlled field is chosen to be resonant with the transition from $|4\rangle$ to $|2\rangle$, and the Raman field is much smaller than $\Delta_r(\Omega_r \ll \Delta_r, \Delta_r = 400\Gamma)$.

simplified as

$$\lambda_{3+} \simeq \frac{\Omega_r^2}{\Delta_r}, |3+\rangle \simeq \frac{\Delta_r}{\sqrt{\Delta_r^2+\Omega_r^2}}|1\rangle - \frac{\Omega_r}{\sqrt{\Delta_r^2+\Omega_r^2}}|3\rangle;$$
$$\lambda_{3-} \simeq -\Delta_r + \frac{\Omega_r^2}{\Delta_r}, |3-\rangle \simeq \frac{\Omega_r}{\sqrt{\Delta_r^2+\Omega_r^2}}|1\rangle + \frac{\Delta_r}{\sqrt{\Delta_r^2+\Omega_r^2}}|3\rangle.$$
$$E_{2+} = \hbar\lambda_{2+}, |2+\rangle = \frac{1}{\sqrt{2}}|2\rangle - \frac{1}{\sqrt{2}}|4\rangle; \quad (6)$$
$$E_{2-} = \hbar\lambda_{2-}, |2-\rangle = \frac{1}{\sqrt{2}}|2\rangle + \frac{1}{\sqrt{2}}|4\rangle.$$

We know that the strength of transition from $|3\pm\rangle$ to $|2\pm\rangle$ dominates the gain spectrum profile. Since we only investigate the gain spectrum for the probe in the vicinity of the Raman resonance $\Delta_p = \Delta_r$, contributions to the gain spectrum by the transition from $|3-\rangle$ to $|2\pm\rangle$ could be neglected. The transitions from $|3+\rangle$ to $|2\pm\rangle$ lead to two peaks in the gain spectrum. The location and transition rate of those two peaks in the spectrum can be derived as

$$\Delta_{p+} = \Delta_r + \lambda_{3+} - \lambda_{2+} = \Delta_r + \frac{\Omega_r^2}{\Delta_r} - \Omega_c,$$
$$P_+ = \frac{2\pi}{\hbar^2}|\langle 3+|\vec{d}\cdot\vec{E}_p|2+\rangle|^2 = \frac{4\pi\Omega_r^2}{\Delta_r^2+\Omega_r^2};$$
$$\Delta_{p-} = \Delta_r + \lambda_{3+} - \lambda_{2-} = \Delta_r + \frac{\Omega_r^2}{\Delta_r} + \Omega_c, \quad (7)$$
$$P_- = \frac{2\pi}{\hbar^2}|\langle 3+|\vec{d}\cdot\vec{E}_p|2-\rangle|^2 = \frac{4\pi\Omega_r^2}{\Delta_r^2+\Omega_r^2}.$$

It can be found that $P_+ = P_-$, leading to two equal gain peaks in the spectrum. The energy spacing ΔE between $|2+\rangle$ and $|2-\rangle$ is $2\hbar\Omega_c$. The controlled field acts in two ways. On one hand, it drives the atoms in state $|2\rangle$ to $|4\rangle$ by its pumping effect, and then atoms in $|4\rangle$ return to $|1\rangle$ by spontaneous emission, maintaining all populations in $|1\rangle$. On the other hand, it creates two dressed states for the Raman-resonance excitation process and dominates the width between the two peaks in the gain spectrum. For small Ω_c, because of narrow dressed-level energy spacing it is too close to be separated for the two peaks, the superposition of which results in only one gain peak. So the dispersive curve of the probe field shows the normal positive slope near $\Delta_p = \Delta_r$, leading to a subluminal effect on the probe pulse propagation. As Ω_c increases, the two peaks are away from each other; then, the probe gain spectrum exhibits double peaks located at $\Delta_p = \Delta_{p\pm}$, which correspond to the energy separation of the two dressed states $|2+\rangle$ and $|2-\rangle$ shown in figure 3. The corresponding dispersion becomes anomalous and exhibits a negative slope near $\Delta_p = \Delta_r$, which leads to superluminal light propagation for the probe pulse.

When we consider a hot atomic system, the Doppler effect must be taken into account. The dressed states $|3\pm\rangle$ for atoms moving with v can be obtained by substituting Δ_r by $\Delta_r - kv$ in the first two equations in equation (6) ($\Delta_r - kv \gg \Omega_r$ can be satisfied since the contributions to the gain spectrum by the atoms moving with large v could be neglected according to the Maxwell velocity distribution). Then the location and transition rate of the two peaks in the gain spectrum corresponding to transitions from $|3+\rangle$ to $|2\pm\rangle$ can be derived:

$$\Delta_{p+}(v) = \Delta_r + \frac{\Omega_r^2}{\Delta_r - kv} - \frac{kv + \sqrt{k^2v^2 + 4\Omega_c^2}}{2};$$
$$\Delta_{p-}(v) = \Delta_r + \frac{\Omega_r^2}{\Delta_r - kv} - \frac{kv - \sqrt{k^2v^2 + 4\Omega_c^2}}{2}.$$
$$P_+(v) = N(v)\cdot\frac{2\pi}{\hbar^2}|\langle 3+|\vec{d}\cdot\vec{E}_p|2+\rangle|^2$$
$$\propto \frac{\Omega_r^2\Omega_c^2 e^{-\frac{v^2}{v_p^2}}}{((\Delta_r - kv)^2 + \Omega_r^2)(\lambda_{2+}^2 + \Omega_c^2)};$$
$$P_-(v) = N(v)\cdot\frac{2\pi}{\hbar^2}|\langle 3+|\vec{d}\cdot\vec{E}_p|2-\rangle|^2$$
$$\propto \frac{\Omega_r^2\Omega_c^2 e^{-\frac{v^2}{v_p^2}}}{((\Delta_r - kv)^2 + \Omega_r^2)(\lambda_{2-}^2 + \Omega_c^2)}. \quad (8)$$

For $v < 0$, as $|v|$ increases, $\Delta_{p+}(v)$ will decrease to $\Delta_{p+}(v) = \Delta_r$, and $P_+(v)$ will rise at first to a maximum and then fall eventually. It means that the gain peaks located at $\Delta_p = \Delta_{p+}(v)$ contributed by atoms moving with $-|v|$ will be closer to the Raman resonance point $\Delta_p = \Delta_r$ with a firstly increasing and finally falling intensity as $|v|$ increases. Meanwhile, $\Delta_{p-}(v)$ deviates from the Raman resonance point $\Delta_p = \Delta_r$ and $P_-(v)$ declines rapidly to zero as $|v|$ increases. So peaks located at $\Delta_p = \Delta_{p-}(v)$ have little contribution to the gain spectrum and could be neglected. The sum of peaks for different $-|v|$ results in the left lower peak in the gain spectrum is shown in figure 2(c) by the solid blue line. For $v > 0$, the situation is reversed. In other words, as $|v|$ increases, one of the two gain peaks gets closer to the two-photon Raman point with a firstly rising and eventually falling strength, and the other one deviates from the Raman resonance point with a quickly vanishing strength. Then, summing over peaks contributed by atoms with $v < 0$ results in the left lower peak and summing over peaks contributed by atoms with $v > 0$ leads to the right higher peak in the gain spectrum as shown in figure 2(c) by the solid blue line. Therefore, a narrowed and amplified gain spectrum depicted in figure 2(c) is attained when the Doppler effect is taken into account. And a corresponding steep dispersion slope near $\Delta_p = \Delta_r$ as shown in figure 2(d) is obtained, leading to the enhanced superluminal

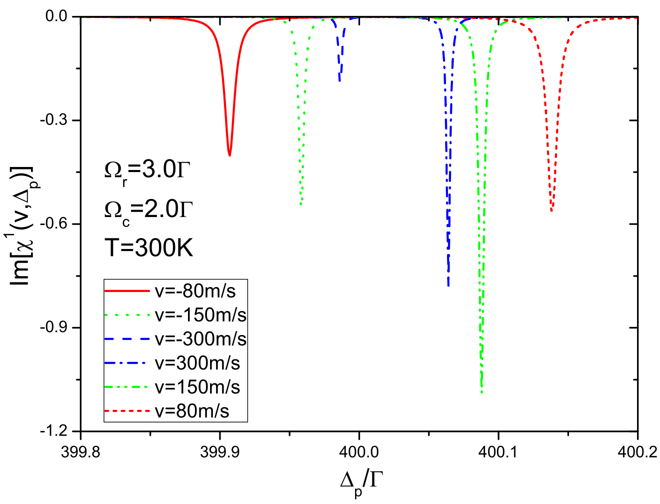

Figure 4. The gain spectrum contributed by atoms with different velocities. Parameters are the same as in figure 2.

or subluminal effect on the probe pulse propagation. For a given $v = |v_0|$, we find $|\Delta_{p+}(-|v_0|) - \Delta_r| < |\Delta_{p-}(|v_0|) - \Delta_r|$, it means that the gain peak located at $\Delta_p = \Delta_{p+}(-|v_0|)$ contributed by atoms moving with $-|v_0|$ will be closer to the Raman resonance point $\Delta_p = \Delta_r$ than that located at $\Delta_p = \Delta_{p-}(|v_0|)$ contributed by atoms at $|v_0|$ as shown in figure 4. Note that the factor $(\Delta_r - kv)^2$ in the denominator of equation (8) at $|v_0|$ is larger than that at $-|v_0|$, which means that the gain peak at $v = |v_0|$ is higher than that at $v = -|v_0|$ as shown in figure 4. Consequently, the integral of v results in two asymmetric peaks in the gain spectrum which has been manifested in figure 2(c).

Next we try to give a quantitative explanation for the results that the linewidth of the gain peak for atoms moving with v is narrowed as $|v|$ increases as shown in figure 4. For $\Delta_r, \Delta_p \gg \Delta_c, \Omega_r, \Omega_c, \gamma_{12}, \gamma_{32}, \gamma_{14}, \gamma_{34}$, equation (3) can be expressed approximately as

$$\rho_{32}^{(1)}(\omega_p) \simeq -\frac{\rho_{13}^{(0)}\Delta_{14}\Omega_r\Omega_p}{\Delta_{32}(\Delta_{12}\Delta_{14} - \Omega_c^2)}. \quad (9)$$

The susceptibility for atoms with v can be calculated for a resonant controlled field:

$$\chi^{(1)}(v,\delta) \simeq -C\frac{\Omega_r^2\,\mathrm{e}^{-\frac{v^2}{v_p^2}}}{(\Delta_r - kv)^2}\frac{\delta - kv - \mathrm{i}\Gamma}{\delta^2 - \delta(kv + \mathrm{i}\Gamma) - \Omega_c^2}, \quad (10)$$

where $C = N_0|\mu_{32}|^2/2\varepsilon_0\hbar\sqrt{\pi}v_p$, and we have set $\delta = \Delta_r - \Delta_p$. Equation (10) can be rewritten as follows:

$$\chi^{(1)}(v,\delta) = -C\frac{\Omega_r^2\,\mathrm{e}^{-\frac{v^2}{v_p^2}}}{2(\Delta_r - kv)^2} \times \left\{\frac{1 + f(v)}{\delta - \frac{kv + \mathrm{i}\Gamma + \sqrt{(kv+\mathrm{i}\Gamma)^2 + 4\Omega_c^2}}{2}} + \frac{1 - f(v)}{\delta - \frac{kv + \mathrm{i}\Gamma - \sqrt{(kv+\mathrm{i}\Gamma)^2 + 4\Omega_c^2}}{2}}\right\}, \quad (11)$$

where

$$f(v) = -\frac{kv + \mathrm{i}\Gamma}{\sqrt{(kv + \mathrm{i}\Gamma)^2 + 4\Omega_c^2}}. \quad (12)$$

When $k^2v^2 + 4\Omega_c^2 \gg \Gamma^2$, equation (11) can be regarded as a sum of two Lorentz-type spectra but with different linewidths:

$$\chi^{(1)}(v,\delta) \simeq -C\frac{\Omega_r^2\,\mathrm{e}^{-\frac{v^2}{v_p^2}}}{2(\Delta_r - kv)^2} \times \left\{\frac{1 + f_1(v)}{\delta - \frac{kv + \sqrt{k^2v^2 + 4\Omega_c^2}}{2} - \frac{\mathrm{i}\Gamma}{2}\left(1 + \frac{kv}{\sqrt{k^2v^2 + 4\Omega_c^2}}\right)} + \frac{1 - f_1(v)}{\delta - \frac{kv - \sqrt{k^2v^2 + 4\Omega_c^2}}{2} - \frac{\mathrm{i}\Gamma}{2}\left(1 - \frac{kv}{\sqrt{k^2v^2 + 4\Omega_c^2}}\right)}\right\}, \quad (13)$$

where

$$f_1(v) = -\frac{kv + \mathrm{i}\Gamma}{\sqrt{k^2v^2 + 4\Omega_c^2}\left(1 + \frac{kv\Gamma}{k^2v^2 + 4\Omega_c^2}\mathrm{i}\right)}. \quad (14)$$

It can be easily found that the first term in equation (13) is related to the transition $|3+\rangle \to |2+\rangle$ for which the linewidth is $\Delta\omega_+ = (1 + kv/\sqrt{k^2v^2 + 4\Omega_c^2})\Gamma$, while the second term refers to the transition $|3+\rangle \to |2-\rangle$ with a linewidth $\Delta\omega_- = (1 - kv/\sqrt{k^2v^2 + 4\Omega_c^2})\Gamma$. For $v = 0$, the two transitions have the same linewidth Γ with same strength as shown in figure 2(c) by the red dashed line for $T = 0$ K. As v increases from 0 to $-\infty$, the linewidth of the peak at the transition $|3+\rangle \to |2+\rangle$ in the gain spectrum will be reduced from Γ to 0. And the linewidth of the other one at the transition $|3+\rangle \to |2-\rangle$ will be broadened from Γ to 2Γ with a rapidly falling strength, it cannot be seen in figure 4 because its strength is too weak. As v increases from 0 to ∞, it results in a reversed version. Then, the sum of all contributions by atoms with different velocities results in the extremely narrowed and amplified gain spectrum as shown in figure 2(c) by the blue solid line, where both the gain peaks are narrowed and amplified. That is to say that the Doppler effect leads to narrowing and amplification to the gain spectrum. The singularity at $\Delta_r - kv = 0$ is avoided since the atoms at $v = \Delta_r/k$ have been pumped away from $|1\rangle$ by the Raman field; then, population for atoms with $v = \Delta_r/k$ vanishes, as has been discussed in [13].

4. Pulse propagation in the hot atomic system

Based on the results obtained above, it is easy to understand that the slow and fast pulse propagation can be enhanced in the four-level *N*-type hot atomic system with ARG. This is quite interesting, since from a traditional point of view, the slow and fast light will always be tampered in the hot atomic system because of the broadened and attenuated spectrum caused by Doppler averaging. We have numerically simulated the probe pulse propagation through a sample with or without the Doppler effect which is shown in figure 5. The Raman and controlled fields are chosen to be continuous waves; only the probe is the pulse-shaped field with a Gaussian profile: $\Omega_p(t) = \Omega_{p0}\,\mathrm{e}^{-(t-t_0)^2/\tau_p^2}$. Obviously the subluminal or superluminal effect with the Doppler effect is much more evident than that without the Doppler effect on the light propagation. Furthermore, there is no absorption and little

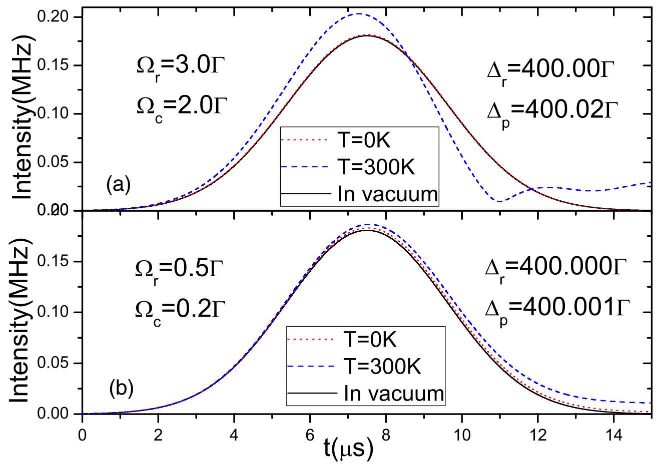

Figure 5. (a) Superluminal light propagation for the probe pulse, the advanced time is $\tau_{ae} \simeq 221.9$ ns and $\tau_a \simeq 11.7$ ns with and without the Doppler effect respectively; (b) subluminal light propagation for the probe pulse, the delayed time is $\tau_{de} \simeq 52.4$ ns and $\tau_d \simeq 17.5$ ns with and without the Doppler effect respectively (dotted lines (red) without the Doppler effect, and dashed lines (blue) with the Doppler effect in both (a) and (b) when the three fields are co-propagating). The parameters are $\gamma_{13} = \gamma_{32} = \Gamma_{32}$, $\gamma_{41} = \gamma_{42} = \Gamma_{42}$, $\gamma_{12} = 0.001\Gamma_{32}$, $\gamma_{34} = \Gamma_{32} + \Gamma_{42}$, $\Delta_c = 0$, $\Gamma_{31} = \Gamma_{32} = \Gamma/2 = \pi \times 5.75$ MHz, $\Gamma_{41} = \Gamma_{42} = \pi \times 6.07$ MHz, $\Omega_{p0} = 0.01\Gamma$, $\tau_p = 3$ μs, $t_0 = 7.5$ μs, $N_0 = 10^{11}$ cm^{-3}, $\lambda_p = 795$ nm, $L = 5$ cm. Others are the same as in figure 2.

pulse distortion. Under the conditions given in figure 5, the group velocity of slow and fast light can be achieved to $v_{gfe} = -c/1330.4$ and $v_{gse} = c/315.4$, compared to $v_{gf} = -c/69.2$ and $v_{gs} = c/106$ without the Doppler effect respectively. It can clearly be seen that the subluminal or superluminal effect on the pulse propagation is enhanced by the Doppler effect in the hot atomic system. It may be useful to optical time delay lines [23, 24], optical storage [25, 26] and so on in the hot atomic system.

5. Conclusions

In summary, we have theoretically demonstrated an *N*-type atomic system with active Raman gain (ARG) in which the spectrum can be changed from one gain peak to double ones by changing the controlled field, and the gain spectrum will be extremely narrowed and amplified when the Doppler effect is considered under selected conditions at room temperature, leading to an enhanced subluminal or superluminal effect on the pulse propagation. It is found that this remarkable result originates from the shift of the dressed levels and the modification to the probability amplitude of transitions between dressed states when the Doppler effect is considered in the hot atomic system. We have also obtained the enhanced subluminal and superluminal pulse propagation in the hot atomic system by numerical simulation, and the probe pulse undergoes no absorption and little pulse distortion.

It should be mentioned that the narrowed and amplified gain spectrum caused by the Doppler effect occurs only when the Raman and probe fields are co-propagating. The two-photon Raman resonance condition will be destroyed if the two lasers are incident in opposite direction, resulting in the normal Doppler-broadening gain spectrum.

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